Physics 101 Final Exam Dec 7 2015

Name: SOLUTIONS

This test is administered under the rules and regulations of the Honor Code of William & Mary.

Signature: ______________________

Problem Session (circle one):

Wed. 9:00 am (Prof. Cooke)
Wed. 3:00 pm (Prof. Aubin)
Thurs. 1:00 pm (Prof. Sher)
Thurs. 3:30 pm (Prof Krakauer)
Tues. 1:00 pm (Prof. Mikhailov)
Thurs. 2:00 pm (Prof. Vahala)

1. ________ (25 points)
2. ________ (25 points)
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7. ________ (25 points)
8. ________ (25 points)

Total ________

Problem 1.

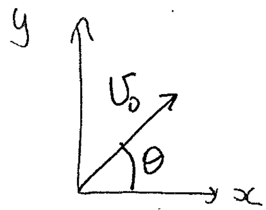
A projectile is launched with initial velocity = 30 m/s at an angle of 70° from the horizontal. Ignore air resistance.

a) How long will it take to reach its maximum height above the ground?

b) How far has it travelled horizontally when it reaches its maximum vertical height?

c) When it reaches its maximum height, it undergoes a chemical explosion, and separates into two equal-mass fragments. Just after the explosion, one of the fragments is observed to be moving at 25 m/s directly downward. What is the velocity vector of the other fragment?

a)



$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

max height $\rightarrow v_y = 0$

$\therefore v_y = v_{0y} - gt$

$0 = v_0 \sin\theta - gt$

$t = \dfrac{v_0 \sin\theta}{g} = \boxed{2.88\,s}$

b)

$x = \cancel{x_0}^{\,0} + v_{0x} t + 0$

$x = v_0 \cos\theta\, t = \boxed{29.5\,m}$

c) Cons. of momentum

$\vec{v}_2 = v_{2x}\hat{i} + v_{2y}\hat{j}$

$\vec{p}_i = m v_x \hat{i} = m v_{0x}\hat{i} = \vec{p}_f = -\frac{m}{2}\left(25\frac{m}{s}\right)\hat{j} + \frac{m}{2}\vec{v}_2$

$\begin{cases} x: \; m v_{0x} = \frac{m}{2} v_{2x} & \therefore v_{2x} = 2 v_0 \cos\theta \\ y: \; 0 = -\frac{m}{2}(25\,m/s) + \frac{m}{2} v_{2y} & \therefore v_{2y} = 25\,m/s \end{cases}$

$\therefore \boxed{\vec{v}_2 = 20.5\frac{m}{s}\hat{i} + 25\frac{m}{s}\hat{j}}$

Problem 2.

A 5 kg object is released, from rest, at a height of $h = 2R_m$ above the surface of the Moon, where $R_m = 1740$ km is the radius of the Moon. The mass of the Moon is 7.3×10^{22} kg.

a) What is its speed when it hits the surface of the Moon? Ignore the gravitational effects of the Earth, the Sun, and the other planets.

b) How much work did the Moon do on the object as it fell?

a) Conservation of Energy : $K_i + U_i = K_f + U_f$

$K_i = 0$ $\quad m = 5\,kg$

$K_f = \frac{1}{2} m v^2$ $\quad M = 7.3\times10^{22}\,kg$

$U(r) = -\frac{GMm}{r} + U_0$

choose: $U(r = R_m) = 0$

$\therefore U_0 = +\frac{GMm}{R_m}$

initial $r = R_m + h$
$= R_m + 2R_m$
$= 3R_m$

$\therefore U_i = -\frac{GMm}{3R_m} + \frac{GMm}{R_m} = \frac{2}{3}\frac{GMm}{R_m}$

$K_i + U_i = K_f + U_f$

$0 + \frac{2}{3}\frac{GM\cancel{m}}{R_m} = \frac{1}{2}\cancel{m}v^2 + 0$ $\quad \therefore v^2 = \frac{4}{3}\frac{GM}{R_m}$

$v = \left(\frac{4}{3}\frac{GM}{R_m}\right)^{1/2}$

$= \left[\frac{4}{3}\frac{(6.67\times10^{-11}\,\frac{Nm^2}{kg^2})(7.3\times10^{22}\,kg)}{1.740\times10^6\,m}\right]^{1/2}$

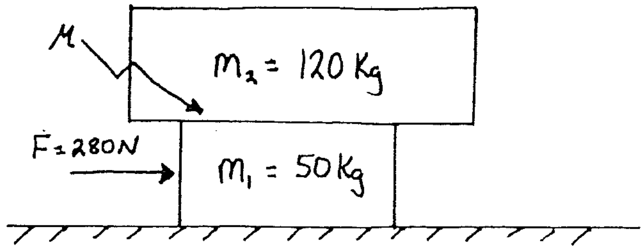
$\boxed{v = 1932\,m/s}$ $= 1.93\,km/s$

b) $W = \Delta K$

(only force on object is moon's gravity)

$W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2 = \frac{1}{2}(5\,kg)(1932\,m/s)^2 = \boxed{9.33\times10^6\,J}$

Problem 3.

