Problem 1.

Prof. Armstrong sits on a (frictionless) stool which is spinning on its axis with an angular speed of 2.00 revolutions per second. His arms are outstretched and he holds a heavy weight in each hand. The moment of inertia of Armstrong, the weights, and the stool is (in total) 10.0 kg m^2. He pulls the weights closer towards his body, so that the moment of inertia decreases to 6.00 kg m^2.

a) What is the new angular speed of the chair and Professor?

b) By how much has the kinetic energy changed?

c) Explain where the kinetic energy came from (if it increased) or where it went to (if it decreased).

a) no external torques on stool + Prof system $\therefore$ $\vec{L}$ = conserved

$\therefore\ L = I\omega = \text{constant}$ $\quad \omega = 2\pi f \quad f_0 = 2\ \text{rev/s}$

$I_0\omega_0 = I_1\omega_1 \qquad \omega_1 = \frac{I_0}{I_1}\omega_0 = \frac{10\ \text{Kgm}^2}{6\ \text{Kgm}^2}\left(2\pi \cdot 2\ \frac{\text{rev}}{s}\right)$

$= \boxed{21\ \text{rad/s}}$ or $\boxed{3.33\ \text{rev/s}}$

b) $K_0 = \frac{1}{2}I_0\omega_0^2 = \frac{1}{2}\left(10\ \text{Kgm}^2\right)\left(4\pi\ \frac{\text{rad}}{s}\right)^2 = \boxed{790\ \text{J}}$

$K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}\left(6\ \text{Kgm}^2\right)\left(21\ \frac{\text{rad}}{s}\right)^2 = \boxed{1316\ \text{J}}$

$\Delta K = K_1 - K_0 = \boxed{+\ 526\ \text{J}}$ increased

c) Prof. Armstrong does work on the weights to pull them inward; this energy comes from $\boxed{\text{chemical energy}}$ (food energy) in his body.

Problem 2.

A bicycle wheel starts off at rest. It is acted on by a constant torque for a time of 15 seconds. During this time, the wheel makes 80 revolutions.

a) What is the magnitude of the angular acceleration of the wheel?

b) Assume that the torque was caused by a force $\vec{F} = 6\text{N}\ \hat{\mathbf{i}} + 5\text{N}\ \hat{\mathbf{j}}$ which acts at a displacement of $\vec{r} = 2\text{m}\ \hat{\mathbf{i}} - 3\text{m}\ \hat{\mathbf{j}}$ from the axle of the wheel.
Calculate the torque vector.

c) What is the moment of inertia of the bicycle wheel (about its axle)?

Rotational Dynamics

a) Constant torque $\therefore \alpha = \text{constant}$; $\omega_i = 0 \quad \theta_i = 0$

$$\theta_f = (80\,\text{rev.})\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right) = 160\pi\,\text{rad}$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$= \frac{1}{2}\alpha t^2 \longrightarrow \alpha = \frac{2\theta_f}{t^2} = \frac{2(160\pi\,\text{rad})}{(15\text{s})^2} = \boxed{4.47\,\frac{\text{rad}}{\text{s}^2}}$$

b) $$\vec{\tau} = \vec{r} \times \vec{F} = (2\text{m}\,\hat{i} - 3\text{m}\,\hat{j}) \times (6\text{N}\,\hat{i} + 5\text{N}\,\hat{j}) = 10\,\text{Nm}\,\hat{k} + (-18\,\text{Nm})(-\hat{k})$$

$$= \boxed{28\,\text{Nm}\,\hat{k}}$$

c) $$\sum \vec{\tau} = I\vec{\alpha} \quad \therefore \quad \tau = I\alpha$$

$$I = \frac{\tau}{\alpha} = \frac{28\,\text{Nm}}{4.47\,\text{rad/s}^2} = \boxed{6.26\,\text{Kgm}^2}$$

Problem 3.

In 1971, one of the Apollo 14 astronauts hit a golf ball while he was standing on the surface of the moon. It is said to have "gone for miles and miles".

a) What would the initial speed of the ball have to have been in order to have allowed it to escape completely from the moon? (It did not).

b) Assume the golf ball started off with less than the escape speed, but instead reached a maximum height of 200 km above the moon's surface. What would the acceleration due to the moon's gravity be at this height?

c) In part b), what was the work done on the ball by gravity?

