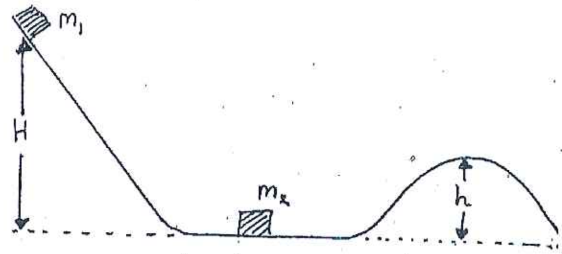
Problem 1.



A roller coaster car of mass $m_1 = 200$ kg starts rolling from rest at the top of a track at a height $H = 50$ m. At the bottom of the hill, it collides with another car of mass $m_2 = 100$ kg, which is initially at rest. The two cars stick together and head for a second hill of height $h = 20$ m. Ignore friction and air resistance.

a) What is the speed of the first car just before the collision?

b) What is the speed of the two combined cars just after the collision?

c) *Show* whether or not they will make it to the top of the second hill.

d) If the collision occurs in a time interval of $t = 0.1$ s, what is the average force exerted on the first car during that collision?

a) Conservation of Mechanical Energy : $\Delta K + \Delta U = 0$

choose $U=0$ @ bottom of hill $\therefore \left(\frac{1}{2}m_1 v_1^2 - 0\right) + \left(0 - m_1 g H\right) = 0$

$\therefore \frac{1}{2}\cancel{m_1} v_1^2 = \cancel{m_1} g H \quad \therefore v_1 = \sqrt{2gH} = \sqrt{2(9.8\frac{m}{s^2})(50m)}$

$= \boxed{31.3 \text{ m/s}}$

b) Inelastic (perfectly) collision $\Rightarrow$ conservation of momentum

$\vec{p}_i = \vec{p}_f$ 1D $\therefore m_1 v_1 = (m_1 + m_2)V \therefore V = \frac{m_1 v_1}{(m_1+m_2)} = \frac{(200\text{Kg})(31.3 \text{ m/s})}{(200\text{ Kg} + 100\text{ Kg})} = \boxed{20.9 \text{ m/s}}$

positive x is to the right

c) Conservation of Energy (again)

$E_i = \frac{1}{2}(m_1+m_2)V^2 \quad E_f = (m_1+m_2)gh + K_f$ "just makes it" if $K_f = 0 \therefore E_f^{min} = (m_1+m_2)gh$

$= \frac{1}{2}(300\text{Kg})(20.9\text{ m/s})^2 = 6.55\times10^4 \text{ J} \quad E_f^{min} = (m_1+m_2)gh = (300\text{Kg})(9.8\frac{m}{s^2})(20m) = 5.88\times10^4 \text{ J}$

$\therefore \boxed{E_i > E_f^{min}}$ $\therefore$ will make it over second hill, with energy to spare

d) impulse $\vec{I} = \Delta\vec{p} = \vec{F}_{av}\Delta t$

1D collision; positive to the right

$F_{av} = \frac{\Delta p}{\Delta t} = \frac{(m_1 V - m_1 v_1)}{\Delta t} = \frac{m_1}{\Delta t}(V - v_1) = \frac{200\text{ Kg}}{0.1s}\left(20.9\frac{m}{s} - 31.3\text{ m/s}\right) = -20.8\times10^3 \text{ N}$

$= \boxed{-20.8 \text{ KN}}$

force is to the $\boxed{\text{left}}$

Problem 2.

Find the location of the center of mass of a system of three masses, $m_1 = 2.0$ kg, $m_2 = 3.0$ kg and $m_3 = 5.0$ kg which are located at

$\vec{r_1} = (-2.0\hat{\imath} + 3.0\hat{\jmath})$ m,

$\vec{r_2} = (2.0\hat{\imath} + 3.0\hat{\jmath})$ m,

and

$\vec{r_3} = (2.0\hat{\imath} - 3.0\hat{\jmath})$ m,

respectively.

$$\vec{r}_{cm} = \frac{1}{M}\sum_i \vec{r}_i m_i \qquad M = \sum m_i = (2+3+5)\,Kg = 10\,Kg$$

$$= \frac{1}{(10\,Kg)}\left\{(-2\hat{i})(2Kg) + (3\hat{j})(2Kg) + (2\hat{i})(3Kg) + (3\hat{j})(3Kg) + (2\hat{i})(5Kg) + (-3\hat{j})(5Kg)\right\}$$

