Problem 1.

A student jumps onto a bathroom scale, which is a simple spring scale with a spring constant of 5.0×10^3 N/m. The needle is observed to oscillate 16 times in 10 seconds (the scale is not very well damped!).

a) What is the period of this oscillation?

b) What is the student's mass?

c) The amplitude of the student's oscillation is 1.5 mm. What is her maximum speed?

a) frequency $f = \frac{16}{10s} = 1.6\ s^{-1} = 1.6\ Hz$ $\quad T = 1/f = 1/1.6Hz$

$$\boxed{T = 0.625\ s}$$

b) Simple Harmonic Motion

$$T = 2\pi\sqrt{m/K} \quad \therefore \left(\frac{T}{2\pi}\right)^2 = \frac{m}{K}$$

$$\therefore m = K\left(\frac{T}{2\pi}\right)^2 = (5.0 \times 10^3\ N/m)\left(\frac{0.625s}{2\pi}\right)^2$$

$$= 49.5\ \frac{Ns^2}{m} = \boxed{49.5\ Kg}$$

c) $v_{max} = \sqrt{K/m}\ A$

$$= \left(\frac{5 \times 10^3\ N/m}{49.5\ Kg}\right)^{1/2}(1.5 \times 10^{-3} m) = 0.015\ m/s$$

$$= \boxed{1.5\ cm/s}$$

Problem 2.

The ear canal of a domestic cat (*felis catus*) can be approximated as a cylinder[1], open at one end and closed at the other (the eardrum). Standing sound waves can be created inside the ear canal. The speed of sound in air is 340 m/s.

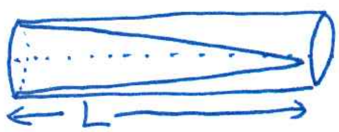
a) The frequency of the longest wavelength standing wave is 4500 Hz. What is the length of the cat's ear canal?

b) What is the frequency of the second harmonic (also known as the first overtone) standing wave?

c) The cat's eardrum has an area of 0.5 cm^2. The sound of a tin of cat food being opened has an intensity of 70 dB at the eardrum. How many joules of sound energy arrive at the eardrum in 30 seconds?

d) If the *amplitude* of the sound wave were to increase by a factor of 10, by how many decibels would the sound level be increased?

a)



fundamental : $L = \lambda/4$

$v = \lambda f$

$\therefore L = \dfrac{v}{4f} = \dfrac{340\ m/s}{4(4500\ Hz)} = \boxed{1.89\ cm}$

b) first overtone (second harmonic) : $L = \dfrac{3\lambda_2}{4}$

$\therefore \lambda_2 = \dfrac{4L}{3}$

$f_2 = \dfrac{v}{\lambda_2} = \dfrac{3v}{4L} = 3f = 3(4500\ Hz) = \boxed{13.5\ kHz} = 1.35\times10^4\ Hz$

c) $\beta = 10\log_{10}(I/I_0)$

$70\,dB = 10\log_{10}(I/I_0) \quad \therefore I/I_0 = 10^7 \quad I = 10^7(10^{-12}\ W/m^2) = 10^{-5}\ W/m^2$

$P = IA$

$= (10^{-5}\ W/m^2)\left(0.5\,cm^2 \times \left(\dfrac{1m}{10^2 cm}\right)^2\right) = 5.0\times10^{-10}\ W$

Energy = Power × time

$= (5.0\times10^{-10}\ W)(30s) = \boxed{1.5\times10^{-8}\ J}$

d) Energy $\propto$ (amplitude)^2 for waves

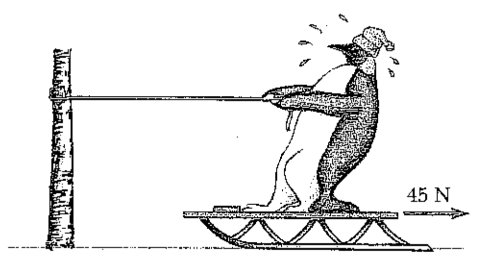
$\therefore$ Intensity " "

$\therefore$ amplitude $\times 10 \rightarrow I \times (10)^2 = 10^{-3}\ W/m^2$

$\beta = 10\log_{10}\left(\dfrac{10^{-3}\ W/m^2}{10^{-12}\ W/m^2}\right) = 90\,dB \quad \therefore \boxed{20\,dB\ \text{increase}}$

[1]Actually, the approximation is not so great, the cat's ear canal is more conical than cylindrical, but we haven't studied standing waves in a conical cavity.

Problem 3.



