THEORETICAL FORMULATIONS AND PHENOMENA

Quantum Electrodynamics

Quantized particles, quantized fields

- spontaneous emission
- photons

    photon statistics, photon bunching, anti-bunching, squeezed states of light, cavity QED
- sometimes simplest description, sometimes very complex. Rarely necessary

Semiclassical Electrodynamics

~~Quantum particles~~, classical fields

- stimulated absorption and emission of light

Phenomenological Electrodynamics

Macroscopic properties of matter, Classical fields

- polarization of materials,

    linear + nonlinear response, frequency doubling, higher harmonic generation, 4-wave mixing, optical phase conjugation, self-focusing, solitons

PART I Maxwell's Equations in Vacuum

A. Effect of electromagnetic fields on charged particles
B. Effect of charged particles on fields in vacuum

Units: MKS (until we change to something else)
Charge - Coulomb
Magnetic Field - Tesla

A. Effect of electromagnetic fields on charged particles

$$F = q(\mathcal{E} + \vec{v} \times \mathcal{B})$$

(Force on a current:

$$F = i\,\vec{l} \times \mathcal{B}\ )$$

B. Effect of charges on fields: Maxwell's Equations in vacuum.

Coulomb-Gauss law $\nabla \cdot \mathcal{E} = \frac{1}{\varepsilon_0}\rho$

No magnetic monopoles $\nabla \cdot \mathcal{B} = 0$

Faraday's law $\nabla \times \mathcal{E} + \frac{\partial \mathcal{B}}{\partial t} = 0$
(changing magnetic field → electric field)

Ampere-Maxwell law $\nabla \times \mathcal{B} - \mu_0 \varepsilon_0 \frac{\partial \mathcal{E}}{\partial t} = \mu_0 \mathcal{J}$
current, or changing electric field produces a magnetic field

Heaviside-Lorentz Units

$\mu_0 = \epsilon_0$

$$\nabla \cdot \mathcal{E} = \rho$$
$$\nabla \cdot \mathcal{B} = 0$$
$$\nabla \times \mathcal{E} + \frac{1}{c}\frac{\partial \mathcal{B}}{\partial t} = 0$$
$$\nabla \times \mathcal{B} - \frac{1}{c}\frac{\partial \mathcal{E}}{\partial t} = \frac{1}{c}\mathcal{J}$$

$$F = q\left[\mathcal{E} + \frac{v}{c} \times \mathcal{B}\right]$$

$\mathcal{E}$ and $\mathcal{B}$ are the "real" electric and magnetic fields that are produced by charges and that act upon charges.

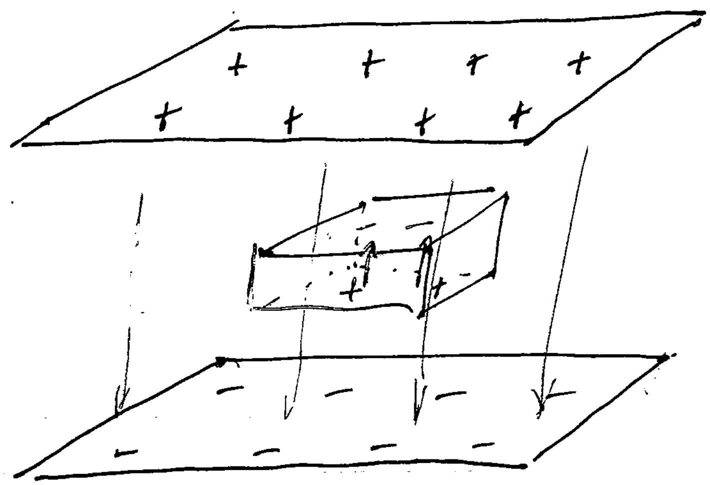
Fields in vacuum: $\rho = 0$ $\quad \mathcal{J} = 0$

$$\nabla \times (\nabla \times \mathcal{E}) + \frac{1}{c}\frac{\partial}{\partial t}(\nabla \times \mathcal{B}) = 0$$
$$\nabla(\underbrace{\nabla \cdot \mathcal{E}}_{=0}) - \nabla^2 \mathcal{E} + \frac{1}{c^2}\frac{\partial^2 \mathcal{E}}{\partial t^2} = 0$$
$$\left(-\nabla^2 + \frac{\partial^2}{\partial t^2}\right)\mathcal{E} = 0$$

Classical Wave Eq.