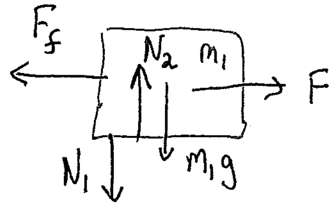


A 50 kg block slides underneath a 120 kg block with an acceleration $a = 2\ \text{m/s}^2$ when an external horizontal force of 280 N is applied. The 50 kg block sits on a horizontal frictionless surface, but there is friction between the two blocks.

a) Find the coefficient of kinetic friction between the blocks.

b) Find the acceleration (magnitude and direction) of the 120 kg block (when it is still in contact with the other block).

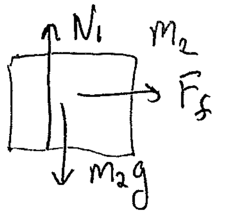
a)



$$\sum \vec{F_1} = m_1 \vec{a_1} \quad \begin{cases} x: F - F_f = m_1 a_1 \\ \therefore F_f = F - m_1 a_1 \end{cases}$$

$$= 280N - (50Kg)(2\tfrac{m}{s^2})$$

$$= 180\ N$$



$$\sum \vec{F_2} = m_2 \vec{a_2} \quad \begin{cases} x: F_f = m_2 a_2 \\ y: N_1 - m_2 g = 0 \quad \therefore N_1 = m_2 g \end{cases}$$

$$\therefore F_f = 180N = \mu_K N_1 = \mu_K m_2 g$$

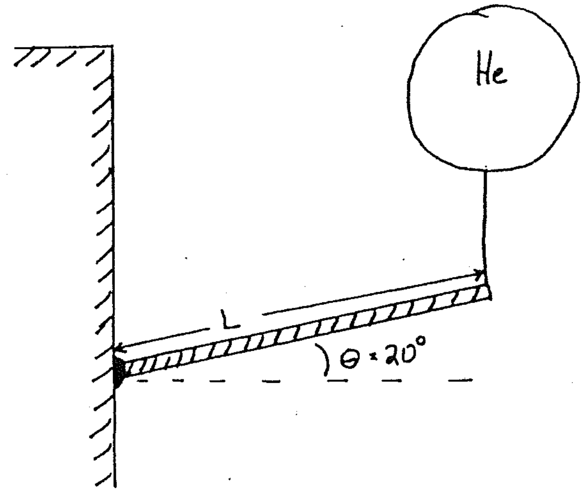
$$\mu_K = \frac{180N}{m_2 g} = \boxed{0.153}$$

b) $F_f = m_2 a_2$

$$\therefore a_2 = \frac{F_f}{m_2} = \frac{180N}{120N} = \boxed{1.50\ m/s^2}\ \boxed{\text{to the right}}$$

Problem 4.

A pole of mass M = 30 kg and length L = 2 m is attached to a vertical wall by a hinge at one end, and is attached via a massless string to a helium-filled balloon at the other end. The pole is at an angle of 20° to the horizontal.



a) What is the tension in the string?

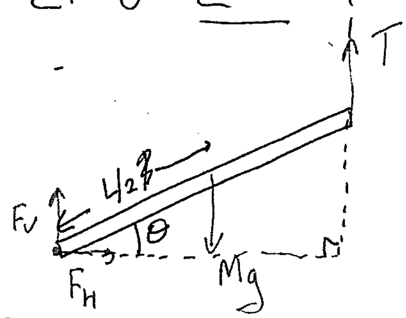
b) What are the horizontal and vertical components of the force exerted by the hinge?

c) Assume that the material of the balloon is massless. The density of helium is $0.18\ \mathrm{kg/m^3}$, and the density of air is $1.3\ \mathrm{kg/m^3}$. What is the radius of the balloon?

