Problem 1.

A ball is thrown horizontally from the roof of Swem Library. The library is 20 m tall. The ball hits the ground a distance of 16 m from the base of the library. Neglect air resistance.

a) Find the initial speed with which the ball was thrown.

b) Find the magnitude and direction of the ball's velocity just before it hits the ground.

\[ a) \text{ Projectile motion. } U_x = 10 \text{ m/s}^2 \text{ horizontally.} \]
\[ y = y_i + U_{iy}t - \frac{1}{2} gt^2 \quad y_f = 0 \quad y_i = 20 \text{ m} \]
\[ t^2 = \frac{20 \text{ m} \times 2}{g} = \frac{40 \text{ m}}{10 \text{ m/s}^2} = 4 \text{ s}^2 \quad \Rightarrow t = 2 \text{ s} \]
\[ x_f = x_i + U_{ix}t 
\]
\[ U_{ix} = \frac{x_f - x_i}{t} = \frac{16 \text{ m} - 0 \text{ m}}{2 \text{ s}} = \boxed{8.0 \text{ m/s}} = U_i \]

\[ b) \quad U_x = U_{ix} = 8.0 \text{ m/s} \]
\[ U_y = U_{iy} - gt = 0 - (10 \text{ m/s}^2 \times 2 \text{ s}) = -20 \text{ m/s} \]
\[ U = |U| = (U_x^2 + U_y^2)^{1/2} = (8^2 + (-20)^2)^{1/2} = \boxed{21.5 \text{ m/s}} \]
\[ \tan \theta = \frac{|U_y|}{|U_x|} \quad \Rightarrow \quad \theta = \tan^{-1} \left( \frac{20}{8} \right) = 68.2^\circ \]
Problem 2.

You have an \( m = 80 \text{ kg} \) grindstone, which is disk-shaped, and has a radius of 50 cm. It is initially turning at 100 revolutions per minute. You press a steel axe against it using a radial force of 2 N. The moment of inertia of a disk, about its center, is \( \frac{1}{2} mR^2 \). The coefficient of kinetic friction between the steel and the grindstone is \( \mu_k = 0.2 \).

a) What is the frictional force between the axe and the grindstone?

b) What torque does this force cause (about the axle of the grindstone)?

c) What is the angular acceleration of the grindstone (in rad/s²)?

d) How long does it take for the grindstone to come to a complete stop?

e) How many revolutions has the grindstone made during that time?

\[ F_x = \mu_k N = \mu_k F = (0.2)(2N) = 0.4N \]

\[ \tau = rF \sin \theta \quad \theta = 90^\circ \]

\[ = (0.5m)(0.4N) \sin 90^\circ = 0.2 \text{ Nm} \]

\[ \tau = I \alpha \quad \alpha = \frac{\tau}{I} = \frac{0.2 \text{ Nm}}{\frac{1}{2}(80 \text{ kg})(0.5 \text{ m})^2} = 0.02 \text{ rad/s}^2 \]

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**Motion under constant angular acceleration**

\[ \omega = \omega_i + \alpha t \]

\[ \omega_i = \left( \frac{100 \text{ rev}}{\text{ min}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) \left( \frac{2\pi \text{ rad}}{\text{ rev}} \right) = 10.5 \text{ rad/s} \]

\[ t = \frac{\omega - \omega_i}{\alpha} = \frac{0 - 10.5 \text{ rad/s}^2}{-0.02 \text{ rad/s}^2} = 524 \text{ s} \]

\( \approx 8 \text{ min, 44 s} \)

\[ \omega = \omega_i + \alpha t + \frac{1}{2} \alpha t^2 \]

\[ = 0 + (10.5 \text{ rad/s}) \left( \frac{524 \text{ s}}{6} \right) + \frac{1}{2} (-0.02 \text{ rad/s}^2) \left( \frac{524 \text{ s}}{6} \right)^2 \]

\[ = 2756 \text{ rad} \]

\[ 2756 \text{ rad} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 439 \text{ revolutions} \]
Problem 3.

Professor Archimedes has a mass of 65 kg. When he is weighed while completely submerged in ordinary water ($\rho = 10^3$ kg/m$^3$), his apparent weight is only 10 N.

a) What is the buoyant force exerted on him by the water?

b) What is his volume?

c) If he was immersed in sea water, which has a density of $1.03 \times 10^3$ kg/m$^3$, would he float or sink? (don't just give the answer, you need to show your calculation).

\[ \text{Bouyancy (Archimedes' Principle)} \]

\[ \text{a) Apparent weight} = mg - B = 10N \]

\[ B = mg - 10N = (65 \text{kg})(9.8 \text{m/s}^2) - 10N = 627N \]

\[ b) B = W_{dip} = \rho_{w}Vg \quad \therefore V = \frac{B}{\rho_{w}g} = \frac{627 N}{10^3 \text{kg/m}^3 \times 9.8 \text{m/s}^2} = 6.40 \times 10^{-2} \text{m}^3 \]

\[ c) \text{He will float if } B > mg \]

His new $B = \rho_{sea}Vg$

\[ = (1.03 \times 10^3 \text{kg/m}^3)(6.40 \times 10^{-2} \text{m}^3)(9.8 \text{m/s}^2) \]

\[ = 646N > 637N = mg \]

\[ \therefore \text{he will float in} \]

\[ \text{sea water (but sinks in fresh water)} \]
Problem 4.