Electromagnetic Fields in Materials

Consider the effect of a static electric field on a material



$\vec{\mathbb{P}}$ = dipole moment / Volume of sample

= − electric field caused by induced charges

$\mathbb{D}$ = original electric field resulting from applied charges.

$\mathbb{E}$ = net electric field in sample.

(averaged over a ~~the~~ small volume)

Here we must be more careful.

- The actual electric field, $\mathcal{E}$, at individual points in the sample is extremely complex.

Eg around each nucleus $\mathcal{E}$ points away from nucleus, toward any localized electron.

- The dipoles are oscillating in the field; initially randomly oriented; gradually settling to equilibrium, they line up

Around each dipole $\mathcal{E}$ is complex.

$$\nabla \cdot \mathcal{E} = \rho \text{ all charges.}$$

We must average over a suf. large volume of sample that we can talk about an average, smoothly varying electric field.

Divide $\rho_{\text{all charges}} = \rho_{\text{applied}} + \rho_{\text{induced}}$

Avg over a region at least 10 nm on each side. A solid contains ~ 10^6 atoms. For a gas we avg over a larger region. We want adjacent regions to be suf alike that we have slowly-varying average quantities.

$$\nabla \cdot \mathcal{E} = \rho_{all}$$

divide into parts

$$= \rho_{app} + \rho_{ind}$$

avg over a region

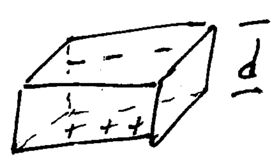
$$\nabla \cdot \underbrace{\bar{\mathcal{E}}}_{\mathbb{E}} = \bar{\rho}_{app} + \bar{\rho}_{ind}$$

DEFINE $\nabla \cdot (\mathbb{D} - \mathbb{P}) = \bar{\rho}_{app} + \bar{\rho}_{ind}$

$$\nabla \cdot \mathbb{D} = \bar{\rho}_{app}$$

$$\nabla \cdot \mathbb{P} = -\bar{\rho}_{\text{induced}}$$

To show: $\mathbb{P} = \text{dpm} / \text{Vol}$



dpm = $q \cdot d$
def: from − to +

$$\int \nabla \cdot \mathbb{P} = -\rho_{ind}$$

$$(\mathbb{P} \cdot n)\, dS = -\int \rho_{ind}\, dV$$

$$-\mathbb{P} A = -q$$

$$\mathbb{P} = \frac{q}{A} = \frac{q\,d}{A\,d} = \frac{q\,d}{V}$$

More generally: applied fields may vary in space and time. Continue to avg over small volumes.

$$\nabla\cdot\mathbb{D}(\mathbf{r},t) = \bar{\rho}_{applied}(\mathbf{r},t)$$

$$\nabla\cdot\mathbb{P}(\mathbf{r},t) = -\bar{\rho}_{induced}(\mathbf{r},t)$$

$$\mathbb{D} = \mathbb{E} + \mathbb{P}$$

$$\nabla\cdot\mathcal{B} = 0 \quad \Rightarrow \quad \nabla\cdot\bar{\mathcal{B}} = 0 \Leftrightarrow \nabla\cdot\mathbb{B} = 0$$

$$\nabla\times\mathcal{E} + \frac{1}{c}\frac{\partial\mathcal{B}}{\partial t} = 0$$

$$\nabla\times\mathbb{E} + \frac{1}{c}\frac{\partial\mathbb{B}}{\partial t} = 0$$

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$$\nabla\times\mathcal{B} - \frac{1}{c}\frac{\partial\mathcal{E}}{\partial t} = \frac{1}{c}\mathbb{J} = \frac{1}{c}(\mathbb{J}_{app} + \mathbb{J}_{ind})$$

$$\underbrace{\nabla\times\bar{\mathcal{B}}}_{(\mathbb{H}+\mathbb{M})} - \frac{1}{c}\underbrace{\frac{\partial\mathbb{E}}{\partial t}}_{(\mathbb{D}-\mathbb{P})} = \frac{1}{c}(\mathbb{J}_{app} + \mathbb{J}_{ind})$$

$$\nabla\times\mathbb{H} - \frac{1}{c}\frac{\partial\mathbb{D}}{\partial t} = \frac{1}{c}\mathbb{J}_{app}$$

$$\nabla\times\mathbb{M} + \frac{1}{c}\frac{\partial\mathbb{P}}{\partial t} = \frac{1}{c}\mathbb{J}_{ind}$$

