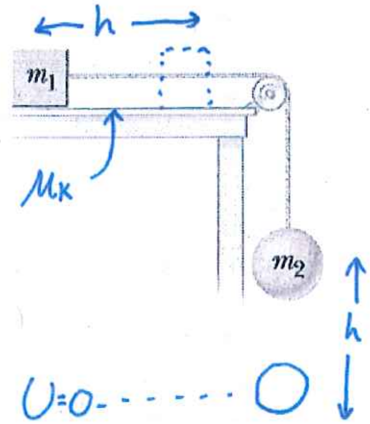
Problem 1.

Two masses, $m_1 = 3.0$ kg and $m_2 = 5.0$ kg are connected via a light rope, as shown in the figure. The system starts at rest. The coefficient of kinetic friction between m_1 and the surface is $\mu_k = 0.400$. Use energy considerations to determine the speed of mass m_2 when it has fallen a distance of $h = 1.50$ m.



Energy Conservation

$$\Delta K + \Delta U + \Delta E_{int} = 0$$

- U of m_1 does not change ; $v_1 = v_2$ if rope doesn't stretch

$$\Delta E_{int} = f_K h = \mu_K m_1 g h \qquad \therefore v_1 = v_2 = v$$

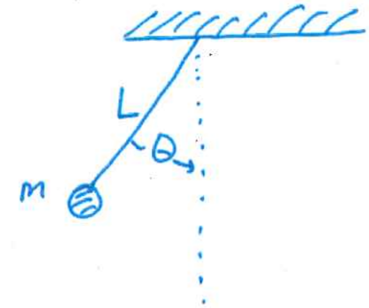
$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_1 v_2^2 + (0 - m_2 g h) + f_K h = 0$$

$$\frac{1}{2}(m_1 + m_2) v^2 = m_2 g h - \mu_K m_1 g h = gh(m_2 - \mu_K m_1)$$

$$v = \sqrt{\frac{2gh(m_2 - \mu_K m_1)}{(m_1 + m_2)}}$$

$$= \sqrt{\frac{2(9.8\,m/s^2)(1.50m)(5.0\,Kg - 0.4(3.0\,Kg))}{(3.0\,Kg + 5.0\,Kg)}}$$

$$= \boxed{3.74\ m/s}$$



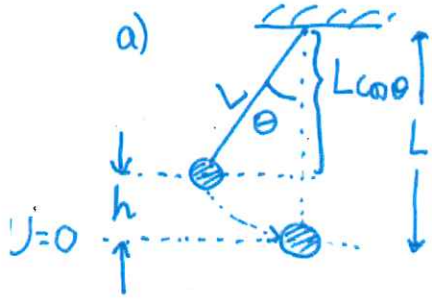
Problem 2.

A pendulum consists of a bob of mass m attached to a massless string of length L. The bob is pulled aside so that the string makes an angle of θ with the vertical, as shown, and is released from rest. Find:

a) The speed v of the bob when it is at the lowest point on its swing.

b) The tension T in the string at that moment.

a)



$$h = L - L\cos\theta = L(1-\cos\theta)$$

Conservation of Energy

$U_i = mgh = mgL(1-\cos\theta)$ $\qquad K_i = 0$

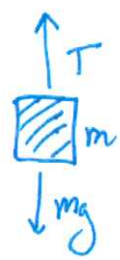
$U_f = 0$ $\qquad K_f = \frac{1}{2}mv^2$

$$\Delta U + \Delta K = 0 \Rightarrow 0 - mgL(1-\cos\theta) + \frac{1}{2}mv^2 - 0 = 0$$

$$\therefore \frac{1}{2}mv^2 = mgL(1-\cos\theta)$$

$$\boxed{v = \sqrt{2gL(1-\cos\theta)}}$$

b) Free Body Diagram :



Centripetal Acceleration

$$a_c = \frac{v^2}{r} = \frac{v^2}{L}$$

$$\Sigma \vec{F} = m\vec{a}$$

$$\therefore T - mg = \frac{mv^2}{L}$$

$$T = mg + \frac{mv^2}{L}$$

$$= m\left(g + \frac{v^2}{L}\right) = m\left(g + \frac{2gL(1-\cos\theta)}{L}\right)$$

$$\boxed{T = mg(3 - 2\cos\theta)}$$

see, for example,
Example 6.6 from the Text

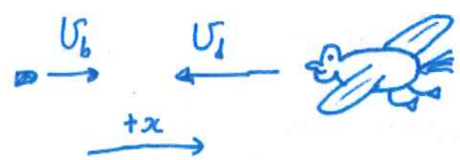
Problem 3.

