Now we have
\[
\log 2 = \text{6-0 term + all others}
\]
\[
\frac{1}{1 - \beta} + \sum_{\text{not all zero}} \left[1 - \beta - \beta \left( \frac{E_{\infty}}{1 - \beta - \beta \left( \frac{E_{\infty}}{1 - \beta} \right)} \right) \right]^{-1}
\]
\[
\langle N \rangle = \frac{2}{1 - \beta} + \sum_{\text{not all zero}} \left( \frac{E_{\infty}}{1 - \beta} \right)
\]

Finally:
\[
E_n = \frac{\hbar^2 k^2}{2\mu} \quad \text{free particles}
\]
\[
E_n = t_0 (\omega x \cos \theta y + \text{tweeze}) \quad \text{particles in traps}
\]

Low density, \( \lambda T, \beta \to 0 \)

High density, \( \lambda T, \beta \to 1 \).

Up to here, everything is exact, for noninteracting particles.

Approx \( \frac{\hbar \omega}{\lambda T} \ll 1 \)

\[
\sum_{\text{not all zero}} F(\omega \lambda x, h\theta y) \to \int_0^\infty F(\omega \lambda x, h\theta y) \, dh \, d\theta \, dx
\]

It is not obvious that we get good results by taking this limit and including the second term separately.
Also not obvious: \( kTc \approx N^{4/3} \mu^2 \). \( \ldots \) We can easily have \( \mu^2/N^{1/3} \ll 1 \) in the range of interest.

\[
N - N_0 = \int_0^\infty \int_0^\infty \int_0^\infty \frac{d\xi d\eta d\kappa}{E + \frac{1}{2} (\xi^2 + \eta^2 + \kappa^2) - 1}
\]

Look near \( z \approx 1 \).

(To test approx., compare with numerical summation)

\[
\text{Change variables } \bar{\xi}_x = \frac{\xi}{\mu} \, \xi_x
\]

\[
\frac{\mu T}{\mu \omega} \bar{\xi}_x = d\xi
\]

\[
(N - N_0) = \left( \frac{kT}{\mu \omega} \right)^3 \int_0^\infty \frac{d\bar{\xi} d\bar{\eta} d\bar{\kappa}}{e + (\bar{\xi}^2 + \bar{\eta}^2 + \bar{\kappa}^2) - 1}
\]

\[
= \left( \frac{kT}{\mu \omega} \right)^3 \zeta(3)
\]

\[
\omega = \sqrt{w_x w_y w_z}
\]

\[
\omega = \left( w_x w_y w_z \right)^{1/3}
\]

\[
\zeta(3) = \frac{1}{\Gamma(3)} \int_0^\infty \frac{x^{5/3}}{e^x + 1} \, dx
\]

Q: At what \( T \) is \( N_0 \approx 0 \)?

\[
\frac{T \omega \mu}{\mu \zeta(3)} N^{4/3} = kT_c
\]

\[
1.94 \, \frac{T \omega \mu}{\mu \zeta(3)} N^{4/3} = kT_c
\]
As $T \to 0$, all particles are in state 0, $N - N_0 \to 0$. At what $T$ is $N - N_0 \sim N$? We have $N \sim \left(\frac{kT}{\hbar \nu \omega m}\right)^3$.

"Thermodynamic Limit": $N \to \infty$ as $\nu m \to 0$. $N \omega g m^3 \approx \text{constant}$.

\[ 1 - \frac{N_0}{N} = \left(\frac{kT}{\hbar \nu \omega m}\right)^3 \leq \alpha \]

\[ \frac{N_0}{N} = \left[ 1 - \left(\frac{kT}{\hbar \nu \omega m}\right)^3 \right] \leq \frac{N_0}{N} \]

[?? \& does not exist??]
It is striking that we get the condensate temp without doing all that -- it comes out trivially, (and the behavior of \( N \) near it does not exist)

Thus we have two important energy scales:

\[
t_i w_{qm} \quad \text{and} \quad k_T / \sim N^{1/3} \quad t_i w_{qm} \times 694.7
\]

See scale:

\[
\alpha_q = (\frac{t_i}{m w})^{1/2}
\]

\[
\alpha_{qm} = (\frac{t_i}{m w_{qm}})^{1/2}
\]

\[
\frac{1}{2} k_T = \frac{1}{2} m w^2 b^2
\]

\[
\frac{N^{1/3} t_i}{m w} = b^2
\]

\[
\left( \frac{N^{1/3} t_i}{m w} \right)^{1/2} = b
\]

The Bose condensate is smaller than the thermal gas by a factor of \( N^{1/6} \).
**Effects of finite \( N \).**

Methods: Evaluate \( \langle N \chi_3, T \rangle \) numerically or fix \( N \) and go to \( Q_N(T) \).

Then derive all thermal properties.

Result:

1. For finite \( N \) there are no phase transitions. (Yang & Lee)
2. Since the universe appears to be finite, there are no phase transitions in the universe.
3. However, as \( N \) gets large, function can approach discontinuities.
4. Numerical result (Ketchen, 1996) \( \nabla \)

**Effects of Dimension**

2D: \[ kT = \frac{\hbar}{\omega_m} N^{\frac{3}{2}} / \sqrt{5(2)} \]

1D: \[ kT = \frac{\hbar}{\omega} \frac{N}{\Lambda(N)} \]

"Thermodynamic limit" \( N \omega \sim \text{constant} \)

Then \( kT \to 0 \)