A uniform ladder of length $L$ rests against a smooth (frictionless) vertical wall. The mass of the ladder is $m$. The coefficient of static friction between the ladder and the ground is $\mu_s = 0.35$.

a) Find the minimum value of the angle $\theta$ such that the ladder does not slip.

b) What is the force that the wall exerts on the ladder? Express your answer in terms of any of the following: $L$, $m$, $\mu_s$, and/or $g$.

\[\sum F = 0\]
\[\sum \tau = 0\]

Consider torques about point $P$:

\[\sum \tau = N_w L \sin \theta - mg \frac{L}{2} \cos \theta = 0\]

\[\therefore \tan \theta = \frac{mgL}{2N_wL} = \frac{mg}{2N_w}\]

Consider forces:

\[\sum \vec{F} = 0\]

\[F_s - N_w = 0\]
\[F_s = N_w\]
\[N_s - mg = 0\]
\[N_s = mg\]

\[F_s = \mu_s N_s = \mu_s mg\]
\[N_w = \mu_s mg\]

\[\therefore \tan \theta = \frac{mg}{2(\mu_s mg)} = \frac{1}{2\mu_s}\]

\[\therefore \theta = \tan^{-1} \left( \frac{1}{2\mu_s} \right) = \tan^{-1} \left( \frac{1}{0.7} \right)\]

\[\theta_{min} = 55^\circ\]
Problem 5.

An innocent duck is flying horizontally to the left at 3.0 m/s. Unfortunately for him, he is struck by a hunter’s bullet, traveling at 100 m/s horizontally to the right. The bullet sticks inside the duck. The mass of the duck is 1.5 kg, and that of the bullet is 50 grams. Neglect any effects due to gravity or air resistance.

a) What is the velocity (magnitude and direction) of the duck after the collision?

b) How much mechanical energy was converted into other forms (heat, sound, chemical...), in the collision?

c) Which is larger: the force the bullet exerted on the duck, or the force the duck exerted on the bullet?

\[ m_b u_b + m_d u_d = (m_b + m_d) \cdot U \]

\[ U = \frac{m_b u_b + m_d u_d}{m_b + m_d} \]

\[ = \frac{(0.05 \text{ kg})(100 \text{ m/s}) + (1.5 \text{ kg})(-3 \text{ m/s})}{(1.5 \text{ kg} + 0.05 \text{ kg})} \]

\[ = + 0.322 \text{ m/s} \] (to the right)

b) \[ \Delta KE = \frac{1}{2}(m_d + m_b)u^2 - \left( \frac{1}{2}m_d u_d^2 + \frac{1}{2}m_b u_b^2 \right) \]

\[ = \frac{1}{2}(1.55 \text{ kg})(0.322 \text{ m/s})^2 - \left( \frac{1}{2}(1.5 \text{ kg})(-3 \text{ m/s})^2 + \frac{1}{2}(0.05 \text{ kg})(100 \text{ m/s})^2 \right) \]

\[ = 0.085 - (6.85 + 250) = 257 \text{ J} \]

\( \approx \) all of mech. energy is lost!

c) Newton’s 3rd Law: The forces are identical in magnitude!

1 No actual ducks were harmed in the crafting of this question.
Problem 6.

A mass \((M = 0.5 \text{ kg})\) is attached to a vertical axis by two light (i.e. massless) strings as shown. Each string has a length of 0.6 m.

a) If the system is rotated about the axis with a frequency of 3 revolutions per second, what are the tensions in each of the strings? *(Hint: they are not the same! Draw a free body diagram!)*

\[
\sum F = Ma
\]

\[
\begin{align*}
\text{x:} & \quad T_1 \cos \theta + T_2 \cos \theta = M a = \frac{M u^2}{R} \\
\text{y:} & \quad T_1 \sin \theta - T_2 \sin \theta - Mg = 0
\end{align*}
\]

\[
(T_1 + T_2) \cos \theta = \frac{M u^2}{L \cos \theta} \quad T_1 - T_2 = \frac{Mg}{\sin \theta}
\]

