Outline of Lectures

0. Fundamentals of Stat Mech of Ideal Gas
1. Statistical Mechanics of Bose Gases
2. Effects of Interactions: NLS — ground state
3. Other applications of NLS — self focusing of light
4. NLS -- dynamics
5. Effects of interactions on condensation.

Starting point:

\[ Z(V,T) = \frac{\sum z^n}{N} \Omega_n(V,T) \]
\[ \Omega_n(V,T) = \sum_{\text{quantum states}} e^{-E_n(V,T)/kT} \]
\[ N = \frac{\alpha}{\beta^2} \log 2 \]
Grundlage:
  - Stat Phys connects micro to macro
    - micro nontrivial
    - macro "connection"

Macroscopic World

There are "thermodynamic states"

One component, one phase system
  what it is
  how much of it
  \( V, T \)

02 State \((V, T)\). 0.2 substances in thermal contact came to same \( T \).
  Can add heat to system & make it do work.

1. Heat is a form of energy and energy is conserved.
   Internal energy \( U \) is a fn of state.
   \( dU = dQ - dW \).

2. There is another function of thermo state called entropy
   \( S(V, T) \)

  only changes can be measured

  to find the change
  - write down the change of state
    \( \Delta U = R_h (V, T) - R_k (V, T) \)
  - find a reversible path that accomplishes that change of state
  - for each isothermal step in that path
    \( \Delta S = \frac{dQ}{T} \)
  - if \( T \) changes continuously
    \( \Delta S = \int \frac{dQ}{T} \)
**First**

To a component one phase system doing only work of expansion and doing that reversibly:

\[ dE = dq - dw = Tds - PdV \]

\[ T = \left( \frac{\partial E}{\partial S} \right)_{V} \]

**Second**

From \( E \) and \( S \) other quantities can be derived:

\[ A = U - TS \]

For a reversible process in which the only work is expansion:

\[ dA = dU - Tds - SdT \]

\[ = dq - dw \]

\[ = Tds - PdV - Tds - SdT \]

\[ dA = -PdV - SdT \]

*If we known \( A(V,S,T) \), we can get all other thermal properties of state \( \epsilon \) all properties involving heat*
Microscopic World

System consists of atoms + molecules that obey Sch. Eq

\[ \Psi(\mathbf{r}_1, \ldots, \mathbf{r}_N, t) \]
\[ \frac{\partial \Psi}{\partial t} = \mathbf{H} \Psi \]
\[ \mathbf{H} = \sum -\frac{\hbar^2}{2m_i} \nabla_i^2 \]
\[ + \sum_i V_i(\mathbf{r}_i) \]
\[ + \sum_{ij} V_{ij}(\mathbf{r}_i - \mathbf{r}_j) \]
\[ \text{etc.} \]

We cannot describe any state in either sense.

Microcanonical ensemble: Specify E. Average over all possibilities consistent with that energy.

To find macroscopic properties we average over all possibilities. Speak Classically.

- Internal energy: \[ U = \sum E \text{ energies of all particles in system} \]

- Pressure: Find the force on the wall exerted by every particle, averaging over all possible positions and velocities of every particle consistent with a fixed total energy. Divide by area of walls.
Classically

\[ S = k \log T(E, V) \]

\[ T = \text{volume of energy shell in phase space} \]

Then undo to get \( E(S, V) \)

Quantumly

Around the specified energy make a little shell

\[ T = \left[ \frac{\text{number of quantum states in that energy shell}}{E} \right] \]

Claim: for a system with lots of particles, there exists a broad range of \( E \) in which we get consistent results.
E: large enough that there are many @ states small enough that we do not see a change in macroscopic system.

E \ll \bar{E}
Canonical Ensemble:

Instead of averaging over all states at fixed energy, average over all states at all energies, with a weighting factor $e^{-\beta E}$.

Partition Fn

Classical

$$Q_N(V, N, T) = \frac{\int e^{-\beta H} \, dq \, dp}{\int N^N \, N!}$$

Quantum

$$Q_N(V, N, T) = \sum_{\text{states}} e^{-\beta E}$$

Postulate

$$\rightarrow \beta \sim \frac{A}{N V S T}$$

Proposition: $\beta$ has properties of the macroscopic temp

$$\beta = \frac{1}{kT}$$

$$A = -kT \log Q_N(V, S, T)$$
Grand Canonical Ensemble
Model of a system interacting with heat bath

| System | Surroundings | Heat bath |

Average also over all imaginary number of particles in the system with a weighting factor $e^{\frac{\mu}{kT}}$.

$z = e^{\frac{\mu}{kT}}$

$GPF$

$Q(z, V, T) = \sum_{n} z^{n} Q_{n}(V, T)$

Propositions:

$A(z, V, T) = + kT \log z - n kT \log z$

$\langle N \rangle (V, T) = z^{\frac{\beta}{2}} \log z$

Scheme: eliminate $z$ between these 2 eqns to get $A(N, V, T)$.

Then all thermo properties follow.
For any operator that is defined in $\mathcal{Q}$

$$\langle \Theta \rangle = \sum_{\text{all states at all } n} \langle \text{state} | (\Theta e^{-i\mathcal{H}}) | \text{state} \rangle e^{E_n}$$