Relativistic Mean Fields in the Early Universe

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by

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Abstract

Our goal is to investigate what changes occur in the standard big bang model for the early universe if we include relativistic mean fields to describe the nuclear interactions. We work in the "nuclear regime" located between 10^{-4} and 10^{-1} seconds after time-zero, where the energy scale is between 200 MeV and 10 MeV. This is the interval, in which the universe was very hot, dense, and filled with baryons, mesons, and leptons coupled with a uniform gas of photons. It is in this regime that we will use quantum hadrodynamics (QHD) to study the effects of relativistic mean fields in several weak and nuclear reactions that have been firmly established by the standard big bang model (SBBM). QHD is an effective field theory for the underlying theory of quantum chromodynamics (QCD). We construct a local lagrangian in infinite nuclear matter with constant scalar and vector fields with no spatial dependence. These new fields of QHD bring about changes in thermal properties of the early universe as well as in interaction cross sections as we shall demonstrate. It is not clear what effects, if any, these changes have on the subsequently cooled cosmos.

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Contents

1 Introduction

Thanks to the advancements made in field theory, cosmology and general relativity, today we are able to draw a reliable picture of what the early universe was like almost back to time zero. In fact, this so-called Standard Big Bang Model (SBBM) has become so sturdy that physicists hardly ever discuss events following the inflationary epoch. Of course, no model is perfect and SBBM is being refined as we obtain more data from the skies. We aim to investigate certain particle interactions of this era by employing an improved model of the nuclear physics, which may potentially have observable consequences for the SBBM.

Specifically, we will investigate some of the reactions that occur before (starting from T = 200 MeV) and during big bang nucleosynthesis, which is around 10^{-2} to 1 seconds after time zero and between the energy scales of 50 MeV to 1 MeV [1]-[4]. At this stage, the universe was a hot and dense soup of protons and neutrons (along with pions and electrons coupled to a sea of photons with neutrinos in the background) that was expanding rapidly. The following weak reactions had begun occuring with the start of hadron era around $T \geq 200\,MeV$:

$$n \leftrightarrow p + e^- + \bar{\nu}$$

$$\nu + n \leftrightarrow p + e^-$$

$$e^+ + n \leftrightarrow p + \bar{\nu}$$

The following reactions mostly belong to the nucleosynthesis era since 200 MeV is too hot for atoms to hold together. However the first reaction below is more relevant to our regime.

$$n + p \leftrightarrow d + \gamma$$

$$d + d \rightarrow He^3 + n$$

$$He^3 + n \leftrightarrow H^3 + p$$

$$He^3 + d \leftrightarrow He^4 + n$$

As mentioned above, we are interested in a soup of interacting neutrons and protons, which means that we will be dealing with many-body nucleon-nucleon (NN) interactions. Traditionally, NN interactions are treated as a non-relativistic many-body problem. In this approach, a static two-body potential obtained from fits to two-body scattering and bound-state data is inserted into the many-particle Schrödinger equation, which is solved in some approximation to give energies and wave functions. However, this method is inadequate for a more detailed picture of nuclear interactions.

A more appropriate set of degrees of freedom for nuclear physics are hadrons — the strongly interacting mesons (a quark, antiquark pair) and baryons (confined quark triplets). The two-nucleon potential is strong, short-ranged, and repulsive at short distances. It is also attractive at intermediate distances. It is the exchange of mesons, the quanta of the nuclear force, that is responsible for the strong interaction between two nucleons. The most important exchange mesons are $(J^\pi, T) = \pi(0^-, 1),\ \sigma(0^+, 0),\ \rho(1^-, 1),\ \omega(1^-, 0)$ where J is spin, π is parity and T is isospin [5]. The necessity for a relativistic framework also comes from the fact that many applications in nuclear physics depend on the behavior of nuclear matter under extreme conditions, such as neutron stars and other condensed stellar objects formed by supernovae explosions. It is therefore essential to have a theory that incorporates from the outset the basic principles of quantum mechanics, Lorentz covariance, and special relativity [5].

The only consistent theoretical framework we have for describing such a relativistic, interacting, many-body system is relativistic quantum field theory based on a local, Lorentz-invariant Lagrangian density. In analogy with quantum electrodynamics (QED), we will refer to relativistic quantum field theories based on hadronic degrees of freedom as quantum hadrodynamics (QHD). Recently, it has been discovered that renormalizable theories in QHD are too restrictive and do not provide enough details for certain nuclear interactions (see [6] and [8]). However, we let our initial simple model that we call QHD-I be renormalizable (see [5]). It is also important to note that quantum chromodynamics (QCD) can be represented by an effective field theory formulated in terms of a few hadronic degrees of freedom at low energies and large distances. QHD does that for us since it is a strong-coupled theory of low-energy scales. Whatever this effective field theory may be, it must be dominated by the linear, isoscalar, scalar and vector interactions of QHD-I. Finally, unlike QED, QHD is not perturbative, which is why we use Relativistic Mean Field Theory (RMFT) to simplify the equations. This effective field theory of nuclear interactions has had substantial success in describing the properties of ordinary, terrestrial nuclei [8].

