Problem 1.

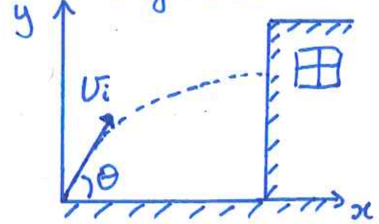
A William & Mary student throws a raw egg at President Reveley's house[1]. The egg's initial speed is 20 m/s. The horizontal distance from the student to the house is 12 m. The egg hits the house 1 second after it is thrown. Assume the egg was released from the student's hand 2 m above the ground.

a) At what initial angle θ (measured from the horizontal) was the egg thrown?

b) At what height is the egg when it hits the house?

c) How fast and in what direction was the egg travelling when it hit the house?

Projectile Motion...



a) $x = x_i + v_{ix}t \qquad x_i = 0$

$= v_{ix}t$

$= v_i \cos\theta\, t \qquad \therefore \cos\theta = \dfrac{x}{v_i t} = \dfrac{12\,\text{m}}{(20\,\frac{\text{m}}{\text{s}})(1\,\text{s})} = 0.6$

$\theta = \cos^{-1}(0.6) = \boxed{53.1^\circ}$

b) $y = y_i + v_{iy}t - \frac{1}{2}gt^2$

$y_i = 2\,\text{m}\,; \quad v_{iy} = v_i \sin\theta \qquad \therefore y = y_i + v_i \sin\theta\, t - \frac{1}{2}gt^2$

$= 2\,\text{m} + (20\,\frac{\text{m}}{\text{s}})(\sin 53.1^\circ)(1\,\text{s}) - \frac{1}{2}(9.8\,\frac{\text{m}}{\text{s}^2})(1\,\text{s})^2 = \boxed{13.1\,\text{m}}$

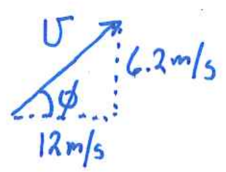
c) $v_x = v_{ix}$ (no x-acceleration)

$v_y = v_{iy} - gt$

$v_x = v_i \cos\theta = (20\,\frac{\text{m}}{\text{s}})(0.6) = 12\,\text{m/s}$

$v_y = v_i \sin\theta - gt = (20\,\frac{\text{m}}{\text{s}})(0.8) - (9.8\,\frac{\text{m}}{\text{s}})(1\,\text{s}) = 6.2\,\text{m/s}$

$v = \sqrt{v_x^2 + v_y^2} = \left((12\,\text{m/s})^2 + (6.2\,\frac{\text{m}}{\text{s}})^2\right)^{1/2} = \boxed{13.5\,\text{m/s}}$



$\phi = \tan^{-1}\left(\dfrac{6.2\,\text{m/s}}{12\,\text{m/s}}\right) = \boxed{27.3^\circ \text{ above horizontal}}$

[1] I don't recommend trying this.

Problem 2.

A car starts from rest and travels in a straight line for 12 seconds with a steady acceleration of 1.5 m/s^2. The driver then applies the brakes, causing her car to slow with a deceleration of 2.0 m/s^2, until it comes to a full stop.

a) What was maximum speed the car reached?

b) How far is the car away from its initial location when it stops?

c) What is the total elapsed time?

1D motion, for a = constant

a) $v_i = 0 \qquad v = v_i + at = at = \left(1.5\frac{m}{s^2}\right)(12s) = \boxed{18\,m/s}$

b) while accelerating : $x_1 = x_i + v_i t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \quad \begin{pmatrix} x_i = 0 \\ v_i = 0 \end{pmatrix}$

$$= \tfrac{1}{2}\left(1.5\frac{m}{s^2}\right)(12s)^2 = 108m$$

while decelerating : $\left.\begin{array}{l} v_i = 18\,m/s \\ v = 0 \text{ for full stop} \\ a = -2.0\,m/s^2 \end{array}\right\} \therefore \; v^2 = v_i^2 + 2a\Delta x$

$$\Delta x = \frac{v^2 - v_i^2}{2a} = \frac{0^2 - (18\,m/s)^2}{2(-2\,m/s^2)} = 81m = x_2$$

Total distance $= x_1 + x_2$

$= 108m + 81m$

$= \boxed{189m}$

c)

while accelerating : $t_1 = 12s$ (given)

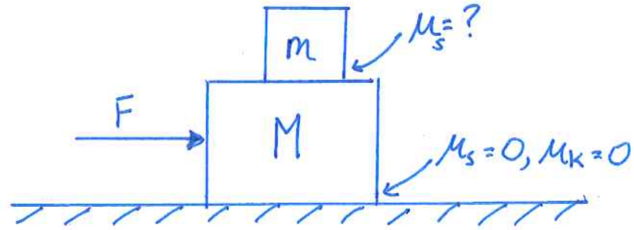
while decelerating : $v = v_i + at_2 \qquad \begin{array}{l} v_i = 18\,m/s \\ v = 0 \end{array}$

$$\therefore t_2 = \frac{v - v_i}{a} = \frac{0 - 18\,m/s}{-2\,m/s^2} = 9s$$

Total time $= t_1 + t_2$

$= 12s + 9s = \boxed{21s}$

Problem 3.