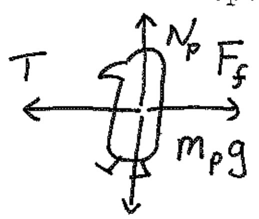
A 5.0 kg penguin sits on a 10 kg sled as shown. A horizontal force of 45 N is applied to the sled, but the penguin holds himself still by holding onto a rope tied to a tree. The coefficient of kinetic friction between the penguin and the sled is 0.20, but there is no friction between the sled and the snow-covered ground. The penguin remains at rest, but the sled slides out from underneath him.

a) Draw a free-body diagram for the penguin, and a separate one for the sled.

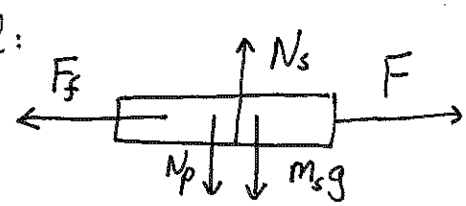
b) What is the magnitude of the acceleration of the sled?

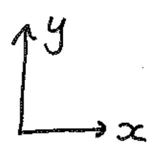
c) What is the tension in the rope?

a) Penguin :



Sled :





$m_p = 5\,\text{Kg}$
$m_s = 10\,\text{Kg}$
$F = 45\,\text{N}$

b) applying : $\vec{F}^{\,p}_{net} = m_p \vec{a}_p = 0$ (penguin not accelerating)

$$\begin{cases} x: & -T + F_f = 0 \quad \therefore T = F_f \\ y: & N_p - m_p g = 0 \quad \therefore N_p = m_p g \end{cases}$$

and : $\vec{F}^{\,s}_{net} = m_s \vec{a}_s$

$$\begin{cases} x: & F - F_f = m_s a_s \quad \therefore a_s = \dfrac{F - F_f}{m_s} \\ y: & N_s - N_p - m_s g = 0 \end{cases}$$

and : $F_f = \mu_K N_p$

$$\therefore a_s = \frac{F - \mu_K N_p}{m_s} = \frac{F - \mu_K m_p g}{m_s}$$

$$= \frac{45\,\text{N} - (0.2)(5\,\text{Kg})(9.8\,\text{m/s}^2)}{10\,\text{Kg}}$$

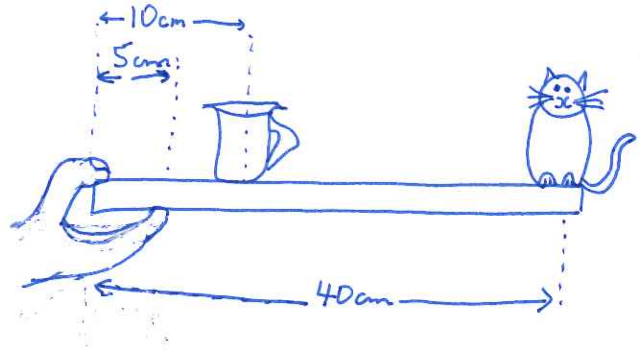
$$= \boxed{3.52\,\text{m/s}^2}$$

c) $T = F_f = \mu_K m_p g$

$$= (0.2)(5\,\text{Kg})(9.8\,\text{m/s}^2)$$

$$= \boxed{9.8\,\text{N}}$$

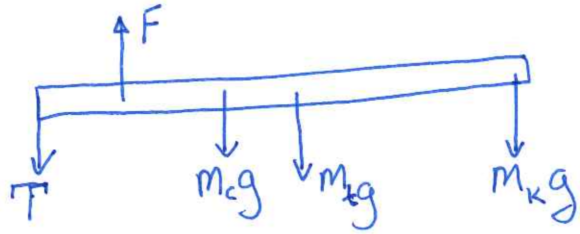
Problem 4.



A lunch tray is being held in one hand as shown. The thumb is pushing down on the end of the tray, and the fingers are pushing up 5 cm from the end. The tray has a mass of 0.2 kg, is 40 cm long, and has its center-of-mass at its geometrical center. A cup of milk with a mass of 0.3 kg sits 10 cm from the end. A kitten, who weighs 1.500 kg jumps onto the tray and sits at the far end. Calculate the force exerted by the thumb, T, and the force exerted by the fingers, F. Compare each force with the weight of the kitten.

