Megan jumps vertically upward with an initial speed of 3.0 m/s, remaining rigid while in the air. At her highest point, Megan throws a 0.25 kg apple horizontally from a height 1.5 m above her feet, attempting to hit Isaac Newton, who is sleeping 5.0 m away horizontally on the ground near a tree.

(a) (15 points) From what height above the ground is the apple thrown?

(b) (15 points) From the time that the apple is thrown, how long does it take for the apple to reach Isaac Newton?

(c) (10 points) How fast must the apple be moving initially in order for it to hit poor Isaac?

(d) (10 points) 0.2 seconds after being thrown, what is the magnitude and direction of the net force on the apple (neglecting air resistance)?

\[
\begin{align*}
\text{Projectile Motion} & \\
\text{a) Highest point, } & \text{At } t_h, \ V_f(t) = 0 \\
& a = -g \\
& \therefore U_f^2 = V_i^2 + 2a\Delta y \\
& \Rightarrow \Delta y = h = \frac{U_i^2}{2g} = \frac{(3.0 \text{m/s})^2}{2(9.81 \text{m/s}^2)} = 0.46 \text{m} \\
& \therefore \text{apple thrown from height } y_i = 1.5 \text{m} + 0.46 \text{m} = 1.96 \text{m}
\end{align*}
\]

\[
\begin{align*}
\text{b) For apple: } & \quad y_f = y_i + V_{iy}t - \frac{1}{2}gt^2 \\
& \text{Horizontal throw: } V_{iy} = 0 \\
& \text{Assume Newton's head } & \text{at } y = 0 \text{ (on ground)} \\
& \therefore y_f = 0 = y_i - \frac{1}{2}gt^2 \\
& \therefore t = \sqrt{\frac{2y_i}{g}} = \left(\frac{2(1.96 \text{m})}{9.81 \text{m/s}^2}\right)^{1/2} = 0.63 \text{s}
\end{align*}
\]

\[
\begin{align*}
\text{c) Needs to travel 5.0m horizontally in 0.63s, steady velocity } & \Rightarrow \text{Initial velocity} \\
& \therefore V_{ix} = \frac{\Delta x}{\Delta t} = \frac{5.0 \text{m}}{0.63 \text{s}} = 7.9 \text{m/s}
\end{align*}
\]

\[
\begin{align*}
\text{d) While in flight, ignoring air resistance, the only force } & = \text{net force} \\
& = \text{gravitational force} \\
& = mg \\
& \therefore \vec{F_{\text{net}}} = (0.25 \text{ kg})(9.81 \text{m/s}^2) \quad \text{downward} \\
& = -2.45 \text{N \hat{j}}
\end{align*}
\]
A certain airplane can fly with a speed of 200 m/s relative to the air. If the
air is moving at 30 m/s relative to the ground in the Northeast direction
(45° North of East), then:

a) (10 points) In what direction should the airplane fly relative to the air
in order for the airplane to move due East relative to the ground?

b) (10 points) What is the magnitude of the airplane's speed while flying
East relative to the ground?

c) (10 points) How much time is required for the airplane to fly a distance
of 1000 km due East?

\[ \vec{V}_{pg} = \vec{V}_{pa} + \vec{V}_{ag} \]
\[ \vec{V}_{ag} = \text{velocity of air relative to ground} \]
\[ \vec{V}_{pg} = \text{"plane" relative to air} \]
\[ \vec{V}_{pa} = \text{"air" relative to ground} \]
\[ x\text{ component: } V_x = V_{pa} \cos \theta + V_{ag} \cos 45^\circ \]
\[ y\text{ component: } 0 = -V_{pa} \sin \theta + V_{ag} \sin 45^\circ \]
\[ \sin \theta = \frac{V_{ag} \sin 45^\circ}{V_{pa}} = \frac{30 \text{ m/s}}{200 \text{ m/s}} = 0.15 \]
\[ \Rightarrow \theta = \sin^{-1}(0.15) = 8.6^\circ \text{ (South of East)} \]

\[ V_{pg} = V_{pa} \cos(6.1^\circ) + V_{ag} \cos 45^\circ \]
\[ = (200 \text{ m/s}) \cos 6.1^\circ + (30 \text{ m/s}) \frac{1}{\sqrt{2}} = 220 \text{ m/s} \]

\[ U_{pg} = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{U_{pg}} = \frac{1000 \times 10^3 \text{ m}}{220 \text{ m/s}} = 4545 \text{ s} \]
\[ = 1.26 \text{ hr} \]
If the orbital period of the Earth around the Sun is 1 Earth year, then what is the orbital period of Jupiter around the Sun (in Earth years)?

Assume that the orbital period of a planet depends only on the mass of the Sun, the radius of the planet’s orbit, and Newton’s constant $G_N$.

Possibly useful information:
The radius of Earth’s orbit is $1.5 \times 10^8$ km.
The radius of Jupiter’s orbit is $7.8 \times 10^8$ km.
Newton’s constant is $G_N = 6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$.

*Hint:* Think about dimensions.

Dimensional Analysis! Use "[Eq. 3]" to indicate dimensions of $x$

$L = \text{Distance} \quad M = \text{mass} \quad T = \text{time}$

$T_E = \text{Earth's orbital period} \quad R_E = \text{Radius of Earth's orbit} \quad M_S = \text{mass of Sun}$

$T_J = \text{Jupiter's } \quad R_S = \text{Jupiter's }$.

Want function that links $T_E$ to $R_E, M_S$ & $G_N$:

$[T_E] = T \quad [R_E] = L \quad [M_S] = M \quad [G_N] = \frac{L^3}{M^{rac{1}{2}} T^2}$

Consider: $T_E = f(R_E, M_S, G_N)$:

to get dimension of $T$ on l.h.s., need $\frac{1}{G_N}$ on r.h.s.

$\left[ \frac{1}{G_N^{\frac{1}{2}}} \right] = T^{L^2} M^{rac{1}{2}}$

$\Rightarrow \frac{1}{G_N^{\frac{1}{2}}} \propto \frac{1}{M_S^{\frac{1}{2}}} \frac{1}{R_E^{\frac{1}{2}}} \frac{1}{L^{\frac{3}{2}}}$

$\Rightarrow T_E \propto \frac{1}{G_N^{\frac{1}{2}}} \frac{1}{M_S^{\frac{1}{2}}} \frac{1}{R_E^{\frac{1}{2}}} \frac{1}{L^{\frac{3}{2}}} \Rightarrow T_J \propto \frac{1}{G_N^{\frac{1}{2}}} \frac{1}{M_S^{\frac{1}{2}}} \frac{1}{R_S^{\frac{1}{2}}} \frac{1}{L^{\frac{3}{2}}}$

Divide these two equations:

$\frac{T_J}{T_E} = \frac{R_S^{\frac{3}{2}}}{R_E^{\frac{3}{2}}}$

or $T_J = T_E \left( \frac{R_J}{R_E} \right)^{\frac{3}{2}} = (1 \text{ Earth year}) \left[ \frac{7.8 \times 10^8 \text{ km}}{1.5 \times 10^8 \text{ km}} \right]^{-\frac{3}{2}}$

$= 11.9 \text{ Earth years}$