Now we have

$$\log \mathcal{Z} = \epsilon = 0 \text{ term} + \text{all others}$$

$$= \frac{1}{1-z} + \sum_{\substack{k_x k_y k_z \\ \text{not all zero}}} \text{[redacted]}\left[1 - z e^{-\beta\,\text{[redacted]}(k_x\omega_x + k_y\omega_y + k_z\omega_z)}\right]^{-1}$$

$$\langle N \rangle = \frac{z}{1-z} + \sum_{\substack{k_x k_y k_z \\ \text{not all zero}}} \frac{z e^{-\beta(\ \epsilon_{\vec{k}}\ )}}{1 - z e^{-\beta(\ \epsilon_{\vec{k}}\ )}}$$

Finally:

$\epsilon_k = \dfrac{\hbar^2 k^2}{2\mu}$ free particles $\qquad$ $\epsilon_k = \hbar(\omega_x k_x + \omega_y k_y + \omega_z k_z)$ particles in traps

Lo density, hiT, $z \to 0$
Hi density, loT, $z \to 1$.

Up to here, everything is exact, for noninteracting particles.

<u>APPROX</u> $\quad \dfrac{\hbar\omega}{kT} \ll 1$

$$\sum_{\substack{k_x k_y k_z \\ \text{not all zero}}} F(k_x k_y k_z) \longrightarrow \int_0^\infty F(k_x k_y k_z)\, dk_x dk_y dk_z$$

not obvious that we get good results by taking this limit and including the zeroth term separately.

Also not obvious: $kT_C \sim N^{1/3}\hbar\omega$. $\therefore$ We can easily have $\hbar\omega/kT \sim 1/N^{1/3} \ll 1$ in the range of interest.

$$N - N_0 = \int_0^\infty\int_0^\infty\int_0^\infty \frac{dk_x\, dk_y\, dk_z}{\left[\frac{1}{z}e^{+\beta\hbar(\omega_x k_x + \omega_y k_y + \omega_z k_z)} - 1\right]}$$

Look near $z \sim 1$.

(To test approx's, compare with numerical summation)

Change vbles $\bar{k}_x = \frac{\hbar\omega_x}{kT} k_x$

[redacted] $\frac{kT}{\hbar\omega_x}\bar{k}_x = dk_x$

[redacted]

$$(N - N_0) = \left(\frac{kT}{\hbar\omega_{gm}}\right)^3 \int_0^\infty \frac{d\bar{k}_x\, d\bar{k}_y\, d\bar{k}_z}{\text{[redacted]}\left[e^{+(\bar{k}_x + \bar{k}_y + \bar{k}_z)} - 1\right]}$$

$$= \left(\frac{kT}{\hbar\omega_{gm}}\right)^3 \zeta(3)$$

Riemann Zeta Fn

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty \frac{x^{s-1}}{e^x + 1}\, dx$$

$$\omega_{gm} = (\omega_x \omega_y \omega_z)^{1/3}$$

Q: At what T is $N_0 \sim 0$?

$$\frac{\hbar\omega_{gm} N^{1/3}}{[\zeta(3)]^{1/3}} = kT_C$$

$$.94\, \hbar\omega_{gm} N^{1/3} = kT_c$$

[redacted]

I 12

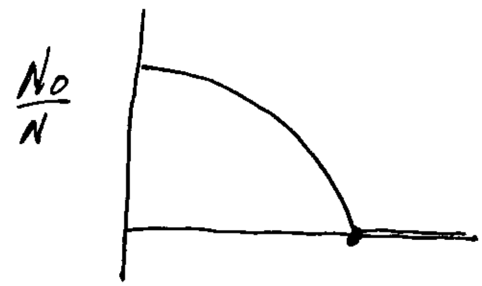
$$N - N_0 \sim (kT)^3$$

As $T \to 0$, all particles are in state 0, $N - N_0 \to 0$.
At what T is $N - N_0 \sim N$? We have $N \sim \left(\frac{kT}{\hbar\omega_{gm}}\right)^3$

"Thermodynamic Limit": $N \to \infty$
$\omega_{gm} \to 0$ $\quad N\,\omega_{gm}^3 \sim$ constant.

$$1 - \frac{N_0}{N} = \left(\frac{kT}{\hbar\omega_{gm}}\right)^3 \zeta$$

or $$\frac{N_0}{N} = \left[1 - \left(\frac{kT}{\hbar\omega}\right)^3 \zeta\right]$$



[P 8 does not exist]

(p 8 does not exist)

It is striking that we get the condensation temp, without doing all that -- it comes out trivially.

(and the behavior of N near it)

— — — —

[redacted]

Thus we have two important energy scales:

$\hbar\omega_{gm}$ and $kT_c \sim N^{1/3}\hbar\omega_{gm} \times [.94]$

Size scale:

$a_{x} = \left(\frac{\hbar}{m\omega_x}\right)^{1/2}$

$a_{gm} = \left(\frac{\hbar}{m\omega_{gm}}\right)^{1/2}$

$\frac{1}{2}kT_c = \frac{1}{2}m\omega^2 b^2$

$\frac{N^{1/3}\hbar\omega}{m\omega^2} = b^2$

$\left(\frac{N^{1/3}\hbar}{m\omega}\right) = b^2$

$N^{1/6}\left(\frac{\hbar}{m\omega}\right)^{1/2} = b$

The Bose condensate is smaller than the thermal gas by a factor of $N^{1/6}$.

Effects of finite N.

Methods: Evaluate $\langle N \rangle (z,T)$, $Q(z,T)$ numerically

or fix N, and go to $Q_N(T)$.

Then derive all thermal properties.

Result:

1) For finite N there are no phase transitions. (Yang + Lee)

2) Since the universe appears to be finite, there are no phase transitions in the universe.

3) However, as N gets large, functions can <u>approach</u> discontinuous fns.

4) Numerical result (Ketterle 1996) Fig 6

Effects of Dimension

2D: $kT = \hbar\omega_{gm} N^{1/2} / [\zeta(2)]^{1/2}$

[redacted]

1D: $kT = \hbar\omega \frac{N}{\ln(2N)}$

"Thermodynamic limit" $N\omega \sim$ constant

Then $kT \to 0$