2 Quantum Hadrodynamics (QHD)

We start with the following fields

- A baryon field for neutrons and protons

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

- A neutral Lorentz scalar field ϕ coupled to scalar density $\bar{\psi}\psi$
- A neutral vector field V_μ coupled to the conserved baryon current $i\bar{\psi}\gamma_\mu\psi$

The following metric will be employed for this work

$$x^\mu = x_\mu = (\mathbf{x}, ix_0) = (\mathbf{x}, it) \tag{1}$$

$$a \cdot b = \mathbf{a} \cdot \mathbf{b} - a_0 b_0 \tag{2}$$

In this metric the gamma matrices are hermitian, and satisfy

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu} \tag{3}$$

The choices above are motivated by several considerations. First, these fields provide the smoothest average nuclear interactions and should describe the dominant features of the bulk properties of nuclear matter. Second, in the static limit of infinitely heavy baryons sources (which we will not assume), these exchanges give rise to an effective NN interaction of the form (given in [5])

$$V_{static} = \frac{g_v^2}{4\pi}\frac{e^{-m_v r}}{r} - \frac{g_s^2}{4\pi}\frac{e^{-m_s r}}{r} \tag{4}$$

With the appropriate choices of coupling constants and masses, this potential describes the main features of the NN interaction: a short-range repulsion due to ω exchange, and a long-range attraction due to σ exchange.

2.1 A Simple Model: QHD-I

This model contains the fields mentioned above. The lagrangian density[1] (where $\hbar =$ c $= 1$) is given by [5]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{1}{2}m_v^2 V_\mu^2 - \frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x_\mu}\right)^2 + m_s^2\phi^2\right]$$

$$-\bar{\psi}\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - ig_v V_\mu\right) + (M - g_s\phi)\right]\psi \tag{5}$$

in which $F_{\mu\nu}$ is the vector field tensor[2] as in QED and $\partial_\mu = \partial/\partial x_\mu$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \tag{6}$$

Now, we apply Hamilton's principle of minimum action to obtain the Euler-Lagrange equations (see [12] and [13] for details). Using $(V_\mu,\ \phi,\ \bar{\psi})$ as field variables yields the field equations for QHD-I as given by [5]-[8]

$$\frac{\partial}{\partial x_\nu}F_{\mu\nu} + m_v^2 V_\mu^2 = ig_v\bar{\psi}\gamma_\mu\psi \tag{7}$$

$$\left[\left(\frac{\partial}{\partial x_\mu}\right)^2 - m_s^2\right]\phi = -g_s\bar{\psi}\psi \tag{8}$$

[1]From now on we will refer to it as the lagrangian

[2]V_μ here is the vector potential not volume. Some texts use A_μ to denote it.

$$\left[\gamma_\mu \left(\frac{\partial}{\partial x_\mu} - ig_v V_\mu\right) + (M - g_s\phi)\right]\psi = 0 \tag{9}$$

The first equation looks like the relativistic form of Maxwell's equations in QED with massive quanta and a conserved baryon current as source; $B_\mu = i\bar{\psi}\gamma_\mu\psi$ with $\partial_\mu B_\mu = 0$. The second equation is the Klein-Gordon equation for the scalar field with the baryon scalar density $\bar{\psi}\psi$ as source. And finally, the third equation is the Dirac equation with scalar and vector fields entering in a minimal fashion.

When quantized, Eqs. 7, 8 and 9 become nonlinear quantum field equations whose exact solutions are very complicated. We also expect the coupling constants in these equations to be large so perturbative solutions are not useful. Fortunately, there is an approximate nonperturbative solution that can serve as a starting point for studying the implications of the lagrangian in Eq. 5. This solution becomes increasingly valid as nuclear density increases.

2.2 Mean Field Theory (MFT)

If we decrease the volume of a system with conserved baryon number the baryon density increases, as do the source terms on the right-hand sides of Eqs. 7 and 8. If the sources are large enough, the scalar and vector field operators can be replaced by their expectation values, which then serve as classical, condensed fields in which the baryons move:

$$\phi \rightarrow \langle\phi\rangle \equiv \phi_0, \quad V_\mu \rightarrow \langle V_\mu\rangle \equiv iV_0 \tag{10}$$

We work in uniform, infinite nuclear matter with equal number of protons and neutrons and no net charge in this case, the classical fields above have no spatial or temporal dependence.

