Physics 101 Test 1 Sept 22 2014

Name: ____________________

This test is administered under the rules and regulations of the Honor Code of William & Mary.

Signature: ____________________

Problem Session Day/time (circle one): Wed 9:00 am

Wed 3:00 pm

Thurs 8:30 am

Thurs 1:00 pm

Thurs 3:30 pm

1. ________ (25 points)

2. ________ (25 points)

3. ________ (25 points)

4. ________ (25 points)

Total ________

Problem 1.

The position of an object of mass 0.2 kg varies in time according $\vec{r} = 10.0t\hat{i} - 7.5t^2\hat{j}$ where $\vec{r}$ is in meters and t is in seconds.

a) Find an expression for the velocity of the object as a function of time.
b) Determine the acceleration of the object as a function of time.
c) Calculate the object's position at $t = 2.0$ s.
d) Calculate the magnitude and direction of the object's velocity at $t = 2.0$ s. Make sure you clearly specify what angle you are using to define the direction.
e) What is the net force on the object at time $t = 2.0$ s?

a) $\vec{r} = 10t\hat{i} - 7.5t^2\hat{j}$ $\quad \vec{v}(t) = \dfrac{d\vec{r}}{dt} = \boxed{10\hat{i} - 15t\hat{j}}$ (in m/s)

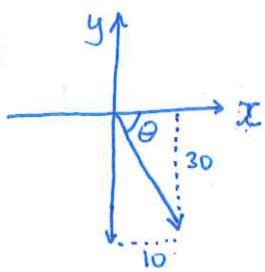
b) $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{r}}{dt^2} = 0\hat{i} - 15\hat{j} = \boxed{-15\hat{j}}$ (in m/s²)

c) $\vec{r}(t=2.0s) = 10(2.0)\hat{i} - 7.5(2.0)^2\hat{j} = \boxed{20\hat{i} - 30\hat{j}}$ (in m)

d) $\vec{v}(t=2.0s) = 10\hat{i} - 15(2.0)\hat{j} = 10\hat{i} - 30\hat{j}$

magnitude : $|\vec{v}| = (v_x^2 + v_y^2)^{1/2} = (10^2 + (-30)^2)^{1/2} = \boxed{31.6 \text{ m/s}}$

direction :



$\tan\theta = \dfrac{|v_y|}{|v_x|}$ $\quad \theta = \tan^{-1}\left(\dfrac{30}{10}\right) = \boxed{71.6^\circ}$

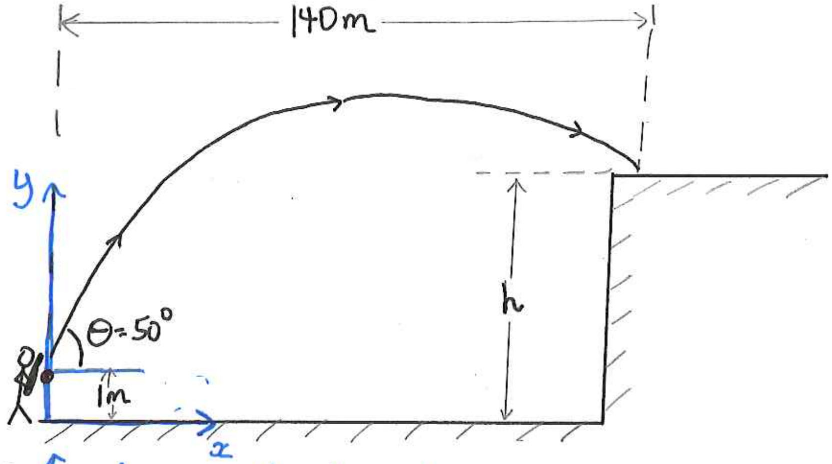
$\boxed{\text{clockwise below } +x \text{ axis}}$

e) $\sum\vec{F} = m\vec{a}$

$= (0.2\,\text{Kg})(-15\hat{j}\ \frac{\text{m}}{\text{s}^2})$

$= \boxed{-3\text{ N}\hat{j}}$

Problem 2.



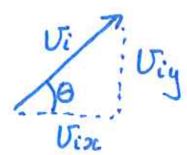
Your favorite Tribe baseball player has hit a home run in which the baseball ended up in the stands. The horizontal distance traveled by the ball is measured to be 140 m (460 ft). It took 5 seconds for the ball to reach the stands. The ball's initial velocity was at 50° above horizontal.

a) What was the initial speed of the ball? ↳ choose coordinate system

b) What is the height h of the stands above the ground? (assume the ball started at 1.0 meter above ground level)

c) What was the speed of the ball at the instant before it hit the stands?

