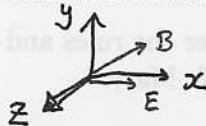


1. Short Answer/Multiple Choice

- a) An electromagnetic wave has, at one location in space and one instant in time, its electric field vector pointing in the $+x$ direction and its magnetic field vector pointing in the $-z$ direction. In what direction is the wave moving?

Answer: $+y$ ($+\hat{j}$)



$$\vec{S} = \vec{E} \times \vec{B} = \hat{i} \times (-\hat{k}) = +\hat{j}$$

- b) What is the maximum efficiency of an engine whose operating temperatures are 530°C and 305°C ? $\hookrightarrow \therefore$ Carnot efficiency

- i) 100%
- ii) 72%
- iii) 42%
- iv) 28%
- iv) 12%

$$E_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{578\text{K}}{803\text{K}} = 0.28$$

- c) The pupil of your eye has a diameter of 3 mm. A car has headlights ($\lambda = 600\text{ nm}$) that are 1.5 m apart. What is the maximum distance away that the car can be at which you can still see the headlights as two distinct objects?

Answer: 6.1 Km

Rayleigh: $\theta_{\text{min}} = 1.22 \frac{\lambda}{D} = 1.22 \left(\frac{600 \times 10^{-9}\text{m}}{3 \times 10^{-3}\text{m}} \right) = 2.44 \times 10^{-4}\text{ rad}$

$$\theta_{\text{min}} = \frac{1.5\text{m}}{L} \quad L = \frac{1.5\text{m}}{2.44 \times 10^{-4}\text{ rad}} = 6148\text{ m}$$

- d) Consider a sample of gas. If work is done by the gas, and its internal energy remains constant, which of the following is true?

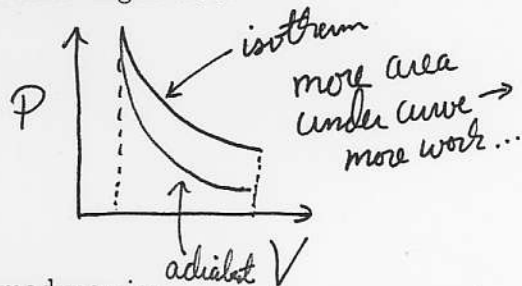
- i) Heat was added to the gas.
- ii) Heat was removed from the gas.
- iii) There was no heat flow.
- iv) The process violates the Second Law of Thermodynamics.

$$Q = \Delta U + W \quad \Delta U = 0 \quad (W \text{ is work done by gas})$$

$$\therefore Q = W \quad \therefore Q = \text{positive}$$

- e) A sample of gas undergoes an expansion from volume V_1 to volume V_2 . Compare the amount of work done by the gas in the case that the expansion was isothermal to the amount done in an adiabatic expansion. Which of the following is true?

- i) The work is larger in the isothermal case.
- ii) The work is larger in the adiabatic case.
- iii) The work is identical in both cases.
- iv) The gas does no work.
- v) Not enough information provided.



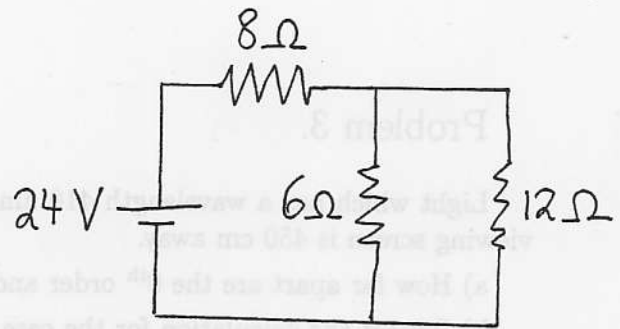
- f) Concisely state one version of the Second Law of Thermodynamics.

The entropy of a closed system can never decrease:

$$\Delta S_{\text{closed}} \geq 0$$

Problem 2.

Consider the circuit shown.