Statics! $\tau^{net} = 0$ $F^{net} = 0$

Free Body Diagram:



$F^{net} = 0$ $\quad F - T - m_c g - m_t g - m_K g = 0$ (2 unknowns: T, F)

$\tau^{net} = 0$ take torques about left end of tray
(∴ thumb exerts no torque about this point)
define clockwise = positive for torques

$$\tau^{net} = -F(5\,cm) + m_c g(10\,cm) + \underbrace{m_t g(20\,cm)}_{\text{center of gravity of tray}} + m_K g(40\,cm) = 0$$

$$\therefore\ F = \frac{[m_c(10\,cm) + m_t(20\,cm) + m_K(40\,cm)]g}{5\,cm}$$

substitute: $m_c = 0.3\,Kg$ $\quad m_t = 0.2\,Kg$ $\quad m_K = 1.5\,Kg$ $\quad \boxed{F = 131\,N}$

use $F^{net} = 0$ to get T

$$T = F - m_c g - m_t g - m_K g = 131N - 19.6N = \boxed{111.4\,N = T}$$

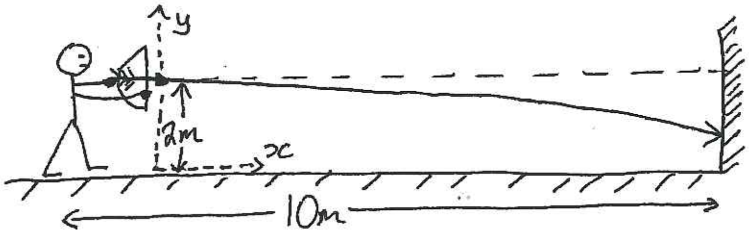
weight of Kitten $= m_K g = 14.7\,N$

Both forces much larger than Kitten's weight
(hard on the hand!)

Problem 45

An archer fires an arrow horizontally at a vertical wall which is 10 m away. The arrow starts at a height of 2 m above the (flat) ground. The initial speed of the arrow is 40 m/s.

a) How long does it take for the arrow to reach the wall?
b) How high above the ground is the arrow when it hits the wall?
c) What are the horizontal and vertical components of the arrow's velocity just before it reaches the wall?
d) What is the direction of the velocity just before it reaches the wall?



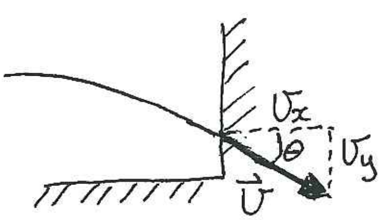
$v_0 = v_{0x} = 40\,m/s$
$v_{0y} = 0$
$y_0 = 2m$
$x_0 = 0$

a) $$t = \frac{\Delta x}{v_{0x}} = \frac{10m}{40m/s} = \boxed{0.25\,s}$$

b) $$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$= 2m + 0 - \tfrac{1}{2}\left(9.8\frac{m}{s^2}\right)(0.25s)^2 = \boxed{1.69m}$$

c) $v_x = v_{0x} = 40m/s$ (no x acceleration)
$$v_y = v_{0y} - gt$$
$$= -gt = \left(-9.8\frac{m}{s^2}\right)(0.25s) = \boxed{-2.45\,m/s}$$

d)



$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-2.45\,m/s}{40\,m/s}\right)$$
$$= -3.5^\circ$$

$\boxed{3.5^\circ \text{ below horizontal}}$

Problem 6

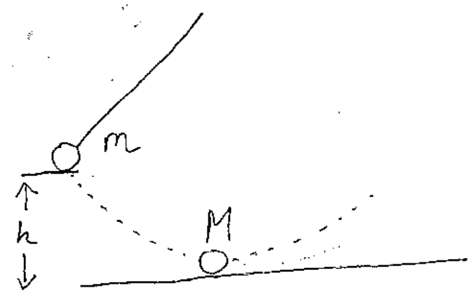
Tarzan's life is endangered by a wild swamp-thing. Jane, standing on a ledge 7.0 meters high, grabs a vine, swoops down, grabs Tarzan at the bottom of the swing, and takes him to safety on another ledge. Jane's mass is 50 kg, Tarzan's is 70 kg.

a) What is her velocity, just before she scoops him up, assuming she starts at rest? Ignore any air resistance.

b) What is their velocity, just after she scoops him up?

c) What is the highest ledge they can swing up to?

This should remind you of ballistic pendulum!



a) energy conservation

$$E_i = E_f$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$v^2 = 2gh \qquad v = \sqrt{2gh} = \boxed{11.7\,m/s}$$

b) Collision (inelastic - they stick together!) ∴ energy not conserved

∴ use momentum conservation

$$p_i = p_f$$

$$mv = (m+M)V \qquad \therefore V = \frac{mv}{(m+M)} = \frac{(50\,Kg)(11.7\,m/s)}{(50\,Kg + 70\,Kg)} = \boxed{4.9\,m/s}$$

c) energy conservation in swing up:

$$E_i = E_f$$

$$\frac{1}{2}(M+m)V^2 = (M+m)gH \qquad \therefore H = \frac{V^2}{2g} = \frac{(4.9\,m/s)^2}{2(9.8\,m/s^2)} = \boxed{1.2\,m}$$

Problem 27

A bargain hunter purchases a crown, allegedly made of gold, at a flea market. After she gets home, she hangs it from a scale and finds its weight to be 7.84 N. She then weighs the crown while it is totally immersed in water and now the scale reads 6.86 N.

a) Determine the buoyant force on the crown, and find its volume.

b) Find the density of the crown. Gold has a density 19.3 times that of water. Is it made of gold?