We will consider cases in which magnetization of sample is negligible: $\mathbb{M} \sim 0$, $\mathbb{H} = \mathbb{B}$

$$\frac{\partial \mathbb{P}}{\partial t} = \frac{1}{c}\mathbb{J}_{ind}$$

$$\frac{d}{dt}\left(\frac{qd}{V}\right) = \frac{d}{dt}\left(\frac{qd}{Ad}\right) = \frac{i}{A}$$

$$= \frac{d}{dt}\sigma = \frac{i}{A} \qquad \text{(Consistent!)}$$

$$\boxed{\begin{array}{ll} \nabla\cdot\mathbb{D} = \rho_{app} & \nabla\times\mathbb{B} - \frac{1}{c}\frac{\partial\mathbb{D}}{\partial t} = \mathbb{J}_{app} \\ \nabla\times\mathbb{E} + \frac{1}{c}\frac{\partial\mathbb{B}}{\partial t} = 0 & \nabla\cdot\mathbb{B} = 0 \end{array}}$$

Macroscopic Maxwell Eqs. $\quad \mathbb{D} = \mathbb{E} + \mathbb{P}$, $\mathbb{H} = \mathbb{B}$

Wave Eq inside a material $\quad \rho_{app} = 0 \quad \mathbb{J}_{app} = 0$

$$\nabla\times\nabla\times E + \frac{1}{c}\frac{\partial}{\partial t}(\nabla\times\mathbb{B})$$

$$\nabla(\nabla\cdot E) - \nabla^2\mathbb{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\mathbb{E}+\mathbb{P}) = 0$$

$$\boxed{\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbb{E} + \text{grad div}\,\mathbb{E} + \frac{1}{c^2}\frac{\partial^2\mathbb{P}}{\partial t^2} = 0}$$

Response of Materials to Electric Fields

At this point we still know precisely nothing, because we do not know how the material responds.
Given $\mathbb{E}(\mathbf{r}, t)$, the local avg electric field in material,
What is $\mathbb{P}(\mathbf{r}, t)$ the local avg dpm/volume?

Proposition: $\mathbb{P}(\mathbf{r}_{here}, t_{now})$ depends on

$\mathbb{E}(\mathbf{r}_{here}, t_{now})$ and on

$\mathbb{E}(\mathbf{r}_{nearby}, t_{recent\ past})$

Phenomenological Electrodynamics uses Measured or Postulated relationships between $\mathbb{E}(\mathbf{r}', t')$ and $\mathbb{E}(\mathbf{r}, t)$ to describe propagation of EM waves in a material.

Example 1: $\mathbb{P}(\mathbf{r}, t) = \chi \quad \mathbb{E}(\mathbf{r}, t)$ SI H-L

instantaneous local response.

Gaussian Units

$$\left(-\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) \mathbb{E} + \text{grad div } E + \frac{4\pi}{c^2}\frac{\partial^2 \mathbb{P}}{\partial t^2} = 0$$

$$\mathbb{P} = \frac{\chi}{4\pi} \mathbb{E} = \frac{\chi}{4\pi} (\mathbb{D} - 4\pi\mathbb{P})$$

$$\nabla \cdot \mathbb{P} = \nabla \cdot \mathbb{D} = 0$$ no applied charges inside

$$= \nabla \cdot \mathbb{E}$$

$$\left(-\nabla^2 + \frac{1+\chi}{c^2}\frac{\partial^2}{\partial t^2}\right) \mathbb{E} = 0$$

$$n^2 = 1 + \chi$$ n = index of refraction

Speed of light in medium = c/n.

Refraction: ~~##~~ all colors bend the same.

Next things to consider:

Anisotropic response: $P_\alpha = \sum_\beta \chi_{\alpha\beta} E_\beta$

Time-delayed response

$$P_\alpha(t) = \int \chi_{\alpha\beta}(t-t')\, E(t')\, dt'$$

Nonlinear response:

$$\begin{aligned} P_\alpha = {} & \chi^{(1)}_{\alpha\beta} E_\beta \\ & + \chi^{(2)}_{\alpha\beta\gamma} E_\beta E_\gamma \\ & + \chi^{(3)}_{\alpha\beta\gamma\delta} E_\beta E_\gamma E_\delta \\ & + \cdots \end{aligned}$$