Problem 1.

A coin of mass = 4 g lies on a copy of your physics textbook, which is being tilted at an angle \( \theta \) with respect to the horizontal. The coefficient of static friction between the coin and the book is \( \mu_s = 0.4 \) and the coefficient of kinetic friction is \( \mu_k = 0.3 \).

a) Find the maximum angle \( \theta \) at which the book can be tilted before the coin will start to slide.

b) Assume the book is tilted at an angle of 73°. What will be the kinetic energy of the coin after it has slid a distance \( l = 15 \text{ cm} \) along the cover of the book?

c) How long did it take for the coin to slide the distance \( l \)?

\[ a) \quad \sum \vec{F} = ma = 0 \begin{cases} x: & mg \sin \theta - F_s = 0 \\ y: & N - mg \cos \theta = 0 \end{cases} \]

\[ F_s \leq \mu_s N \rightarrow \text{max angle} : \quad F_s = \mu_s N \]

\[ F_s = \mu_s mg \cos \theta = mg \sin \theta \]

\[ \mu_s = \tan \theta \quad \theta = \tan^{-1}(\mu_s) = 21.8^\circ \]

\[ b) \quad W = \Delta K = F \Delta x = (mg \sin \theta - \mu_k mg \cos \theta)l \]

\[ = mg \sin \theta - \mu_k mg \cos \theta \]

\[ = 4.94 \times 10^{-3} \text{ J} \]

\[ \therefore K_i = 0 \]

\[ K_f = 4.94 \times 10^{-3} \text{ J} \]

\[ c) \quad a = \frac{F_s}{m} = \frac{mg \sin \theta - \mu_k mg \cos \theta}{m} = g (\sin \theta - \mu_k \cos \theta) = 8.22 \text{ m/s}^2 \]

\[ x_f = x_i + vt + \frac{1}{2}at^2 \]

\[ x_i = 0 \]

\[ \ell = 0 + 0 + \frac{1}{2}at^2 \]

\[ t = \frac{(2\ell)}{a} = 0.19 \text{ s} \]
Problem 2.

A child on rollerblades catches a ball that has a mass of \( m_1 = 0.25 \) kg. The initial velocity of the ball is \( v_1 = (10 \text{ m/s})i \) and the initial velocity of the child is zero. The mass of the child (including her rollerblades) is \( m_2 = 30 \) kg. Ignore gravity and air resistance.

a) What is the velocity of the child immediately after she catches the ball?

b) What is the change in the total mechanical energy when the child catches the ball?

\[
\begin{align*}
\text{c)} \quad & \vec{p}_i = \vec{p}_f \quad \text{(no external net force)} \\
& x \text{ only: } m_1 v_1 + 0 = (m_1 + m_2) \vec{v} \\
& \vec{v} = \frac{m_1 v_1}{m_1 + m_2} = 0.083 \text{ m/s} \\
& \vec{v} = 0.083 \text{ m/s} \\
\end{align*}
\]

\[
\begin{align*}
\text{b)} \quad & K_i = \frac{1}{2} m_1 v_1^2 = 12.5 \text{ J} \\
& K_f = \frac{1}{2} (m_1 + m_2) \vec{v}^2 = 0.103 \text{ J} \\
& \Delta K = K_f - K_i = -12.4 \text{ J} \\
& \text{(energy lost)}
\end{align*}
\]
Problem 3.

An object is projected from the surface of the Earth with an initial speed equal to three times the escape velocity. What is the speed of the object when it is very far from the Earth? Neglect air resistance. The radius of the Earth is 6370 km, its mass is $5.98 \times 10^{24}$ kg, $G = 6.67 \times 10^{-11}$ Nm$^2$/kg$^2$.

\[ a) \text{ What is } U_E? \text{ Conservation of Energy } \quad E_i = E_f \]
\[ \frac{1}{2} m U_E^2 - \frac{GMm}{R_E} = 0 + 0 \]
\[ \text{choose } U_0 \text{ to be } U(r=\infty) \quad \therefore U_0 = 0 \]
\[ U_E = \left( \frac{2GM}{R_E} \right)^{1/2} \]

\[ U_i = 3U_E \]
\[ K_i = \frac{1}{2} m (3U_E)^2 \]
\[ \frac{1}{2} m (3U_E)^2 - \frac{GMm}{R_E} = \frac{1}{2} m U_f^2 \]
\[ \frac{1}{2} m (3U_E)^2 - \frac{GM}{R_E} = \frac{1}{2} U_f^2 \]
\[ \frac{1}{2} \frac{9}{2} \frac{GM}{R_E} - \frac{GM}{R_E} = \frac{1}{2} U_f^2 \]
\[ \frac{1}{2} \frac{9}{2} \left( \frac{2GM}{R_E} \right) = \frac{1}{2} U_f^2 \]
\[ 8 \frac{GM}{R_E} = \frac{U_f^2}{2} \quad U_f = \left( \frac{16 GM}{R_E} \right)^{1/2} = 31,652 \text{ m/s} \]
Problem 4.

