Effects of Interactions I Gnd State

$N = 1000$ $\Psi(x_1\, y_1\, z_1 \cdots x_{100}\, y_{1000}\, z_{100})$

we have a wave defined in a 3000 dimensional space.

To tabulate it, with 10 pts / dimension, we need 10^{3000} points.

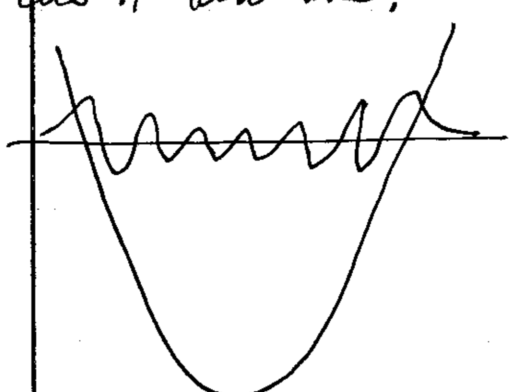
For 1 bit numbers 10^{3000} bits

8 bit " 10^{3001} bits

100 bit " 10^{3002} bits.

$$\sum_i \left\{ -\frac{\hbar^2}{2m}\nabla_{r_i}^2 + V_{trap}(x_i) + \frac{1}{2}\sum_{j\neq i} V_{ij}(|\vec{r}_i - \vec{r}_j|) \right\} \Psi = \begin{cases} i\hbar\, \partial\Psi/\partial t & \\ E\,\Psi & \text{stat states} \end{cases}$$

What does it look like?



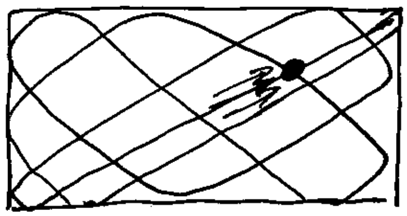
• [redacted] Consider it as a fn of x_1 with all other x_i, y_i, z_i fixed

Traveling wave in both directions

For any given state in the trap (harmonic) there is a well defined E_x E_y E_z

For each x there is a classically corresponding momentum $\frac{p(x)^2}{2m} + \frac{k_x}{2} x^2 = E_x$

These momenta give a semiclassical approx to the wave fn.

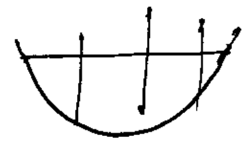


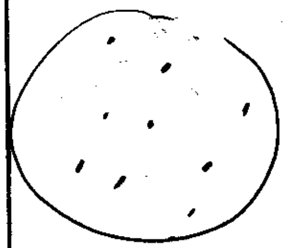
At any point there is a corresponding vector momentum (actually 4 possibilities)

Now suppose that there happens to be another particle at that point. Then the wave fn near there actually looks like an incoming p.w. + outgoing spherical wave

There's (say) 10^3 particles in the enclosure. Near each there is such a scattering process.

Analogy: As a function of $x_1 y_1$ (holding $z_1 = 0$)
$\psi(x_1 y_1 \; z_1 = 0 \; ; \quad x_2 y_2 z_2 \cdots x_3 y_3 \; z_3 \cdots)$

Imagine the surface of water in a bowl, 
with lots of small scatterers.
What are the standing waves in this system?



Approx: Ψ is $\mathcal{S}\left[\phi_1(\vec{r}_1) \cdots \phi_N(\vec{r}_N)\right] = \phi(\vec{r}_1) \cdots \phi(\vec{r}_N)$

What is the <u>best</u> wave fn of this form?

lowest energy

Hartree

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{trap}(r)\right]\phi_1(\vec{r}_1)$$

$$+(N-1)\int V(\vec{r}-\vec{r}\,')\,|\phi(r')|^2 d^3r' \quad \phi_1(\vec{r}_1) \quad = \epsilon\, \phi_1(\vec{r}_1)$$

(proby)
density of particles at r'

interaction of particle at r with one at r'

avg p.e. of interaction of particle 1
with all others.

proof: δ-process.

Approx: $V(r - r')$ is short range compared to the [redacted] range over [redacted] which $\phi(r)$ varies significantly.

$$V(r-r') \simeq g\, \delta(\vec{r}-\vec{r}\,')$$ ★ Caution here.

$$\left[\frac{\hbar^2}{2m} \nabla_r^2 + V_{trap} \right] \phi(r) + Ng|\phi(r)|^2 \phi(r) = \epsilon\, \phi(r)$$

Stationary Nonlinear Sch. Eq. GP Gross Pitaevskii

$g < 0$ "attractive"

$g > 0$ "repulsive"

★ This approx makes sense if we claim that the range of the interatomic PE is small compared to the wavelength of $\phi(r)$. [redacted]

That is true for the asymptotic wavelength ($|\vec{r}_i - \vec{r}_j|$ large), but it is NOT true when the particles are close together.

