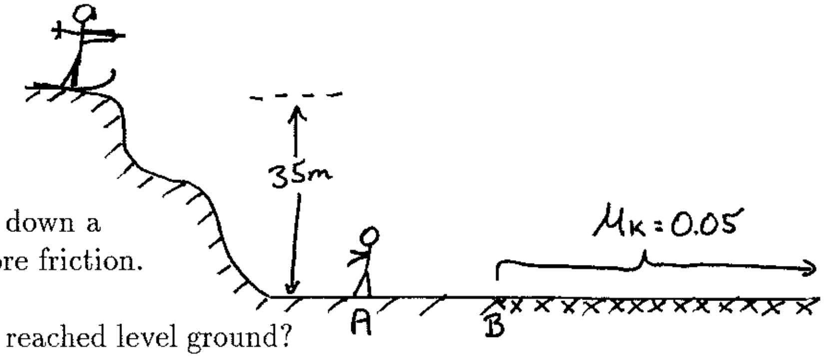
Problem 1.



A snow skier (mass = 60 kg) slides down a 35 m high hill, starting from rest. Ignore friction.

a) What is her speed when she has reached level ground?

b) At location 'A', in an impressive stunt, she grabs a 25 kg child, who is initially at rest, and the two of them continue to slide without friction. What is the new speed of the skier+child system?

c) From location 'B' onwards they encounter a rough patch, so that now there is friction with the ground ($\mu_K = 0.05$). How far do they travel from 'B' before coming to rest?

a) $mgh = \frac{1}{2}mv^2$ $\quad v = \sqrt{2gh} = \sqrt{2(9.8\,m/s^2)(35m)} = \boxed{26.2\,m/s}$

(energy conservation)

b) inelastic collision : momentum conservation

$$mv = (m+M)V \qquad V = \frac{mv}{(m+M)} = \frac{(60Kg)(26.2\,m/s)}{(60Kg + 25Kg)} = \boxed{18.5\,m/s}$$

c) Frictional force $= \mu_K N = \mu_K(m+M)g$

Work done by friction $= -Fd = \Delta KE = -\frac{1}{2}(m+M)v^2$

$$\therefore \mu_K(m+M)gd = \frac{1}{2}(m+M)v^2$$

$$d = \frac{v^2}{2\mu_K g} = \frac{(18.5\,m/s)^2}{2(0.05)(9.8\,m/s^2)} = \boxed{349m}$$

Problem 2.

An intravenous (IV) system is supplying saline solution to a patient at a rate of 0.150 cm^3/s through a needle of radius 0.120 mm and length 2.5 cm. The blood pressure in the patient's vein is 7.00 mm Hg. The viscosity of the saline solution is 1.0×10^{-3} N·s/m^2. Note: 760 mm of Hg = 10^5 Pa.

a) What pressure is needed at the entrance to the needle to cause this flow?
b) What is the speed of the fluid in the needle (in m/s)?
c) How high would the IV bottle that supplies this flow have to be elevated to provide this pressure? You can assume that the tubing connecting the needle to the IV bottle has a large enough radius that there is negligible pressure drop due to viscosity in the tubing.
d) Show that the flow will not be turbulent (the density of the solution = 1025 kg/m^3.)

a) $$F = \frac{(P_2 - P_1)\pi r^4}{8\eta l} \quad \therefore \quad P_2 = \frac{8\eta l F}{\pi r^4} + P_1 = \frac{8\left(10^{-3}\frac{Ns}{m^2}\right)(0.025m)\left(0.15\times10^{-6}\frac{m^3}{s}\right)}{\pi(0.12\times10^{-3}m)^4} + P_1$$

$$P_1 = 7.0\,mmHg \times \frac{10^5 Pa}{760\,mmHg} = 921\,Pa$$

$$= 4.60\times10^4 Pa + 921\,Pa$$

$$= \boxed{4.70\times10^4 Pa}$$

b) $$F = vA$$

$$v = F/A = \frac{F}{\pi r^2} = \frac{0.15\times10^{-6} m^3/s}{\pi(0.12\times10^{-3}m)^2} = \boxed{3.3 m/s}$$