A flagpole of mass $m$ and length $L$ is hinged at a wall and supported in a horizontal position by a massless cable attached to the free end (see diagram). The cable makes an angle $\theta$ with the horizontal.

a) What is the tension in the cable?
b) What is the (vector) force exerted on the flagpole by the hinge?
c) What is the direction of the torque (about the hinge point) that the tension in the cable exerts on the flagpole?

\[ \Sigma \vec{F} = 0 \rightarrow \begin{cases} x: F_H - T \cos \theta = 0 \\ \Sigma \vec{\tau} = 0 \\ y: F_v - mg + T \sin \theta = 0 \end{cases} \]

\[ \text{taken torques about "p"} \]
\[ -mg \frac{L}{2} + T \ell \sin \theta = 0 \]
\[ T = \frac{mg}{2 \sin \theta} \]

\[ F_H = T \cos \theta = \frac{mg \cos \theta}{2 \sin \theta} \]
\[ F_v = mg - T \sin \theta = mg - mg \frac{\sin \theta}{2 \sin \theta} = \frac{mg}{2} \]
\[ \vec{F}_{\text{Hinge}} = \vec{F}_H \hat{\imath} + \vec{F}_v \hat{j} \]

\[ \vec{\tau} = \vec{F} \times \vec{r} \]
\[ \text{(RIGHT HAND RULE)} \]
Problem 5.

A transverse wave has the form

\[ y = 10 \sin(0.01x - 20t) \]

where \( x \), \( y \) have units of meters, and \( t \) has units of seconds.

a) What is the wavelength of this wave? What is the frequency?
b) What are the magnitude and direction of the wave velocity?
c) What is the value of \( y \) for \( x = 20 \) m at time \( t = 0.05 \) s?
d) What are the maximum values of \( \frac{dy}{dt} \) for \( x = 0 \), and for \( \frac{dy}{dx} \) at \( t = 0 \)?

\[
A \sin(kx - \omega t)
\]

a) \( A = 10 \) m
\( \lambda = \frac{2\pi}{k} \)
\( \omega = 20 \)
\( \frac{\lambda}{2\pi} = \frac{20}{2\pi} = \frac{10}{\pi} \) s

b) \( \frac{k}{\lambda} = \frac{10}{(10 \text{ m/s})} = 20 \) m/s

\[
\text{for the right hand side} \quad (kx = \omega t)
\]

\[
y = 10 \sin(0.01 \cdot 20 - 20 \cdot 0.05) = 10 \sin(-0.8) = -7.17 \text{ m}
\]

d) \( \frac{dy}{dt} = -\omega A \cos(0.01x - 20t) \)

\[
\frac{dy}{dt}(x=0) = -\omega A \cos(-20t) = 2 \cos(20 \text{ s}^{-1}) \text{m/s} = 2 \text{ m/s}
\]

\[
\frac{dy}{dx}(t=0) = k A \omega \cos(kx)
\]

\[
\frac{dy}{dx}(t=0) = k A \omega \cos(0.01 \cdot 0) = 2 \text{ m/s}
\]
Problem 6.

A police car, traveling at 120 km/hr, passes a pedestrian. The siren is on, emitting a sound of frequency 360 Hz. The speed of sound in air is 330 m/s. What is the frequency of sound heard by the pedestrian?

a) when the police car is approaching the pedestrian?

\[ f' = f \left( \frac{v \pm u_0}{v \mp u_s} \right) \]

Source toward observer

\[ u_0 = 0 \]

\[ f' = f \left( \frac{v}{v - u_s} \right) = 360 \text{ Hz} \left( \frac{330 \text{ m/s}}{330 \text{ m/s} - 33 \text{ m/s}} \right) = 400 \text{ Hz} \]

b) when the police car is receding from the pedestrian?

\[ f' = f \left( \frac{v}{v + u_s} \right) = 328 \text{ Hz} \]
Problem 7.

