

Problem 1.

Your clothes washer is rotating at a rate of 800 revolutions per minute during the spin cycle. The power is switched off, and it then smoothly slows down to come to a complete stop; this takes 10 seconds.

- What is the magnitude of its angular acceleration?
- How many rotations did it complete after the power was switched off?

Rotational Kinematics

$$a) \quad \omega_i = 800 \frac{\text{rev.}}{\text{min.}} \times \frac{2\pi \text{ rad}}{\text{rev.}} \times \frac{1 \text{ min}}{60 \text{ s}} = 83.8 \text{ rad/s} \quad ; \quad \omega_f = 0$$

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - 83.8 \text{ rad/s}}{10 \text{ s}} = -8.38 \text{ rad/s}^2 \quad \boxed{|\alpha| = 8.38 \text{ rad/s}^2}$$

$$b) \quad 0 = \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \quad \therefore \Delta\theta = \frac{\omega_i^2}{-2\alpha} = \frac{(83.8 \text{ rad/s})^2}{2(8.38 \text{ rad/s}^2)} = 419 \text{ rad}$$

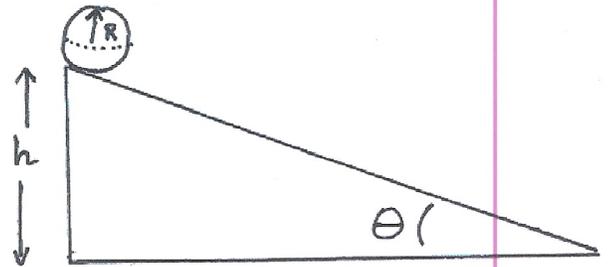
$$\# \text{ rotations} = 419 \text{ rad} \times \frac{1 \text{ rotation}}{2\pi \text{ rad}} = \boxed{66.7}$$

$$\underline{\underline{\theta}} \quad \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$= 0 + (83.8 \frac{\text{rad}}{\text{s}})(10 \text{ s}) + \frac{1}{2}(-8.38 \frac{\text{rad}}{\text{s}^2})(10 \text{ s})^2 = 419 \text{ rad} = 66.7 \text{ rotations} \\ \text{(as above)}$$

Problem 2.

A solid bowling ball of mass M and radius R rolls (without slipping, and starting from rest) down an incline with an angle θ from an initial height of h . The moment of inertia of a solid sphere about an axis through its center is $I = \frac{2}{5}MR^2$.



a) What is the magnitude of the angular velocity of the sphere when it reaches the bottom of the incline? Give your answer in terms of any of these: M , R , θ , h , g .

b) If, instead, a basketball of the same radius R was rolled down the incline (without slipping), would it reach the bottom of the incline faster, slower, or in the same time as the bowling ball? Briefly explain why. The moment of inertia of a hollow sphere about an axis through its center is $I = \frac{2}{3}MR^2$.

Conservation of Energy

a) $\Delta K + \Delta U = 0$ $U_i = Mgh$ $U_f = 0$ (choosing $U=0$ @ bottom of incline)

$K_i = 0$ $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

rolling $\therefore v = R\omega \longrightarrow$ $= \frac{1}{2}Mv^2 + \frac{1}{2} \frac{2}{5}MR^2 \left(\frac{v}{R}\right)^2$

$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$

$\therefore \Delta K + \Delta U = 0$

$\frac{7}{10}Mv^2 - Mgh = 0$

$v = \sqrt{\frac{10}{7}gh}$

$\omega = \frac{v}{R} = \sqrt{\frac{10gh}{7R^2}}$

b) The hollow sphere (basketball) has a larger I than the solid sphere (bowling ball) \therefore at a given U more of the energy is rotational K than is translational K $\therefore v$ will be slower

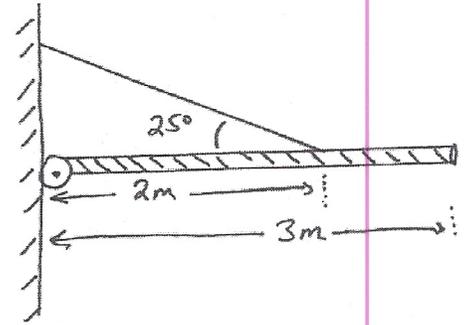
Explicitly, following same calculation as above but with $I = \frac{2}{3}MR^2$

we get: $v = \sqrt{\frac{6}{5}gh} < \sqrt{\frac{10}{7}gh}$

$= 1.095\sqrt{gh} < 1.195\sqrt{gh}$ (about 10% slower)

Problem 3.