2 equations in 2 unknowns \((T_1 \& T_2)\)

\[
\begin{align*}
T_1 + T_2 &= \frac{M u^2}{L \cos \theta} \\
T_1 - T_2 &= \frac{Mg}{\sin \theta}
\end{align*}
\]

\[
\begin{align*}
2T_1 &= M \left(\frac{u^2}{L \cos \theta} + \frac{g}{\sin \theta}\right) & T_1 &= \frac{M}{2} \left(\frac{u^2}{L \cos \theta} + \frac{g}{\sin \theta}\right) = 58.2 \text{ N} \\
2T_2 &= -M \left(\frac{u^2}{L \cos \theta} - \frac{g}{\sin \theta}\right) & T_2 &= \frac{M}{2} \left(\frac{u^2}{L \cos \theta} - \frac{g}{\sin \theta}\right) = 48.4 \text{ N}
\end{align*}
\]

b) \[
\vec{I} = \vec{I}^2 = (MR^2) \left(\frac{u}{R}\right) \hat{j}
\]

\[
= M u R = M u L \cos \theta
\]

\[
= (0.5 \text{ kg})(9.79 \text{ m})(0.6 \text{ m}) \cos 30^\circ
\]

\[
= 2.54 \text{ kg m}^2/\text{s}
\]

\[
\text{Direction} = \hat{\mathbf{y}} \quad \text{(or positive y-direction)}
\]

\[
\text{from right-hand rule}
\]
A string of mass \( m = 2 \times 10^{-3} \text{ kg} \) and length \( L = 3 \text{ m} \) is held fixed at both ends, and oscillates in a standing wave of one and a half wavelengths, at a frequency of 60 Hz.

a) What is the tension in the string?

b) What would be the velocity of a traveling wave on this string?

c) What would be the lowest possible frequency of a standing wave on this string, \( i.e. \) what is the fundamental frequency?

\[ a) \quad U = \sqrt{T/\mu} \quad \therefore \quad T = \mu U^2 \quad \therefore \quad U = 2f \]
\[ \therefore \quad L = \frac{3 \lambda}{2} \quad \therefore \quad \lambda = \frac{2}{3} L \]
\[ T = \mu \left( \frac{2Lf}{3} \right)^2 = \frac{4}{9} \mu L^2 f^2 \]
\[ \therefore \quad U = \frac{2Lf}{3} \quad \mu = m/L \]
\[ \therefore \quad \frac{4}{9} \frac{m}{L} L^2 f^2 = \frac{4}{9} mL f^2 = \frac{4}{9} \frac{m} {L} \frac{(2\times10^{-3} \text{ kg})(3 \text{ m})}{(60 \text{ Hz})^2} \]
\[ = 9.6 \text{ N} \]

\[ b) \quad U = 2f = \frac{2Lf}{3} = \frac{2}{3}(3 \text{ m})(60 \text{ Hz}) = 120 \text{ m/s} \]

\[ c) \quad \frac{L}{2} \quad \lambda = 2L \]

\[ \therefore \quad f = \frac{U}{\lambda} = \frac{U}{2L} = \frac{120 \text{ m/s}}{2(3 \text{ m})} = 20 \text{ Hz} \]

\[ \Omega \quad \text{frequency in part a) is 3rd harmonic} = \omega_3 \]
\[ \therefore \quad \text{fundamental} \quad f_1 = \frac{\omega_3}{3} = \frac{60 \text{ Hz}}{3} = 20 \text{ Hz} \]
Problem 8.

Captain Jean-Luc Picard is standing on the surface of the previously unknown planet Xantac. He knows that Xantac has a radius of 2000 km, but he does not know its mass. To determine the planet’s mass, he suspends a 0.1 kg yo-yo from a massless string, and makes it into a pendulum with a length of 1/3 of a meter. He measures the period of this pendulum to be 2.1 seconds.

a) What is the acceleration due to gravity at Xantac’s surface?

b) What is Xantac’s mass?

c) Picard observes that Xantac has a moon which orbits the planet once every 12 hours. He recalls having studied Kepler in Starfleet Academy. How far away is that moon from Xantac?

\[ T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow \frac{g}{L} = \frac{(2\pi)^2}{T^2} \Rightarrow g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2 (\frac{1}{3} \text{ m})}{(2.1 \text{ s})^2} = 3 \text{ m/s}^2 \]

\[ \text{Weight: } mg = \frac{GMm}{R^2} \Rightarrow M = \frac{mgR^2}{G} = \frac{(3 \text{ m/s}^2)(2 \times 10^6 \text{ m})}{6.67 \times 10^{-11} \text{ Nm}^2/\text{Kg}^2} = 1.80 \times 10^{23} \text{ Kg} \]

\[ \text{Kepler's 3rd law: } T^2 = \frac{4\pi^2 R^3}{GM} \]

\[ R = \left( \frac{T^2 GM}{4\pi^2} \right)^{\frac{1}{3}} = \left( \frac{(12 \text{ hr} \times 3600 \text{ s})}{2 \pi} \left( \frac{6.67 \times 10^{-11} \text{ Nm}^2/\text{Kg}^2}{1.80 \times 10^{23} \text{ Kg}} \right) \right)^{\frac{1}{3}} \]

\[ = 8.28 \times 10^6 \text{ m} = 8280 \text{ Km} \]