A cylindrical rod is formed from cork (specific gravity = 0.25) and aluminum (specific gravity = 2.7) cylindrical sections. The lengths of the sections are 20.0 cm and 1.0 cm respectively. The rod is allowed to float in Lake Matoaka, with the heavy end downwards.

a) In equilibrium, how much of the cylinder's length is underwater?

b) [Extra credit question - do not attempt this part unless you have finished the rest of the exam!] The cylinder is given a push, so that it begins to bob up-and-down in the water. What is the frequency of the resulting oscillation?

\[ F_{\text{B}} = W_{\text{dip}} \]
\[ = \pi R_{h_{0}}^{2} \rho_{h_{0}} g \]
\[ = \pi l_{1} l_{2} \rho_{c} + \pi l_{2} l_{2} \rho_{a} \]
\[ x_{h_{0}} = l_{1} \rho_{c} + l_{2} \rho_{a} \]
\[ \frac{x}{h_{0}} = \frac{l_{1} \rho_{c} + l_{2} \rho_{a}}{h_{0}} \]
\[ x = \frac{l_{1} \rho_{c} + l_{2} \rho_{a}}{h_{0}} \]
\[ x = (0.25)(20\text{cm}) + (0.27)(1\text{cm}) = 7.7\text{cm} \]

b) SHM? Push down by \( y \) - deviation from equilibrium
\[ F = \Delta F_{\text{B}} = -h_{0} y \rho_{a} g = ma \]
\[ F = -Kx \rightarrow x = y \]
\[ K = h_{0} \rho_{a} g \]
\[ \omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{h_{0} \rho_{a} g}{l_{1} l_{2} \rho_{c} + l_{2} l_{2} \rho_{a}}} \]
\[ f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \left( \frac{\sqrt{\frac{h_{0} \rho_{a} g}{l_{1} l_{2} \rho_{c} + l_{2} l_{2} \rho_{a}}}}{\sqrt{\frac{\rho_{c} l_{1} + \rho_{a} l_{2}}{A}}} \right) \]
\[ = 1.795 \text{ Hz} \]
Problem 8.

Earth's polar ice caps contain $2 \times 10^{19}$ kg of ice. Since this ice is located at the poles, its contribution to Earth's moment of inertia (or rotational inertia) is negligible. As a weird practical joke, the Klingon empire arranges to have all of this ice moved to a single pile on the equator. The mass of the Earth is $6 \times 10^{24}$ kg, its radius is 6370 km, and the moment of inertia of a sphere about an axis through its center is $2MR^2/5$.

a) Would the length of the day become longer or shorter? Why?
b) By how much?

\[
L = I \omega = I' \omega' \quad I = \frac{2}{5} MR^2 \\
I' = \frac{2}{5} MR^2 + m_{\text{ice}} R^2 \\
\omega' = \frac{I \omega}{I'} = \left(\frac{\frac{2}{5}M}{\frac{2}{5}M + m_{\text{ice}}}\right) \frac{\omega}{R^2} \\
\omega = \frac{2M}{\left(\frac{2}{5}M + m_{\text{ice}}\right)} \approx 0.999991667 \ \omega \\
\omega' \approx 0.999991667 \ \omega \\

A\omega = \frac{\omega - \omega'}{\omega} = 8.33 \times 10^{-6} \quad \text{T is longer} \\

\omega = \frac{2\pi}{T} \quad \Delta T = \left(8.33 \times 10^{-6}\right) \left(3600 \text{ s} \times \frac{2 \pi \text{ rad}}{\text{day}}\right) = \boxed{0.725 \text{ s}} \quad \text{larger by} \approx 3/4 \text{ s}
Problem 1.

A projectile is launched with initial velocity $= 30 \, \text{m/s}$ at an angle of $70^\circ$ from the horizontal. Ignore air resistance.

a) How long will it take to reach its maximum height above the ground?

b) How far has it travelled horizontally when it reaches its maximum vertical height?

c) When it reaches its maximum height, it undergoes a chemical explosion, and separates into two equal-mass fragments. Just after the explosion, one of the fragments is observed to be moving at $25 \, \text{m/s}$ directly downward. What is the velocity vector of the other fragment?

\begin{align*}
a) & \quad \begin{align*}
    U_{ox} &= U_0 \cos \theta \\
    U_{oy} &= U_0 \sin \theta \\
    \max \text{ height} \rightarrow U_y &= 0 \\
    \therefore \quad U_y &= U_{oy} - gt \\
    0 &= U_{oy} \sin \theta - gt \\
    t &= \frac{U_0 \sin \theta}{g} = 2.88 \, \text{s} \\
\end{align*} \\

b) & \quad \begin{align*}
    x &= x_0 + U_{ox} t + 0 \\
    x &= U_0 \cos \theta t = 29.5 \, \text{m} \\
\end{align*} \\

c) & \quad \begin{align*}
    \vec{p}_i &= mU_x \hat{i} = mU_{ox} \hat{i} = \vec{p}_f = -\frac{m(25 \, \text{m/s})}{2} \hat{j} + \frac{m \vec{U}_2}{2} \\
    \left\{ \begin{array}{l}
    x: \quad mU_{ox} = \frac{m}{2} U_{2x} \\
    y: \quad 0 = -\frac{m(25 \, \text{m/s})}{2} + \frac{m}{2} U_{2y} \\
    \end{array} \right. \\
    \therefore \quad U_{2x} &= 2U_0 \cos \theta \\
    U_{2y} &= 25 \, \text{m/s} \\
    \therefore \quad \vec{U}_2 &= \frac{20.5 \, \text{m/s}}{\sqrt{2}} \hat{i} + \frac{25 \, \text{m/s}}{\sqrt{2}} \hat{j} \\
\end{align*}
Problem 3.