- What is the total equivalent resistance?
- What is the current in the $8\ \Omega$ resistor?
- What is the potential difference across the $12\ \Omega$ resistor?
- If the $8\ \Omega$ resistor were replaced by an $8\ \mu\text{F}$ capacitor, and one waited for a long time, what current would be passing through the $12\ \Omega$ resistor?

a) $6\ \Omega$ & $12\ \Omega$ in parallel $\therefore \frac{1}{R_p} = \frac{1}{6\ \Omega} + \frac{1}{12\ \Omega}$ $R_p = 4\ \Omega$

This combo is in series with $8\ \Omega$ $\therefore R_{\text{Total}} = 8\ \Omega + 4\ \Omega = \boxed{12\ \Omega}$

b) $I_{8\ \Omega} = I_{\text{Battery}}$ (in series)

$\therefore I_{8\ \Omega} = I_B = \frac{24\text{V}}{R_{\text{Total}}}$

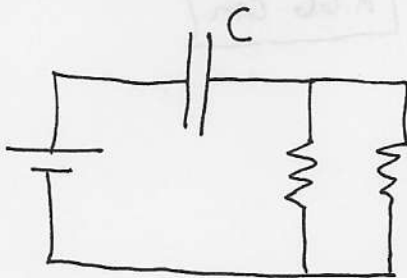
$= \frac{24\text{V}}{12\ \Omega} = \boxed{2.0\text{A}}$

c) $V_{8\ \Omega} + V_{12\ \Omega} = 24\text{V}$ (loop rule)

$V_{8\ \Omega} = (I_{8\ \Omega})(8\ \Omega) = (2.0\text{A})(8\ \Omega) = 16\text{V}$

$\therefore V_{12\ \Omega} = 24\text{V} - 16\text{V} = \boxed{8\text{V}}$

d) Circuit would now be:



Since after a long time the capacitor would be fully charged (to $V=24\text{V}$) there would be no current anywhere in circuit

$\boxed{I_{12\ \Omega} = 0}$

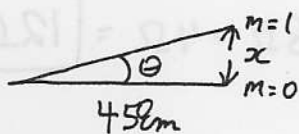
Problem 3.

Light which has a wavelength 410 nm in air falls on two slits 5.20×10^{-2} mm apart. A viewing screen is 450 cm away.

- How far apart are the 0th order and 1st order bright fringes on the screen?
- Repeat the calculation for the case in which the entire system (slits, viewing screen, space inbetween) is immersed in water, which has an index of refraction of 1.33.

a) Two slit interference : $m\lambda = d \sin \theta$

$$m=1: \theta_1 = \sin^{-1}\left(\frac{1 \cdot \lambda}{d}\right) = \sin^{-1}\left(\frac{410 \text{ nm}}{5.20 \times 10^{-5} \text{ m}}\right) = \sin^{-1}\left(7.88 \times 10^{-3}\right) = 0.45^\circ = 7.88 \times 10^{-3} \text{ rad}$$



$$\theta = \frac{x}{450 \text{ cm}}$$

($\sin \theta \approx \tan \theta \approx \theta$)
here

$$x = (450 \text{ cm})(7.88 \times 10^{-3} \text{ rad})$$

$$= \boxed{3.55 \text{ cm}}$$

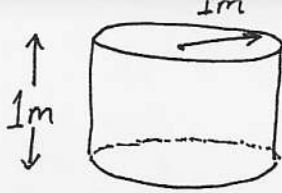
$$b) \lambda' = \frac{\lambda}{n} = \frac{410 \text{ nm}}{1.33} = 308 \text{ nm}$$

repeat a) with λ' : $\theta_1 = \frac{\lambda'}{d} = 5.92 \times 10^{-3} \text{ rad}$

$$x = (450 \text{ cm})(5.92 \times 10^{-3} \text{ rad})$$

$$= \boxed{2.66 \text{ cm}}$$

Problem 4.



Al Gore's hot tub is cylindrical in shape, has a radius of 1 m and a depth of 1 m. It is filled with tap water at 15°C. The tub has a 2 kW heater. Possibly useful properties of water: latent heat of vaporization = 22.6×10^5 J/kg, latent heat of fusion = 3.33×10^5 J/kg, specific heat = 4186 J/kg·°C, density = 1.00 g/cm³.