A box of mass m is sitting on top of another box of mass M, which sits on a (frictionless) layer of ice. There is friction between the two boxes. A horizontal force of magnitude F is applied to the lower box (see diagram).

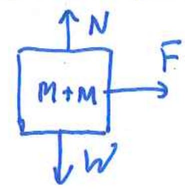
a) Assume that the static friction is such that the two boxes will move together. What is the acceleration of the system?

b) What is the minimum coefficient of static friction μ_s between the two boxes such that they move together?

c) Now, assume that something is exerting an additional vertical (downwards) force F_2 on the upper box. What would the minimum μ_s now be such that the two boxes still move together?

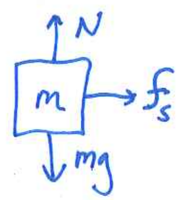
a) Consider the two boxes as one system

FBD : y, x



$\sum \vec{F} = (m+M)\vec{a}$ $\quad x:$ $\boxed{a = \frac{F}{(m+M)}}$

b) Consider mass m as the system ; FBD :



y, x

$f_s \leq \mu_s N$

Minimum $\mu_s \rightarrow f_s = \mu_s N$

$$\sum \vec{F} = m\vec{a} \quad \begin{cases} x: & f_s = ma \\ y: & N - mg = 0 \end{cases}$$

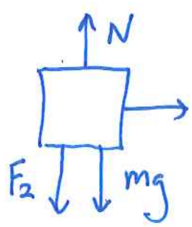
$\therefore N = mg$

$\therefore \quad f_s = \mu_s N = \mu_s \cancel{m} g = \cancel{m} a$

$\therefore \mu_s = a/g = \boxed{\frac{F}{g(m+M)}}$

since "a" is found in part a)

c) FBD is now :



$$\sum \vec{F} = m\vec{a} \quad \begin{cases} x: & f_s = ma \\ y: & N - mg - F_2 = 0 \end{cases}$$

$\therefore N = F_2 + mg$

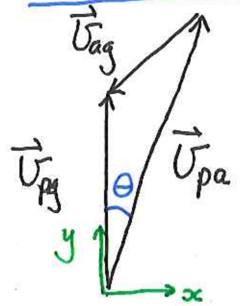
$\therefore f_s = \mu_s N = \mu_s(F_2 + mg) = ma$

$\boxed{\therefore \mu_s = \frac{mF}{(m+M)(mg+F_2)}}$

Problem 4.

An airplane pilot wishes to travel directly North from her starting location, and to maintain a speed of 200 km/hr with respect to the ground. However, there is a strong wind blowing from the Northeast to the Southwest; the wind speed is 30 m/s. In what direction must she head, and with what airspeed (i.e. speed of the airplane relative to the air) in order to do this?

Relative Velocity



$$\vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}$$

A = plane = p
B = ground = g
C = air = a

$$\vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$$

v_{pg} = plane with respect to ground → must be due North

v_{ag} = air with respect to ground = wind

v_{pa} = plane with respect to air (what question asks for)

$$\therefore \vec{v}_{pa} = \vec{v}_{pg} - \vec{v}_{ag}$$

convert 30m/s to Kph:

$$v_{ag} = \left(30\,\frac{m}{s}\right) \times \left(\frac{3600\,s}{hr}\right) \times \left(\frac{1\,km}{1000\,m}\right) = 108\ \frac{km}{hr}$$

Consider components:
(units of km/hr)

$$\begin{array}{lccc}
 & & x & y \\
 & v_{pg} & 0 & 200 \\
- & [\,v_{ag} & -108\sin 45^\circ & -108\cos 45^\circ\,] \\
\hline
= & v_{pa} & v_{pa}\sin\theta & v_{pa}\cos\theta
\end{array}$$

2 Eqns, 2 unknowns (v_{pa}, θ)

$$x: \quad v_{pa}\sin\theta = \frac{108}{\sqrt{2}}\ \frac{km}{hr} = 76.4\ km/hr$$

$$y: \quad v_{pa}\cos\theta = \left(200 + \frac{108}{\sqrt{2}}\right)\frac{km}{hr} = 276.4\ km/hr$$

$$\tan\theta = \frac{76.4}{276.4} \quad \therefore \boxed{\theta = 15.4^\circ}$$

$$v_{pa} = \left(\left(76.4\ \frac{km}{hr}\right)^2 + \left(276.4\ \frac{km}{hr}\right)^2\right)^{1/2}$$

$$= \boxed{287\ km/hr}$$

She must fly 15.4° E of N @ 287 Kph

[See example done in class, Sept 9]