Projectile Motion



a) $x_f = x_i + v_{ix} t = 0 + v_i \cos\theta \, t$

$$\therefore v_i = \frac{x_f}{\cos\theta \, t} = \frac{140\,m}{(\cos 50°)(5\,s)} = \boxed{43.6\,m/s}$$

b) $y_f = y_i + v_{iy} t - \frac{1}{2} g t^2$

$$= 1m + v_i \sin\theta \, t - \tfrac{1}{2} g t^2$$

$$= 1m + \left(43.6\frac{m}{s}\right)(\sin 50°)(5\,s) - \tfrac{1}{2}\left(9.81\,m/s^2\right)(5\,s)^2 = \boxed{45.4\,m}$$

c) $v_{fx} = v_{ix} = v_i \cos\theta = \left(43.6\frac{m}{s}\right)\cos 50° = 28.0\,m/s$

$$v_{fy} = v_{iy} - gt = v_i \sin\theta - gt = \left(43.6\frac{m}{s}\right)\sin 50° - \left(9.81\frac{m}{s^2}\right)(5\,s) = -15.6\,m/s$$

$$v_f = \left(v_{fx}^2 + v_{fy}^2\right)^{1/2} = \left(\left(28.0\frac{m}{s}\right)^2 + \left(-15.6\frac{m}{s}\right)^2\right)^{1/2} = \boxed{32.1\,m/s}$$

Problem 3.

A student swims across the James River. She wishes to arrive at a location due West of her starting point. The river has a current which runs from North to South at this location, and has a speed of 0.2 m/s. The swimmer can swim at 0.4 m/s relative to still water.

a) In which direction should she swim?

b) If the river is 3/4 of a mile across, how long will it take her to reach the other side? One mile = 1.609 km.

Relative Velocity

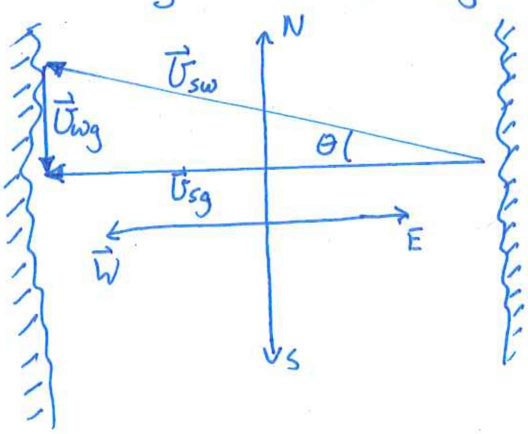
$\vec{v}_{sw}$ = velocity of swimmer with respect to water

$\vec{v}_{wg}$ = velocity of water with respect to ground ($\equiv$ current)

$\vec{v}_{sg}$ = velocity of swimmer with respect to ground

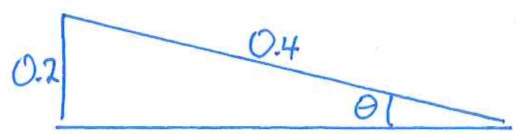
$$\therefore \quad \vec{v}_{sg} = \vec{v}_{sw} + \vec{v}_{wg} \qquad (\vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB})$$

a) want $\vec{v}_{sg}$ due West ; $\vec{v}_{wg}$ is North to South



Given : $|\vec{v}_{wg}| = 0.2\,m/s$

$|\vec{v}_{sw}| = 0.4\,m/s$



$\therefore \theta = \tan^{-1}\left(\frac{0.2}{0.4}\right)$ $\boxed{\theta = 30^\circ \text{ North of West}}$

b) $v_x = \frac{\Delta x}{\Delta t} \quad \therefore \quad \Delta t = \frac{\Delta x}{v_x} \qquad v_x = v_{sg} = (0.4)\cos 30^\circ = 0.346\,m/s$

$$= \frac{\left[(3/4\ \text{mile})\left(\frac{1.609\ \text{Km}}{\text{mile}}\right)\left(\frac{10^3\,m}{\text{Km}}\right)\right]}{(0.346\ m/s)} = \boxed{3484\,s}$$

$= 58$ min

Problem 4.

While traveling at 90 km/hr, a train engineer suddenly notices a car stopped on the tracks at a level crossing, 700 m ahead of the train. He immediately applies the breaks, slowing the train down with a constant deceleration of $a = -0.5\text{m/s}^2$.

a) How far apart will the train and the car be when the train comes to a complete stop?

b) Assume, instead, that the train was initially traveling faster than 90 km/hr, but that the deceleration was still $a = -0.5\text{m/s}^2$. What is the maximum initial speed of the train such that it does not hit the car?

1D Motion with constant acceleration

a) $v_f = 0$ $\quad v_f^2 = v_i^2 + 2a\Delta x$

$0 = \quad$ " $\quad$ "

$$\therefore \Delta x = -\frac{v_i^2}{2a}$$

$$= -\frac{\left(90\,\frac{\text{Km}}{\text{hr}} \times \frac{10^3\text{m}}{\text{Km}} \times \frac{1\text{hr}}{3600\text{s}}\right)^2}{2(-0.5\,\text{m/s}^2)}$$

$$= 625\text{m}$$

$\therefore$ train will be $700\text{m} - 625\text{m} = \boxed{75\text{m}}$ from car

b) Now we want $v_f = 0$ @ $\Delta x = 700\text{m}$
(train just reaches car)

$v_f^2 = v_i^2 + 2a\Delta x$

$0 = \quad$ " $\quad$ "

$$v_i^2 = 0 - 2a\Delta x$$

$$= -2(-0.5\text{m/s}^2)(700\text{m})$$

$$= 700\text{m}^2/\text{s}^2$$

$$\therefore v_i = \left(700\,\frac{\text{m}^2}{\text{s}^2}\right)^{1/2} = \boxed{26.5\text{ m/s}}$$

(or 95.2 Km/hr)