- How many hours does it take to heat the water to 35°C?
- If the water is plugged into a 110 V outlet, what RMS current passes through it?
- What is the resistance of the heater?
- If electricity costs 8 cents per kW-hr, how much does it cost to heat the tub?
- If the water in the tub starts at 100°C, how long would it take the same heater to boil all the water away?

$$a) V = \pi r^2 d = \pi (1\text{m})^2 (1\text{m}) = 3.14\text{m}^3$$

$$m = \rho V = \left(\frac{1\text{g}}{\text{cm}^3} \times \frac{10^6\text{cm}^3}{\text{m}^3} \times \frac{1\text{Kg}}{10^3\text{g}} \right) (3.14\text{m}^3) = 3140\text{Kg}$$

$$Q = mc\Delta T = (3.14 \times 10^3\text{Kg}) (4186 \frac{\text{J}}{\text{Kg}\cdot^\circ\text{C}}) (35^\circ\text{C} - 15^\circ\text{C}) = 2.63 \times 10^8\text{J}$$

$$P = \frac{\text{energy}}{\text{time}} = \frac{Q}{t} \quad t = \frac{Q}{P} = \frac{2.63 \times 10^8\text{J}}{2 \times 10^3\text{J/s}} = \boxed{1.31 \times 10^5\text{s}} = 36.5\text{ hours}$$

$$b) P = I_{\text{rms}} V_{\text{rms}} \quad I_{\text{rms}} = \frac{P}{V_{\text{rms}}} = \frac{2000\text{W}}{110\text{V}} = \boxed{18.2\text{A}}$$

$$c) P = I_{\text{rms}}^2 R \quad R = \frac{P}{I_{\text{rms}}^2} = \frac{2000\text{W}}{(18.2\text{A})^2} = \boxed{6.05\Omega}$$

$$d) (36.5\text{hrs})(2\text{kW}) = 73\text{ kW}\cdot\text{hrs} \times \frac{\$0.08}{\text{Kw}\cdot\text{hr}} = \boxed{\$5.84}$$

$$e) Q = mL_v = (3140\text{Kg}) (22.6 \times 10^5 \frac{\text{J}}{\text{Kg}}) = 7.10 \times 10^9\text{J}$$

$$t = \frac{Q}{P} = \frac{7.10 \times 10^9\text{J}}{2 \times 10^3\text{W}} = \boxed{3.55 \times 10^6\text{s}} = 986\text{ hrs} = 41\text{ days!}$$

Problem 5.



A single circular loop of wire, of radius 1.0 cm, is located inside a uniform and constant magnetic field. The magnetic field has a magnitude of 3.0×10^{-3} T and is oriented perpendicular to the area of the coil.

- What is the magnitude of the magnetic flux through the coil?
- If the coil is rotated about its diameter at 60 Hz, what is the peak value of the EMF that is generated in the coil?
- Assume that the magnetic field is being generated by a tightly-wound solenoid (i.e. the coil is located inside the solenoid). The solenoid consists of 4000 turns of wire, has a radius of 2.0 cm, and carries a current of 100 mA. How long is the solenoid?

$$\begin{aligned}
 \text{a) } \phi_B &= \int \vec{B} \cdot d\vec{A} = \int B dA \cos 0^\circ = BA = (3 \times 10^{-3} \text{ T}) (\pi (1.0 \text{ cm})^2) \\
 &= 9.42 \times 10^{-7} \text{ T} \cdot \text{m}^2 = \boxed{9.42 \times 10^{-7} \text{ Wb}}
 \end{aligned}$$

$$\text{b) } \mathcal{E}^{\text{max}} = NBA\omega$$

$$N = 1 \text{ turn}$$

$$\omega = 2\pi f = 2\pi (60 \text{ Hz})$$

$$\mathcal{E}^{\text{max}} = (1)(3 \times 10^{-3} \text{ T})(2\pi (60 \text{ Hz}))(\pi (1 \times 10^{-2} \text{ m})^2) = 3.55 \times 10^{-4} \text{ V}$$

