Physics 621 Midterm Test
due Saturday November 14 2009, 17:00

Ground rules: you can use your notes, the textbook, mathematical tables, programs like Maple, MathCAD, Mathematica, etc. but *not* other quantum mechanics textbooks. No discussions with your colleagues.

1. [20 points] Consider a particle in a 1 dimensional "box", *i.e.* where

   $V(x) = 0 \quad \text{for } 0 < x < a$

   and

   $V(x) = \infty \quad \text{elsewhere.}$

   Calculate the uncertainty product $\Delta x \Delta p$ for the n^{th} eigenstate, and compare to the Heisenberg uncertainty principle. Comment.

2. [15 points] For the 1-D harmonic oscillator, evaluate $<x^4>$ for the n^{th} energy eigenstate.

3. [10 points] Merzbacher Problem 6.5 (page 112).

4. Consider a particle of mass m moving along the x-axis in the potential $V(x)$ where

   $V(x) = +\infty \quad \text{for } x < 0$

   and

   $V(x) = \frac{1}{2}m\omega^2 x^2 \quad \text{for } x > 0$

   a) [10 points] Use the variational principle with the trial wavefunction $\psi(x) = 2\lambda^{3/2} x e^{-\lambda x}$ (for $x < 0$) and $\psi(x) = 0$ (for $x > 0$) to estimate the ground state energy.

   b) [5 points] What are the *exact* stationary-state energy spectrum and wavefunctions?

5. The deuteron (the nucleus of deuterium) is a bound state of a neutron and a proton. A useful approximation for the neutron-proton interaction is the potential

   $V(r) = -Ae^{-r/a}$

   where $A = 32$ MeV, $a = 2.2$ fm, and r is the neutron-proton separation.

   a) [15 points] Use the variational principle to estimate the deuteron ground-state energy. Take a trial wavefunction $\Psi = e^{-\alpha r/2a}$ where α is a variational parameter. This is a 3D problem, but you can assume spherical symmetry; remember to use the proper form for the Laplacian in spherical coordinates. Note you will need to use the reduced mass of the neutron-proton system as the mass in your Hamiltonian. You may need to use a numerical or graphical method to minimize the variation.

   b) [5 points] Compare your result to the experimental value for the deuteron binding energy, which is 2.22 MeV.

(over)

6. Consider a particle with mass m in a two-dimensional square box (infinite potential walls) with sides of length L. There is a weak potential in the box given by

$$V(x,y) = V_0 L^2 \delta(x - x_0)(y - y_0)$$

a) [5 points] Determine the first-order correction to the ground state energy.

b) [10 points] Write the expression for the first-order estimate of the energy of the first excited state. For the special case that $x_0 = y_0 = L/4$, what is the splitting of this energy level?

c) [5 points] For what choices of the point x_0, y_0 would the first excited state remain degenerate, to this order in the perturbation? Explain this result in terms of the symmetry of the problem.