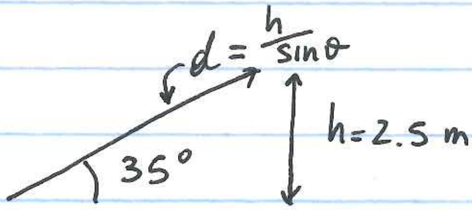
①

A 70 kg skier with initial speed $v_i = 12\ \frac{m}{s}$ coasts up a 2.5 m high rise.
Find final speed at the top if the coefficient of friction, μ_k, equals 0.08.



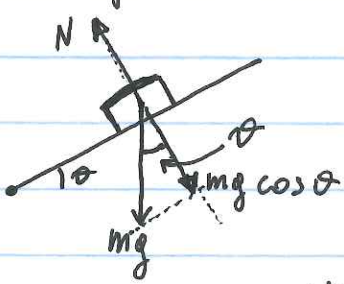
① Forget friction

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = PE_i - PE_f = 0 - mgh$$

$\times \frac{2}{m}$

$$v_f^2 = v_i^2 - 2gh = 95\ \frac{m^2}{s^2} \qquad v_f = 9.75\ \frac{m}{s}$$

② Include friction



$$F_{friction} = N \cdot \mu_k$$

$$N = mg\cos\theta \quad \text{(Normal force)}$$

$$W_{friction} = -mg\cos\theta\, \mu_k \cdot d$$

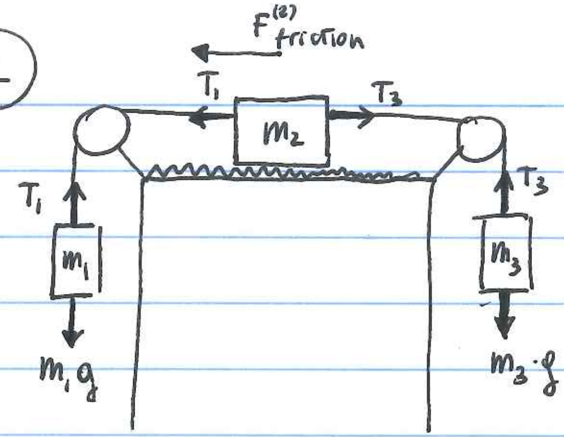
$$= -mg\,\mu_k \cdot h \cdot \text{coten}\,\theta$$

$$KE_f - KE_i = PE_i - PE_f + W_{friction}$$

$\times \frac{2}{m}$

$$v_f^2 = \underbrace{v_i^2 - 2gh}_{95\ \frac{m^2}{s^2}} \underbrace{- 2 g \mu_k h\, \text{coten}(\theta)}_{-5.60\ \frac{m^2}{s^2}} = 89.4\ \frac{m^2}{s^2}; \quad v_f = 9.45\ \frac{m}{s}$$

(2)



$m_1 = 3.4$ kg
$m_2 = 14$ kg
$m_3 = 17$ kg

$F^{(2)}_{friction} = 32N$

Assume the system initially at rest. What's the speed (after release) of m_3 after it has dropped by 3 m ?

- All work done by Tensions cancels out!
- Write Energy conservation for 3 bodies

$$KE_f - KE_i = \frac{1}{2}(m_1 + m_2 + m_3)\, v_f^2 - 0$$

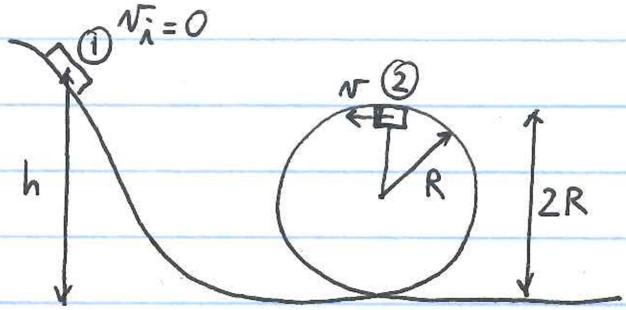
$$\begin{cases} PE_i^{(1)} - PE_f^{(1)} = -m_1 g h \\ PE_i^{(2)} - PE_f^{(2)} = 0 \\ PE_i^{(3)} - PE_f^{(3)} = m_3 g h \end{cases}$$

$$KE_f - KE_i = PE_i^{tot} - PE_f^{tot} + W_{friction}$$

$$\frac{1}{2}(m_1 + m_2 + m_3)\, v^2 = -m_1 g h + m_3 g h - \overbrace{F_{friction}}^{32N} h$$

$v = \ldots\ldots\ldots$ (solve Eq. for v!)

③



What's minimal "h" such that the object will be able to complete the circle?

- Find velocity at ② by using Energy conservation

$$\begin{cases} PE_i - PE_f = mgh - mg\,2R \\ KE_f - KE_i = \frac{1}{2} m v^2 - 0 \end{cases}$$

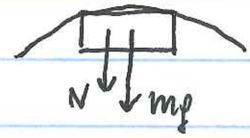
$$KE_f - KE_i = PE_i - PE_f$$

$\times \frac{2}{m}$

$$v^2 = 2gh - 4gR \quad (\star)$$

- To stay "attached" the normal force must be greater than 0.

Both forces, mg and N, point down



$$m\frac{v^2}{R} = mg + N \Rightarrow N = \cancel{m}\frac{v^2}{R} - \cancel{m}g \geq 0$$

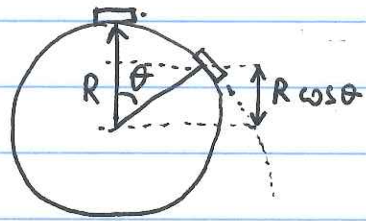
a_c

condition to stay attached

By using $(\star)$, we get:

$$\frac{2\cancel{g}h}{R} - 4\cancel{g} - \cancel{g} \geq 0 \qquad \boxed{h \geq \frac{5}{2}R}$$

(4)



Starting from rest, at what angle does the penny leave the ball? (assume no friction)

- Use conservation of energy to find the velocity as a function of θ

$PE_i = mg2R \qquad PE(\theta) = mgR + mgR\cos\theta$

$KE_i = 0 \qquad KE(\theta) = ?$

$KE(\theta) - KE_i = PE_i - PE(\theta)$

$\frac{1}{2}\cancel{m}v^2 = \cancel{m}g2R - \cancel{m}gR - \cancel{m}gR\cos\theta$

(☆) $v^2 = 2gR - 2gR\cos\theta \Leftarrow$ velocity^2 as a function of angle θ

- For the penny to stay attached, there must be a normal force $\Rightarrow$ write 2nd Law

a_c: the penny moves on a circle as it slides on ball

$m\frac{v^2}{R} =$ Sum of forces directed toward the center $\stackrel{!}{=} mg\cos\theta - N$

use (☆)

$2mg - 2mg\cos\theta = mg\cos\theta - N$

(Turn page)

(continuation of problem 4)

$$2\cancel{mg} - 2\cancel{mg}\cos\theta - \cancel{mg}\cos\theta = -N < 0$$

condition for penny to stay attached

$$2 - 3\cos\theta \leq 0$$

$$\cancel{\cos\theta \leq \frac{3}{2}}$$

$$\cos\theta \geq \frac{2}{3}$$

$\theta \leq 48.2°$ condition for the penny to stay attached directly in terms of angle.