$$= \boxed{0.355 \text{ mV}}$$

$$\text{c) Solenoid: } B = \mu_0 n I$$

$$n = \frac{\text{turns}}{\text{length}} = \frac{N}{L}$$

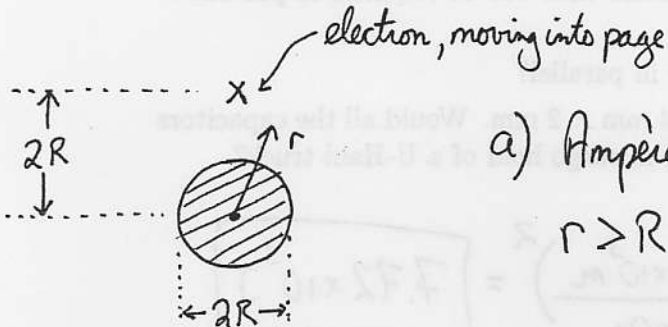
$$\therefore B = \frac{\mu_0 N I}{L} \Rightarrow L = \frac{\mu_0 N I}{B} = \frac{(4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}}) (4000) (10^{-3} \text{ A})}{3 \times 10^{-3} \text{ T}}$$

$$= 0.168 \text{ m} = \boxed{16.8 \text{ cm}}$$

Problem 6.

A long wire of circular cross section and radius R carries a total current I , which is distributed uniformly through the wire. The current flows into the page in the sketch below.

- Find the magnitude of the magnetic field, due to this current, for the regions $0 \leq r \leq R$ and $r > R$.
- Plot $B(r)$ vs. r for $0 \leq r \leq 3R$.
- An electron is moving into the page with speed v , parallel to and directly above the wire, at a distance $r = 2R$. What is the magnitude and direction of the force on the electron?



a) Ampère's law! $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$

$r > R$: consider Amperian loop with radius $r > R$, centered on wire. All of I penetrates area defined by loop: $I_{\text{encl}} = I$. B same everywhere along loop (symmetry)

$$\therefore \int \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad r > R$$

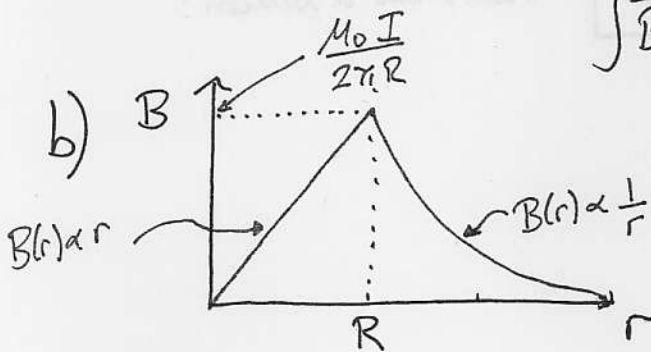
$$0 \leq r \leq R$$



$I_{\text{encl}} = I \left(\frac{\pi r^2}{\pi R^2} \right)$ since is distributed uniform \therefore current proportional to area

$$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B 2\pi r = \mu_0 I \left(\frac{\pi r^2}{\pi R^2} \right)$$

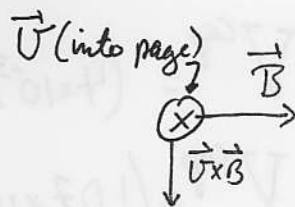
$$B = \frac{\mu_0 I r}{2\pi R^2}$$



c) $\vec{F} = q \vec{v} \times \vec{B}$

$$F = q v B \sin \theta \quad \theta = 90^\circ$$

$$= q v B = q v \frac{\mu_0 I}{2\pi(2R)} = \frac{\mu_0 q v I}{4\pi R} = F$$



$\vec{v} \times \vec{B}$ is down page (towards wire) but $q =$ negative (electron)

$$\vec{F} = \text{up page (away from wire)}$$

Problem 7.

You are working for Toyota, and have been tasked with designing a new electric car. Instead of using a battery to power the car, you decide to use capacitors.