Archimede's principle (see Example 10.10 from text!)

a) $F_B = W_{disp}$ $\qquad F_B = 7.84\,N - 6.86\,N = \boxed{0.98\,N}$

(Bouyant force is difference between weight in air & submerged weight)

$\therefore\ W_{disp} = 0.98N$

$\rho_{H_2O} V g = \ ''$ $\qquad \therefore\ V = \dfrac{0.98N}{\rho_{H_2O}\, g} = \dfrac{0.98\,N}{(1000\,Kg/m^3)(9.8\,m/s^2)} = \boxed{1\times10^{-4}\,m^3}$

$(= 100\,cm^3)$

b) Weight in air $= mg = 7.84\,N$

(ignoring bouyancy of air, which is tiny!)

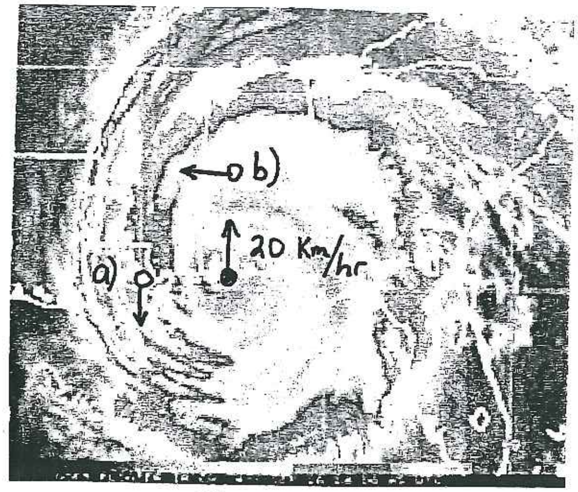
$\therefore\ m = \dfrac{7.84N}{g} = 0.8\,Kg$

$\rho = \dfrac{m}{V} = \dfrac{0.8\,Kg}{1\times10^{-4}\,m^3} = \boxed{8000\,Kg/m^3}$

$= 8\times$ density of water

$\therefore$ she was ripped off: it's not gold!

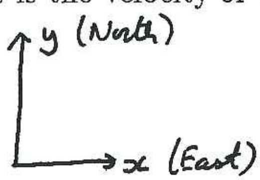
Problem 8.



The eye (or center) of Hurricane Ivan is moving directly north with a speed (relative to the ground) of 20 km/hr.

a) An intrepid meterologist is located west of the eye. At her location, the wind speed relative to the eye of the hurricane is 120 km/hr, and the wind is blowing to the south. What is the velocity (magnitude and direction) of the wind she measures relative to the ground?

b) Another meterologist is located north of the eye. At his location, the wind's speed relative to the eye is 120 km/hr, blowing to the west. What is the velocity of the wind he measures relative to the ground?



Relative velocity...

a) $\vec{V}_{wg} = \vec{V}_{we} + \vec{V}_{eg}$

$\vec{V}_{eg}$ → eye relative to ground

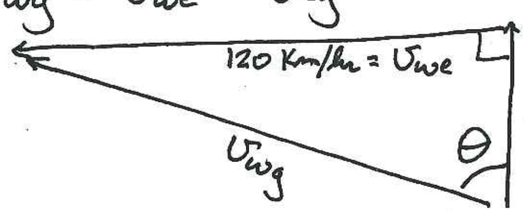
$\vec{V}_{we}$ → wind relative to eye

$\vec{V}_{wg}$ → wind relative to ground

for this part all vectors are along y-axis ∴ easy

$$V_{wg} = -120\ \frac{Km}{hr} + 20\ \frac{Km}{hr} = -100\ \frac{Km}{hr} \quad \therefore \quad \boxed{100\ \frac{Km}{hr}\ \text{South}}$$

b) $\vec{V}_{wg} = \vec{V}_{we} + \vec{V}_{eg}$



$$V_{wg} = \sqrt{\left(120\ \frac{Km}{hr}\right)^2 + \left(20\ \frac{Km}{hr}\right)^2} = \boxed{121.6\ Km/hr}$$

$$\theta = \tan^{-1}\left(\frac{120\ Km/hr}{20\ Km/hr}\right) = \boxed{80.5^\circ\ \text{West of North}}$$

(or 9.5° North of West)