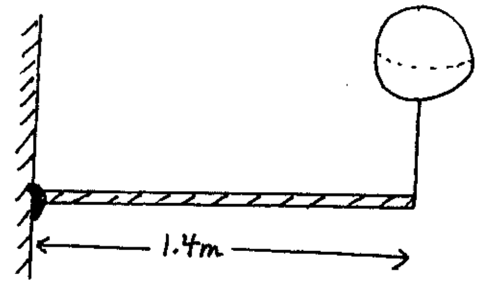
c) $$\Delta P = \rho g h = 4.7\times10^4 Pa$$

$$h = \frac{4.7\times10^4 Pa}{\left(1025\frac{Kg}{m^3}\right)(9.8 m/s^2)} = \boxed{4.67 m}$$

d) $$N_R = \frac{2\rho v r}{\eta} = \frac{2\left(1025\frac{Kg}{m^3}\right)(3.3 m/s)(0.12\times10^{-3}m)}{10^{-3}\frac{Ns}{m^2}} = 812$$

Reynolds Number < 2000
∴ not turbulent

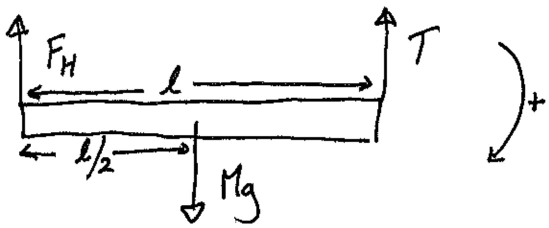
Problem 3.



A uniform 0.5 kg bar of length 1.4 m is held at rest in a horizontal position; it is attached at one end to fixed wall by a hinge, and at the other end by a string attached to a balloon.

a) What is the tension in the string?
b) What is the force (magnitude and direction) that the hinge exerts on the bar?
c) Assume the (spherical) balloon is filled with helium (density=0.18 kg/m^3). What is the radius of the balloon? You can ignore the mass of the rubber of the balloon. Note: the volume of a sphere is $\frac{4}{3}\pi r^3$.

a) Statics FBD:



take torques about hinge:

$$\tau^{net} = Mg\frac{l}{2} - Tl = 0 \qquad \therefore\ T = \frac{Mg}{2} = \boxed{2.45\text{ N}}$$

b) $$F^{net} = 0 = F_H + T - Mg \qquad \therefore\ F_H = Mg - T = \boxed{2.45\text{ N upward}}$$

(no x-forces)

c) $$F_B = W_{disp} \quad \text{(Archimedes!)}$$

$$F_{net} = T = F_B - mg \qquad V = \frac{4}{3}\pi r^3$$

$$\rho_{air} V g - \rho_{He} V g = T$$

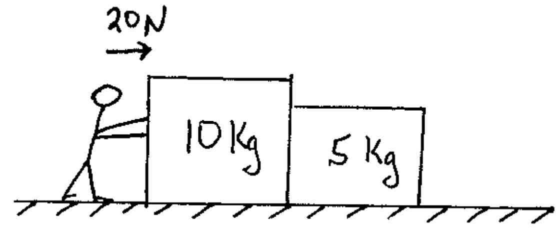
$$\tfrac{4}{3}\pi r^3 g(\rho_{air} - \rho_{He}) = T$$

$$r^3 = \frac{3T}{4\pi g(\rho_{air} - \rho_{He})} = \frac{3(2.45\text{ N})}{4\pi(9.8\frac{m}{s^2})(1.29\frac{Kg}{m^3} - 0.18\frac{Kg}{m^3})} = 0.0538\text{ m}^3$$

$$\boxed{r = 0.377\text{ m}}$$

Problem 4.

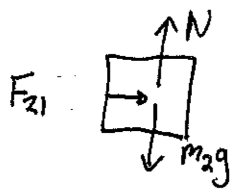
Two boxes are located side by side on a frictionless, level floor as shown. The box on the left has a mass of 10 kg and the box on the right has a mass of 5 kg. You push to the right on the 10 kg box with a force of 20 N, and the boxes move together.



a) What is the acceleration of the boxes?
b) What is the force (magnitude & direction) that the 5 kg box exerts on the 10 kg box?
c) What is the force (magnitude & direction) that the 10 kg box exerts on the 5 kg box?
d) What is the force (magnitude & direction) that the 10 kg box exerts on you?

a) $F^{net} = (m_1+m_2)a \qquad a = \frac{F^{net}}{(m_1+m_2)} = \frac{20\,N}{15\,Kg} = \boxed{1.33\,m/s^2}$ (to the right)

b) FBD on 5 Kg:



$F_{21} = m_2 a$
$= (5\,Kg)(1.33\,m/s^2)$
$= 6.67\,N$

Newton's third law: $F_{21} = -F_{12}$

$\therefore F_{12} = \boxed{-6.67\,N}$ (force is to left)

c) $F_{21} = \boxed{6.67\,N}$ (to right)
[see above]

d) Third law again : $\boxed{20\,N}$ to the left

Problem 5.