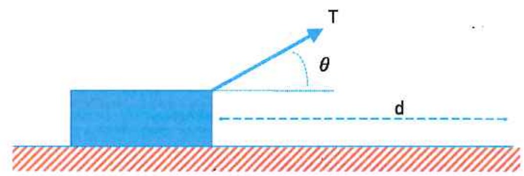
$$= \frac{1}{(10\,Kg)}\left\{(-4+6+10)\,Kg\,m\,\hat{i} + (6+9-15)\,Kg\,m\,\hat{j}\right\}$$

$$= \frac{1}{(10\,Kg)}\left\{(12\hat{i} + 0\hat{j})\,Kg\,m\right\} = \boxed{1.2\hat{i}\,m + 0\hat{j}}$$

See, for example Problem 9-45 (third problem on Homework 8)

Problem 3.

You are dragging a box across the floor by pulling on it with a rope attached to the box. The box slides a distance of $d = 5$ m along the floor, and the tension in the rope is $T = 40$ N. The box moves with constant speed (there is friction between the box and the floor).



a) If you have done a total of 100 J of work on the box as it slides that distance, what is the angle θ of the rope (measured from the horizontal)?

b) If the box requires 5 seconds to travel that distance, what is the minimum power you must be expending to move the box in this way?

a) $$W = \int \vec{F} \cdot d\vec{r} = \int \vec{T} \cdot d\vec{r} = \int_0^d T\cos\theta \, dx = T\cos\theta \int_0^d dx = Td\cos\theta$$

(T, $\cos\theta$ are constant)

$$\therefore \quad \cos\theta = \frac{W}{Td} = \frac{100\,J}{(40N)(5m)} = 0.5$$

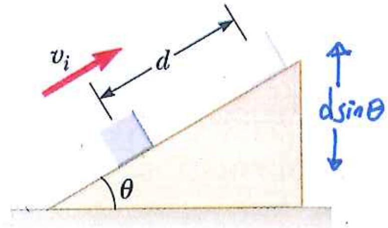
$$\therefore \quad \theta = \cos^{-1}(0.5) \qquad \boxed{\theta = 60^\circ}$$

b) $$P = \frac{dW}{dt} = \frac{\Delta W}{\Delta t} = \frac{100\,J}{5\,s} = \boxed{20\,W}$$

($\dot{W}$ = constant)

Or $$P = \vec{F} \cdot \vec{v} = Fv\cos\theta = Tv\cos\theta = (40N)\left(\frac{5m}{5s}\right)\cos 60^\circ = 20W$$

Problem 4.



A 5.00 kg mass is set into motion up an inclined plane with an initial speed of 8.0 m/s. The block comes to rest after it has slid a distance of $d = 3.0$ m up the plane. The angle of the incline is $\theta = 30°$, as shown.

a) What is the change in kinetic energy of the block?

b) What is the change in potential energy of the block?

c) What is the magnitude of the frictional force exerted by the incline on the block?

d) What is the coefficient of kinetic friction between the block and the incline?

a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}mv_i^2 = -\frac{1}{2}(5\,\text{kg})\left(8\frac{\text{m}}{\text{s}}\right)^2 = \boxed{-160\,\text{J}}$

b) $\Delta U = U_f - U_i = mgh - 0 = mgh = (5\,\text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(3\,\text{m})\sin 30° = \boxed{73.5\,\text{J}}$

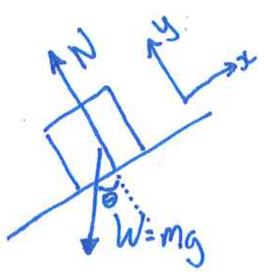
$h = d\sin\theta$

c) $\Delta K + \Delta U + \Delta E^{int} = 0$ $\quad \Delta E^{int} = f_K d$

$\therefore \Delta K + \Delta U + f_K d = 0$

$f_K = -\dfrac{(\Delta K + \Delta U)}{d} = -\dfrac{(-160 + 73.5)\,\text{J}}{3\,\text{m}} = \boxed{28.8\,\text{N}}$

d)



$\sum \vec{F} = m\vec{a}$

consider y-direction : $a_y = 0$

$\therefore \sum F_y = 0$

$\therefore N - mg\cos\theta = 0 \quad \therefore N = mg\cos\theta$

Note: this is Problem 8-23 from the text which was done in Problem Session the week of Oct. 3

$f_K = \mu_K N$

$\mu_K = \dfrac{f_K}{N} = \dfrac{f_K}{mg\cos\theta} = \dfrac{28.8\,\text{N}}{(5\,\text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\cos 30°}$

$= \boxed{0.679}$