a) Static Equil. $\sum \vec{F} = 0$ $\underline{\sum \vec{\tau} = 0}$

Take torques about hinge

FBD:



$$\sum \vec{\tau} = 0 = -Mg\frac{L}{2}\cancel{\cos\theta} + TL\cancel{\cos\theta}$$

$$T = \frac{Mg}{2} = \frac{(30\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{2}$$

$$\boxed{T = 147\ N}$$

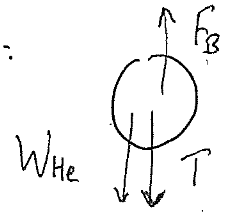
b) $\sum \vec{F} = 0$ $\begin{cases} x: F_H = 0 \\ y: F_V + T - Mg = 0 \end{cases}$ $\therefore$ $F_V = Mg - T = Mg - \frac{Mg}{2} = \frac{Mg}{2}$

$$= 147N$$

$$\vec{F}_{HINGE} = 147N\,\hat{j}$$

$$\boxed{F_V = 147\hat{j} \quad F_H = 0\hat{i}}$$

c) FBD on balloon:



$$F_B = W_{disp} = \rho_{air} V g$$

$$\sum F: F_B - W_{He} - T = 0$$

$$\rho_{air} V g - T - \rho_{He} V g = 0$$

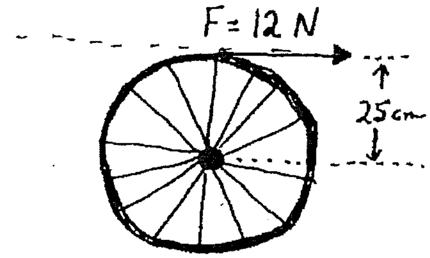
$$V = \frac{4}{3}\pi r^3$$

$$Vg(\rho_{air} - \rho_{He}) = T$$

$$\frac{4}{3}\pi r^3 = \frac{T}{g(\rho_{air} - \rho_{He})} \qquad r = \left[\frac{3T}{4\pi g(\rho_{air} - \rho_{He})}\right]^{1/3} = \boxed{1.48m}$$

Problem 5.

A bicycle wheel with a moment of inertia of 30 kg·m^2 rotates with an initial angular speed of 5.0 rad/s. A tangential force of 12.0 N is applied at a distance of 25 cm from the center of the wheel, in such a way that the rotation rate decreases.



a) How long will it take the wheel to stop?

b) How many revolutions will the wheel have made by the time it stops (from the instant the tangential force is first applied)?

c) What is the direction of the torque caused by the tangential force? Pick one of the following: i) to the left, ii) to the right, iii) out of the page, iv) into the page, v) none of the these.

a) $\tau = I\alpha = rF$ $\qquad r = 0.25m \quad F = 12N \quad I = 30Kg \cdot m^2$

$$\alpha = \frac{rF}{I} = \frac{(0.25m)(12N)}{30Kg\,m^2} = 0.1\,rad/s^2 \qquad \alpha = -0.1\,rad/s^2$$

with respect to ω_i

$$\omega_f = \omega_i + \alpha t \quad \therefore \quad t = \frac{0 - \omega_i}{\alpha} = \frac{-5\,rad/s}{-0.1\,rad/s^2} = \boxed{50s}$$

b) $\omega_f = 0$; $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

$$\Delta\theta = -\frac{\omega_i^2}{2\alpha} = -\frac{(5\,rad/s)^2}{2(-0.1\,rad/s^2)} = 125\,rad$$

$$\Delta\theta = 125\,rad \times \frac{1\,rev}{2\pi\,rad} = \boxed{19.9 \text{ revolutions}}$$

c) Right Hand Rule : $\vec{\tau}$ is $\boxed{\text{into the page}}$

Problem 6.

Earth's polar ice caps contain 2×10^{19} kg of ice. Since this ice is located at the poles, its contribution to the Earth's moment of inertia is negligible. As a weird practical joke, the Klingon empire arranges to have all of this ice moved to a single pile, located at the equator. The mass of the Earth is 6.0×10^{24} kg, its radius is 6370 km, and the moment of inertia of a sphere about an axis through its center is $\frac{2}{5}MR^2$; you can treat the Earth as a perfect sphere.

a) Would the length of the day become shorter or longer? Why?

b) By how much?