A disk-shaped playground merry-go-round (radius = 1.8 m) has a mass of 120 kg and is rotating with an angular velocity of 0.400 rev/s (the moment of inertia of a disk = $\frac{1}{2}Mr^2$).

a) What is the initial rotational kinetic energy of the merry-go-round?
b) What is the initial angular momentum of the merry-go-round?
c) What is its angular velocity after a 27.0 kg child gets onto it by grabbing its outer edge? The child is initially at rest. Consider the child to be a point mass.
d) After the child has grabbed on, a 5.0 N frictional force is now applied tangentially, at the edge of the merry-go-round. How long does it take for the system to come to rest?

a) $KE = \frac{1}{2} I \omega_i^2 = \frac{1}{2}(194\ \text{Kgm}^2)(2.5\ \text{rad/s})^2 = \boxed{612\ \text{J}}$

$\omega_i = \frac{0.4\ \text{rev}}{\text{s}} \times \frac{2\pi\ \text{rad}}{\text{rev}} = 2.5\ \text{rad/s}$

$I = \frac{1}{2}Mr^2 = \frac{1}{2}(120\ \text{Kg})(1.8\ \text{m})^2 = 194.4\ \text{Kgm}^2$

b) $L = I\omega_i = (194\ \text{Kgm}^2)(2.5\ \frac{\text{rad}}{\text{s}}) = \boxed{485\ \text{Kg}\frac{\text{m}^2}{\text{s}}}$

c) conservation of angular momentum (no net external torques on system)

$\therefore\ L_i = L_f = (I + mr^2)\omega_f$

$\therefore\ \omega_f = \frac{L_i}{I + mr^2} = \frac{485\ \text{Kg m}^2/\text{s}}{\underbrace{(194\ \text{Kgm}^2 + (27\ \text{Kg})(1.8\ \text{m})^2)}_{282\ \text{Kgm}^2}} = \boxed{1.72\ \text{rad/s}}$

d) $\tau = rF = (1.8\ \text{m})(5.0\ \text{N}) = 9\ \text{Nm}$

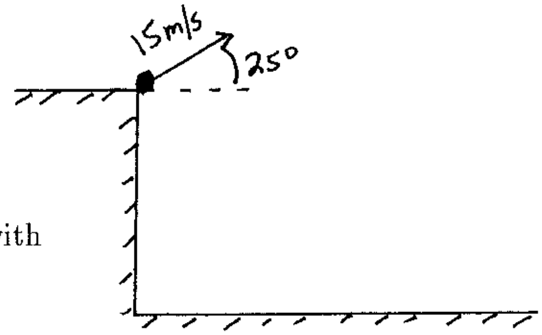
$\alpha = \frac{\tau}{I} = \frac{9\ \text{Nm}}{282\ \text{Kgm}^2} = -0.032\ \text{rad/s}^2$

$\omega - \omega_0 = \alpha t$

(motion under constant angular acceleration)

$t = \frac{\omega - \omega_0}{\alpha} = \frac{-1.72\ \text{rad/s}}{-0.032\ \text{rad/s}^2} = \boxed{54\ \text{s}}$

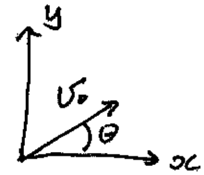
Problem 6.



A ball is thrown upward from the top of a building at an angle of 25° to the horizontal, with an initial speed of 15 m/s.

a) What are the components (horizontal and vertical) of the initial velocity?
b) The ball hits the ground after being in the air for 3 seconds. How tall is the building?
c) What are the components (horizontal and vertical) of the ball's velocity just before it hits the ground?
d) What is the magnitude and direction of this velocity?

a)
$$v_{0x} = v_0 \cos 25^\circ = (15\,m/s)(\cos 25^\circ) = \boxed{13.6\,m/s}$$
$$v_{0y} = v_0 \sin 25^\circ = \text{"}\ (\sin 25^\circ) = \boxed{6.34\,m/s}$$



b)
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$0 = h + v_{0y}t - \frac{1}{2}gt^2 \qquad h = \frac{1}{2}gt^2 - v_{0y}t$$
$$= \frac{1}{2}(9.8\,m/s^2)(3s)^2 - (6.34\,m/s)(3s) = \boxed{25.1\,m}$$

c)
$$v_y = v_{0y} - gt = \boxed{-23.1\,m/s}$$
$$v_x = v_{0x} = \boxed{13.6\,m/s}$$

d)
$$v = \left[v_x^2 + v_y^2\right]^{1/2} = \boxed{26.8\,m/s}$$
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \boxed{-59.5^\circ}$$

below horizontal