A 50 kg block slides underneath a 120 kg block with an acceleration $a = 2 \text{ m/s}^2$ when an external horizontal force of 280 N is applied. The 50 kg block sits on a horizontal frictionless surface, but there is friction between the two blocks.

a) Find the coefficient of kinetic friction between the blocks.

b) Find the acceleration (magnitude and direction) of the 120 kg block (when it is still in contact with the other block).

\[ \sum F = m\ddot{a} \]

\[ x: \ F - F_f = m_1\ddot{a}_1 \]

\[ F_f = 280 N - (50 \text{ kg})(2 \text{ m/s}^2) \]

\[ = 180 \text{ N} \]

\[ \sum F = m_2\ddot{a}_2 \]

\[ x: \ F_f = m_2\ddot{a}_2 \]

\[ \dot{y}: \ N - m_2 g = 0 \quad \Rightarrow \quad N = m_2 g \]

\[ F_f = 180 N = m_k N = m_k m_2 g \]

\[ m_k = \frac{180 N}{m_2 g} = 0.153 \]

\[ \ddot{a}_2 = \frac{F_f}{m_2} = \frac{180 N}{120 N} = 1.50 \text{ m/s}^2 \text{ to the right} \]
Problem 4.

A pole of mass $M = 30 \text{ kg}$ and length $L = 2 \text{ m}$ is attached to a vertical wall by a hinge at one end, and is attached via a massless string to a helium-filled balloon at the other end. The pole is at an angle of $20^\circ$ to the horizontal.

a) What is the tension in the string?

b) What are the horizontal and vertical components of the force exerted by the hinge?

c) Assume that the material of the balloon is massless. The density of helium is $0.18 \text{ kg/m}^3$, and the density of air is $1.3 \text{ kg/m}^3$. What is the radius of the balloon?

\[\Sigma F = 0 \quad \Sigma \tau = 0\]

**FBD:**

\[
\begin{align*}
F_H & \quad \cos \theta - Mg \\
F_v & \quad \sin \theta + T
\end{align*}
\]

\[\tau = 0 = -\frac{MgL}{2} \cos \theta + TL \cos \theta\]

\[T = \frac{Mg}{2} \cdot \frac{1}{g(\text{air})} = 147 \text{ N}\]

\[F_H = 147 \text{ N}\]

\[
\begin{align*}
\Sigma F = 0 & \Rightarrow F_H = 0 \\
\Sigma \tau = 0 & \Rightarrow F_v = Mg - T = Mg - \frac{Mg}{2} = \frac{Mg}{2}
\end{align*}
\]

\[
\begin{align*}
F_{\text{Hinge}} & = 147 \text{ N} \uparrow \\
V & = \frac{4}{3} \pi r^3
\end{align*}
\]

\[
\begin{align*}
\frac{4}{3} \pi r^3 & = \frac{T}{g(\text{air})} \\
r & = \frac{3T}{4\pi g(\text{air})} = 1.48 \text{ m}
\end{align*}
\]
Problem 5.

A bicycle wheel with moment of inertia of 30 kg·m² rotates with an initial angular speed of 5.0 rad/s. A tangential force of 12.0 N is applied at a distance of 25 cm from the center of the wheel, in such a way that the angular speed decreases.

a) How long will it take for the wheel to stop?

b) What is the direction of the torque caused by the tangential force? Pick one of the following: i) to the left, ii) to the right, iii) out of the page, iv) into the page, v) none of these.

\[ \alpha = \frac{rF}{I} \]
\[ \alpha = \frac{(0.25\text{m})(12\text{N})}{30\text{ kgm}^2} = 0.1\text{ rad/s}^2 \text{ into the page} \]
\[ \alpha = -0.1\text{ rad/s}^2 \text{ with respect to } \omega_i \]

\[ \omega = \omega_i + \alpha t \]
\[ t = \frac{0 - \omega_i}{\alpha} = \frac{0 - 5\text{ rad/s}}{-0.1\text{ rad/s}^2} = 50\text{s} \]