An innocent duck is flying horizontally to the left at 3.0 m/s. Unfortunately for him, he is struck by a hunter's bullet, traveling at 100 m/s horizontally to the right. The bullet sticks inside the duck. The mass of the duck is 1.5 kg, and that of the bullet is 50 grams. Neglect any effects due to gravity or air resistance[1].

a) What is the velocity (magnitude *and* direction) of the duck after the collision?

b) How much mechanical energy was converted into other forms (heat, sound, chemical...) in the collision?

c) Which is larger: the force the bullet exerted on the duck, or the force the duck exerted on the bullet?



a) Conservation of momentum

$\vec{p}_i = \vec{p}_f$ 1D problem

$$m_b v_b + m_d v_d = (m_b + m_d) v \quad \therefore \quad v = \frac{m_b v_b + m_d v_d}{(m_b + m_d)}$$

$$= \frac{(0.05\,\text{kg})(100\,\tfrac{\text{m}}{\text{s}}) + (1.5\,\text{kg})(-3\,\text{m/s})}{(1.5 + 0.05)\,\text{kg}}$$

$$= \boxed{+0.322\ \text{m/s} \quad (\text{to the right})}$$

b) $$\Delta KE = \frac{1}{2}(m_d + m_b)v^2 - \left(\frac{1}{2}m_b v_b^2 + \frac{1}{2}m_d v_d^2\right)$$

$$= \frac{1}{2}(1.55\,\text{kg})(0.322\,\tfrac{\text{m}}{\text{s}})^2 - \left(\frac{1}{2}(1.5\,\text{kg})(-3\,\tfrac{\text{m}}{\text{s}})^2 + \frac{1}{2}(0.05\,\text{kg})(100\,\tfrac{\text{m}}{\text{s}})^2\right)$$

$$= 0.08\,\text{J} - (6.8\,\text{J} + 250\,\text{J}) = \boxed{-257\,\text{J}}$$

$\therefore \simeq$ all of mech. energy lost to other forms.

c) Newton's 3rd Law: the forces are $\boxed{\text{identical}}$ in magnitude (opposite in direction)

[1] No actual ducks were harmed in the crafting of this question.

Problem 4.

A constant force $\vec{\mathbf{F}} = (3.0\text{N})\hat{\mathbf{i}} - (2.0\text{N})\hat{\mathbf{j}} + (4.0\text{N})\hat{\mathbf{k}}$ is exerted on an object which moves from $r_1 = (1.0\text{m})\hat{\mathbf{i}} - (3.0\text{m})\hat{\mathbf{j}}$ to $r_2 = (4.0\text{m})\hat{\mathbf{i}} - (5.0\text{m})\hat{\mathbf{j}}$.

a) How much work is done by this force on the object?

b) What is the angle between the force $\vec{\mathbf{F}}$ and the displacement vector?

a) $\vec{F}$ = constant $\therefore W = \vec{F} \cdot \Delta\vec{r}$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (4m\hat{i} - 5m\hat{j}) - (1m\hat{i} - 3m\hat{j}) = 3m\hat{i} - 2m\hat{j}$$

$$W = (3N\hat{i} - 2N\hat{j} + 4N\hat{k}) \cdot (3m\hat{i} - 2m\hat{j})$$
$$= 9Nm + 4Nm$$
$$= 13\ Nm = \boxed{13\ J}$$

See Example 7.3 from the text

b) $W = \vec{F} \cdot \Delta\vec{r} = F \Delta r \cos\theta$

$$F = |\vec{F}| = \left((3N)^2 + (-2N)^2 + (4N)^2\right)^{1/2} = \sqrt{29}N = 5.38\ N$$

$$\Delta r = |\Delta\vec{r}| = \left((3m)^2 + (-2m)^2\right)^{1/2} = \sqrt{13}m = 3.60m$$

$$\cos\theta = \frac{W}{F\Delta r} = \frac{13\ J}{(\sqrt{29}\ N)(\sqrt{13}\ m)} = 0.670$$

$$\theta = \cos^{-1}(0.670)$$
$$\boxed{\theta = 47.9^\circ}$$