- Assume the car has a total mass of 2000 kg, and that the capacitors need to store sufficient energy to bring the car from rest to a speed of 100 km/hr (about 62 mph). What is the total energy that must be stored in the capacitors?
- You will charge the capacitors using a 12 V power supply. What is the total capacitance that will be required to store this energy?
- What is the minimum number of 10 μF capacitors that will be required to provide this total capacitance?
- Would you connect the capacitors in series or in parallel?
- The typical volume for a capacitor is 4 mm \times 4 mm \times 2 mm. Would all the capacitors from part c) fit into the 1.5 m \times 1.5 m \times 2.0 m cargo hold of a U-Haul truck?

$$a) \quad K = \frac{1}{2} m v_f^2 = \frac{1}{2} (2000 \text{ kg}) \left(\frac{100 \times 10^3 \text{ m}}{3600 \text{ s}} \right)^2 = \boxed{7.72 \times 10^5 \text{ J}}$$

$$b) \quad U = \frac{1}{2} C V^2 = K \quad \therefore C = \frac{2K}{V^2} = \frac{2(7.72 \times 10^5 \text{ J})}{(12 \text{ V})^2} = 10,717 \text{ F} \\ = \boxed{10.7 \text{ KF}} \quad !!$$

$$c) \quad \frac{10.7 \text{ KF}}{10 \mu\text{F}} = \frac{1.07 \times 10^4 \text{ F}}{1.0 \times 10^{-5} \text{ F}} = \boxed{1.07 \times 10^9} \quad \text{more than a billion!}$$

$$d) \quad \boxed{\text{Parallel}} \quad \therefore C_{\text{eq}}^{\text{parallel}} = C_1 + C_2 \dots \\ (\text{Series would reduce capacitance})$$

$$e) \quad V_{\text{cap. Volume}} = V^{\text{cap}} = (4 \times 10^{-3} \text{ m}) \times (4 \times 10^{-3} \text{ m}) \times (2 \times 10^{-3} \text{ m}) = 3.2 \times 10^{-8} \text{ m}^3$$

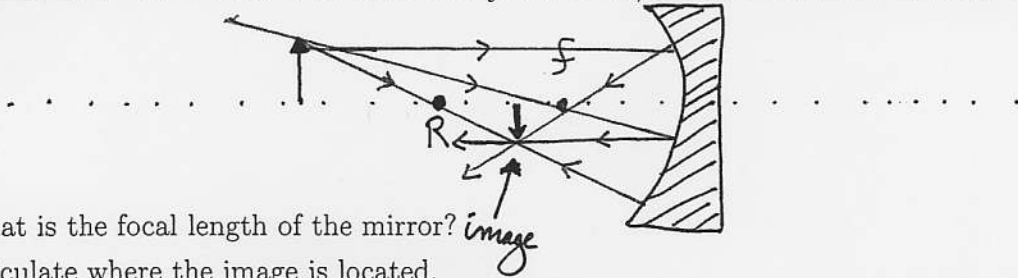
$$\text{all caps: } V^{\text{Total}} = V^{\text{cap}} \times (1.07 \times 10^9) = 34.2 \text{ m}^3$$

$$\text{Trucks: } V^{\text{U-Haul}} = (1.5 \text{ m}) \times (1.5 \text{ m}) \times (2.0 \text{ m}) = 4.5 \text{ m}^3 \ll V^{\text{Total}}$$

No would need \approx 8 trucks ... not a practical electric car ...
Toyota fires you and you are hired by GM ...

Problem 8.

A spherical mirror has a radius of curvature of 80 cm. The center of curvature is indicated by the dot labelled "R". A candle is indicated by the arrow, and is located 130 cm from the mirror.



- What is the focal length of the mirror?
- Calculate where the image is located.
- What is the magnification of the image? Is it real or virtual? Is it upright or inverted?
- On the figure, sketch a couple of rays that could be used to confirm the location of the image.

$$a) \quad f = R/2 = \frac{80\text{cm}}{2} = 40\text{cm}$$

$$b) \quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \therefore \quad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{40\text{cm}} - \frac{1}{130\text{cm}}$$

$$d_i = +57.8\text{cm}$$

positive \therefore on same side of mirror
as object \therefore real image

$$c) \quad m = -\frac{d_i}{d_o} = -\frac{57.8\text{cm}}{130\text{cm}} = \boxed{-0.44}$$

inverted image

d) See sketch...