Possibly useful data: $G = 6.67 \times 10^{-11}$ Nm^2/kg^2, the moon's radius is 1740 km, its mass is 7.35×10^{22} kg, the radius of the Moon's orbit around the Earth is 3.84×10^5 km, and the mass of a typical golf ball is 75 gm.

a) Conservation of Energy. To just escape entirely, we want $v = 0$ @ $r = \infty$

$$K_i + U_i = K_f + U_f$$

$$U(r) = -\frac{GMm}{r} \quad \left[\text{choose } U_0 \text{ such that } U(\infty) = 0\right]$$

$$\therefore \frac{1}{2} m v_E^2 - \frac{GMm}{R_m} \geq 0 + 0$$

M = mass of moon
R_m = radius of moon
m = mass of golf ball

$$v_E = \left(\frac{2GM}{R_m}\right)^{1/2} = \left(\frac{2\left(6.67\times10^{-11}\frac{Nm^2}{Kg^2}\right)\left(7.35\times10^{22}Kg\right)}{1.74\times10^{6}m}\right)^{1/2} = \boxed{2374 \text{ m/s}}$$

No golfer could hit it that fast!

b) $$\text{Weight} = \cancel{m}g = \frac{GM\cancel{m}}{r^2}$$

$r = R_m + h$ where $h = 2\times10^5 m$ $(= 200 \text{ Km})$

$$\therefore \text{"}g\text{"} = \frac{GM}{(R_m+h)^2} = \frac{\left(6.67\times10^{-11}\frac{Nm^2}{Kg^2}\right)\left(7.35\times10^{22}Kg\right)}{\left(1.74\times10^6 m + 2\times10^5 m\right)^2} = \boxed{1.30 \text{ m/s}^2}$$

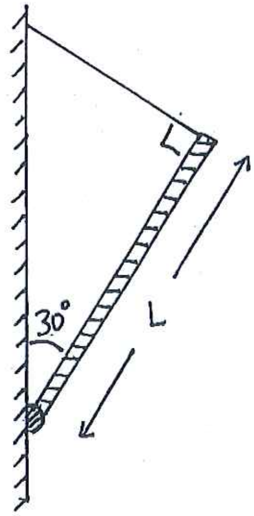
c) $$\underset{\text{(work)}}{W} = -\Delta U = -(U_f - U_i) = U_i - U_f = -\frac{GMm}{R_m} - \left(-\frac{GMm}{R_m}\right) = GMm\left(\frac{1}{R_m+h} - \frac{1}{R_m}\right)$$

$$= \left(6.67\times10^{-11}\frac{Nm^2}{Kg^2}\right)\left(7.35\times10^{22}Kg\right)\left(0.075Kg\right)\left(\frac{1}{1.94\times10^6 m} - \frac{1}{1.74\times10^6 m}\right)$$

$$= \boxed{-2.18\times10^4 \text{ J}}$$

gravity does negative work (reduces Kinetic energy)

Problem 4.



A uniform beam of mass $M = 5$ kg and length $L = 2$ m is attached at one end to a wall via a hinge. The other end of the beam is supported by a rope. The beam makes an angle of 30° to the vertical (see diagram). The angle between the rope and the beam is 90°. The system is at rest.

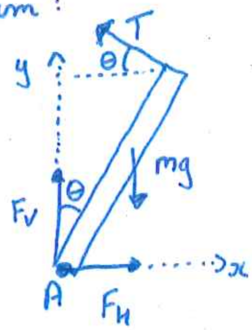
a) What is the tension in the cable?

b) What is the force that the hinge exerts on the beam?

c) If the cable were cut, and the beam were to start to fall, what would be the direction of the resulting angular momentum vector?

a) Static Equilibrium

FBD on beam:



Consider torques about "A" (the hinge)

ccw = positive

$$\sum \tau_A = 0 = -mg\sin\theta \frac{L}{2} + TL = 0$$

$$\therefore T = \frac{mg\sin\theta}{2} = \frac{(5\,\text{Kg})(9.81\,\frac{m}{s^2})\sin 30^\circ}{2}$$

$$= \boxed{12.3\text{ N}}$$

b) $\sum \vec{F} = 0$ $\begin{cases} x: & F_H - T\cos\theta = 0 \\ y: & F_V + T\sin\theta - mg = 0 \end{cases}$

$$\therefore F_H = T\cos\theta = (12.3\text{N})\cos 30^\circ = \boxed{10.6\text{ N}}$$

$$F_V = mg - T\sin\theta = (5\text{Kg})(9.81\,\frac{m}{s^2}) - (12.3\text{N})\sin 30^\circ = \boxed{42.9\text{N}}$$

$$\therefore \text{Hinge Force} = \boxed{10.6\text{N}\,\hat{i} + 42.9\text{N}\,\hat{j}}$$

c) rotation, due to torque caused by gravity would be clockwise (looking @ the page) $\therefore$ direction of $\vec{\omega}$ and thus of $\vec{L}$ vector would be **into the page**, using the right-hand rule.