A uniform thin rod of length 3 m and mass 15 kg is attached to a wall by a hinge and is held horizontal and stationary by a rope attached to the rod a distance of 2 m from the wall, at an angle of 25° .



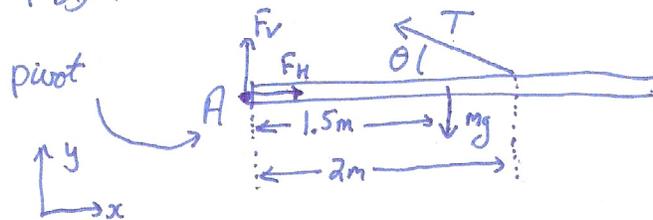
a) Find the magnitude of the tension in the rope.

b) Find the force (horizontal and vertical components) exerted on the rod by the hinge.

c) If the rope is suddenly cut, find the initial angular acceleration (magnitude *and* direction) of the rod as it begins to rotate about the hinge. Note: the moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$.

a) Static Equilibrium $\therefore \sum \vec{F} = 0 \quad \sum \vec{\tau} = 0$

FBD :



$$L = 3m \quad l = 2m$$

$$\sum \tau_A = -mg \frac{L}{2} + T \sin(180^\circ - \theta) l = 0$$

$$T = \frac{mgL}{2l \sin \theta} = \frac{(15 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})(3 \text{ m})}{2(2 \text{ m}) \sin 25^\circ}$$

$$T = 261 \text{ N}$$

$$b) \quad \sum \vec{F} = 0 \quad \begin{cases} x: F_H - T \cos \theta = 0 \quad \therefore F_H = T \cos \theta \\ y: F_V + T \sin \theta - mg = 0 \end{cases} = \boxed{236 \text{ N}} \text{ upwards}$$

$$\therefore F_V = mg - T \sin \theta = \boxed{36.8 \text{ N}} \text{ to the right}$$

$$c) \quad \tau = I \alpha$$

Only mg provides torque about pivot after rope is cut.

$$\tau = mg \frac{L}{2}$$

$$\therefore \alpha = \frac{\tau}{I} = \frac{mg \frac{L}{2}}{\frac{1}{3} ML^2} = \frac{3g}{2L} = \frac{3(9.8 \text{ m/s}^2)}{2(3 \text{ m})} = \boxed{4.9 \text{ rad/s}^2}$$

Problem 4.

One day, humanity may colonize Mars.

a) Assume you wish to place a communications satellite in orbit around Mars. What must the radius of the satellite's orbit be such that it could stay fixed in the Martian sky (above a particular location on the surface of Mars) as Mars rotates? In other words, so that it orbits Mars once per Martian day.

b) What is the value of the gravitational acceleration " g_{mars} " (due to Mars' gravity) that this satellite would experience while in orbit?

Possibly useful values: Radius of Mars = 3400 km; mass of Mars = 6.4×10^{23} kg; length of the Martian day = 24.6 hours; length of the Martian year = 687 days; mass of the satellite = 42 kg; revenue of the film "The Martian" in its first weekend of release = \$55 million.

$$\begin{aligned} \text{a) Kepler's 3rd Law} &: T^2 = \left(\frac{4\pi^2}{GM}\right)R^3 \\ \therefore R^3 &= \frac{GMT^2}{4\pi^2} & T &= 24.6 \text{ hr} \times \frac{3600 \text{ s}}{\text{hr}} = 8.86 \times 10^4 \text{ s} \\ R &= \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} = \left[\frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(6.4 \times 10^{23} \text{ kg})(8.86 \times 10^4 \text{ s})^2}{4\pi^2}\right]^{1/3} \\ &= \boxed{2.04 \times 10^7 \text{ m}} = 20,400 \text{ km} \end{aligned}$$

$$\begin{aligned} \text{b) } F = m g_{\text{mars}} &= \frac{GMm}{R^2} & \therefore g_{\text{mars}} &= \frac{GM}{R^2} \\ & & &= \frac{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(6.4 \times 10^{23} \text{ kg})}{(2.04 \times 10^7 \text{ m})^2} \\ & & &= \boxed{0.102 \text{ m/s}^2} \end{aligned}$$