$M = 6\times10^{24}$ Kg (Earth)
$m = 2\times10^{19}$ Kg (ice)

a) Conservation of Angular Momentum (no external torques on Earth)

$\vec{L}_{total} = \text{constant}$

$L_i = L_f$

$I\omega = I_f\omega_f$

$I = \frac{2}{5}MR^2$

$I_f = \frac{2}{5}MR^2 + mR^2$

$$\omega_f = \frac{I\omega}{I_f} = \frac{2/5 M\cancel{R^2}}{\left(\frac{2}{5}M+m\right)\cancel{R^2}}\omega = \frac{2/5\, M}{\left(\frac{2}{5}M+m\right)}\omega$$

$= 0.9999917\,\omega$

$\boxed{\omega_f \text{ gets smaller}}$

$\therefore$ $\boxed{\text{day is longer}}$

b) $$\frac{\Delta\omega}{\omega} = \frac{\omega_f-\omega}{\omega} = -8.3\times10^{-6}$$

$\omega = \frac{2\pi}{T}$ $\quad\therefore\ \Delta T = +8.3\times10^{-6}$

$$= (8.3\times10^{-6})\left(3600\,\frac{s}{hr}\times 24\,\frac{hr}{day}\right) = \boxed{0.72\,s}$$

Day increases by $\approx$ 3/4 of a second

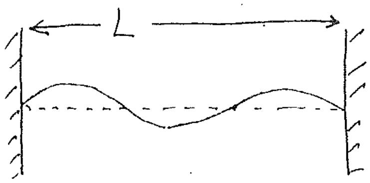
Problem 7.

A string of mass $m = 2.0 \times 10^{-3}$ kg, and length $L = 3$ m oscillates in a standing wave, at a frequency of 60 Hz. There is one and a half wavelengths between the two ends of the string.

a) What is the tension in the string?

b) If one end of the string is plucked, how long would it take until the other end of the string reacts?

a)



$$L = \frac{3\lambda}{2} \quad \therefore \lambda = \frac{2L}{3} = \frac{2(3m)}{3} = 2m$$

$$f = 60\,Hz$$

$$\therefore v = \lambda f = (2m)(60Hz) = 120\,m/s$$

$$\mu = \frac{M}{L} = \frac{2\times10^{-3}\,Kg}{3m} = 6.67\times10^{-4}\,Kg/m$$

$$v = \sqrt{T/\mu} \quad \therefore T = \mu v^2 = \left(6.67\times10^{-4}\,\frac{Kg}{m}\right)\left(120\,m/s\right)^2 = \boxed{9.60\,N}$$

b)

$$v = \frac{\Delta x}{\Delta t} \quad \therefore \Delta t = \frac{\Delta x}{v} = \frac{3m}{120\,m/s} = \boxed{2.5\times10^{-2}\,s} = 25\,ms$$

Problem 8.

A 0.2 kg mass is held vertically by a spring. The mass is pulled down and released, and it then oscillates vertically, with a time dependence given by $y = (0.5)\cos(\pi t)$, where y is in meters and t is in seconds.

a) What are the amplitude, angular frequency and period of the motion?

b) What is the spring constant of the spring?

c) Write the equation for the velocity of the mass, $v(t)$.

d) Now, instead, a penny is placed on top of the mass before it is pulled down. The frequency of the oscillation the same as before, but the amplitude is larger. What is the maximum amplitude with which the mass can be released, such that the penny will not fly off the top of the mass during the oscillation?

Simple Harmonic Motion

a) Compare to : $x = A\cos(\omega t + \phi)$

we see : $\boxed{A = 0.5m}$ $\boxed{\omega = \pi\ rad/s}$ $\phi = 0$

$\omega = 2\pi f = \frac{2\pi}{T}$ $\therefore$ $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi\ rad/s} \Rightarrow \boxed{T = 2s}$

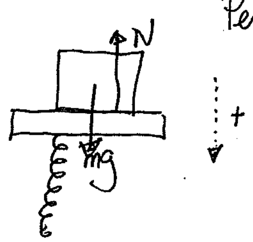
b) $\omega = \sqrt{K/m}$ $\therefore$ $K = m\omega^2 = (0.2Kg)(\pi\ rad/s)^2 = \boxed{1.97\ Kg/s^2}$

c) $v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t) = -\pi(0.5m)\sin(\pi t)$

$\boxed{v(t) = -\pi/2\ \sin(\pi t)}$ units of m/s

d) FBD on penny, while @ top of oscillation

Penny will leave mass when $N = 0$



$\Sigma F = mg - N = ma$ $\therefore$ leaves when $a \geq g$

$a(t) = -\omega^2 A\cos(\omega t)$

$|a_{max}| = \omega^2 A$

$g = \text{'' ''}$ $\therefore$ $A \geq \frac{g}{\omega^2} = \boxed{0.993m}$