Two Remarkable Results:

(1) $g = 4\pi \hbar^2 a/m$ a = scat. length

(2) This eq (GP) becomes an exact description of the one-particle density matrix in a certain limit.

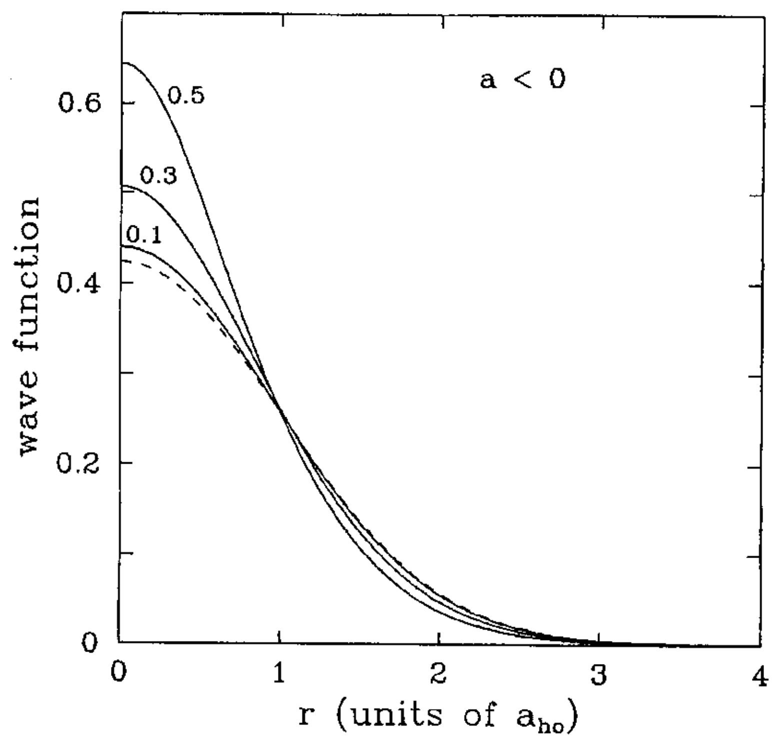


FIG. 8. Condensate wave function, at $T=0$, obtained by solving numerically the stationary GP Eq. (39) in a spherical trap and with attractive interaction among the atoms ($a<0$). The three solid lines correspond to $N|a|/a_{\rm ho}=0.1,0.3,0.5$. The dashed line is the prediction for the ideal gas. Here the radius r is in units of the oscillator length $a_{\rm ho}$ and we plot $(a_{\rm ho}^3/N)^{1/2}\phi(r)$, so that the curves are normalized to 1 [see also Eq. (40)].

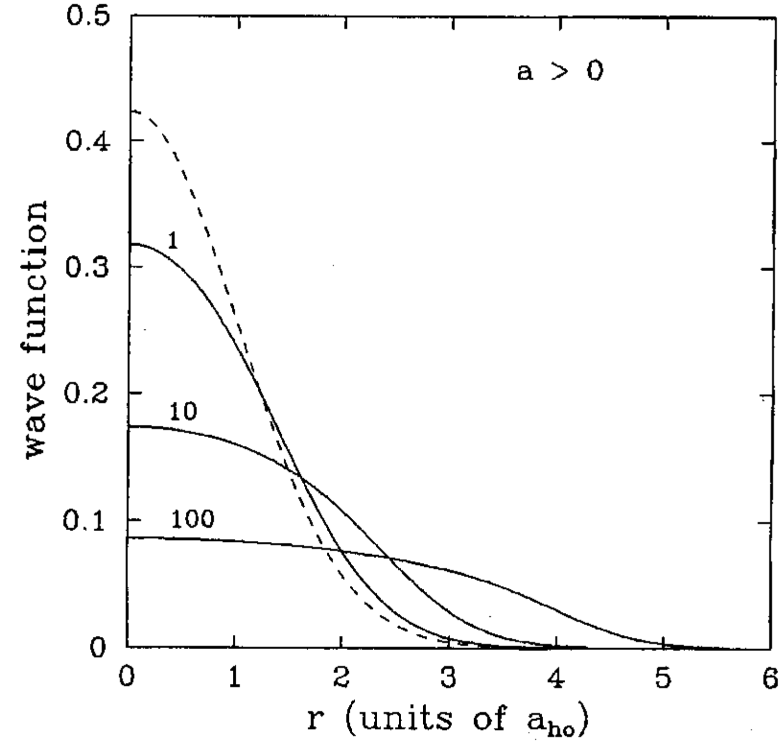


FIG. 9. Same as in Fig. 8, but for repulsive interaction ($a>0$) and $Na/a_{\rm ho}=1,10,100$.