With the substitution of the new fields in Eq. 5, the new lagrangian reads

$$\mathcal{L}_{MFT} = \frac{1}{2}m_v^2 V_0^2 - \frac{1}{2}m_s^2\phi_0^2 - \bar{\psi}\left[\gamma_\mu \frac{\partial}{\partial x_\mu} + \gamma_4 g_v V_0 + M^*\right]\psi \tag{11}$$

in which M^* is the *effective mass* of the nucleon and is defined by

$$M^* = M - g_s\phi_0 \tag{12}$$

Substituting the new fields into the Euler - Lagrange equations (Eqs. 7 and 8) and solving for the fields, we obtain

$$\phi_0 = \frac{g_s}{m_s^2}\langle\bar{\psi}\psi\rangle \quad and \quad V_0 = \frac{g_v}{m_v^2}\langle\psi^\dagger\psi\rangle \tag{13}$$

The new Dirac equation looks like

$$\left[\gamma_\mu \frac{\partial}{\partial x_\mu} + \gamma_4 g_v V_0 + M^*\right]\psi(\mathbf{x}, t) = 0 \tag{14}$$

We use the usual normal-mode, plane-wave solution to the Dirac equation of the form $\psi = U(\mathbf{p}, \lambda)e^{i\mathbf{p}\cdot\mathbf{x} - iEt}$, in which $U(\mathbf{p}, \lambda)$ is a 4-component Dirac spinor and λ denotes the spin and isospin index. Once the substitution is complete, we obtain the following energy eigenvalue equation [5]

$$E = g_v V_0 \pm (\mathbf{p}^2 + M^{*2})^{1/2} \tag{15}$$

which can be compared to the corresponding solution for a free Dirac particle[3], $E = \pm(p^2 + M^2)^{1/2}$. We will refer to the eigenvalues in Eq. 15 as $E_\pm$. Overall, we see that the condensed scalar field ϕ_0 shifts the mass of the baryons whereas the condensed vector field V_0 shifts the energy (or the frequency) of the solutions. This causes the kinematics of the system to change, as well as the density of states, as we will see in Section 3. These are the modifications that we will apply to the nuclear reactions of the early universe. Our goal is to see if these mass and energy shifts have any large-scale effect on our standard depiction of the early universe.

2.3 The QHD Hamiltonian

Since the meson fields are classical, only the fermion field needs to be quantized. This is done in depth in [5], [6], [12] and [13] so we will not get into the details here.

[3]The reader might be more familiar with $E^2 = \mathbf{p}^2c^2 + m^2c^4$

The solutions to the Dirac equation provide a complete basis in which we expand the quantum field operator for baryons. In the Schrödinger picture, where the operators are independent of time, the baryon field operator is given by

$$\hat{\psi}(\mathbf{x}) = \frac{1}{\sqrt{\Omega}} \sum_{\mathbf{k}\lambda} \left[U(\mathbf{k}\lambda) A_{\mathbf{k}\lambda} e^{i\mathbf{k}\cdot\mathbf{x}} + V(-\mathbf{k}\lambda) B^{\dagger}_{\mathbf{k}\lambda} e^{-i\mathbf{k}\cdot\mathbf{x}} \right] \tag{16}$$

in which the spinors $U(\mathbf{k}\lambda)$ and $V(-\mathbf{k}\lambda)$ correspond to E_+ and E_-, respectively. A,$A^\dagger$ and B, $B^\dagger$ are particle and antiparticle annihilation and creation operators, respectively, satisfying the standard anticommutation relations:

$$\{A_{\mathbf{k}\lambda}, A^{\dagger}_{\mathbf{k}'\lambda'}\} = \{B_{\mathbf{k}\lambda}, B^{\dagger}_{\mathbf{k}'\lambda'}\} = \delta_{kk'}\delta_{\lambda\lambda'} \tag{17}$$

Everything else anticommutes. The Hamiltonian density is given by

$$\mathcal{H} = \delta\mathcal{H} + \frac{1}{2}m_s^2\phi_0^2 - \frac{1}{2}m_v^2 V_0^2 + g_v V_0 \hat{\rho}_B$$
$$+ \frac{1}{\Omega}\sum_{\mathbf{k}\lambda} \sqrt{\mathbf{k}^2 + M^{*2}} (A^{\dagger}_{\mathbf{k}\lambda} A_{\mathbf{k}\lambda} + B^{\dagger}_{\mathbf{k}\lambda} B_{\mathbf{k}\lambda}) \tag{18}$$

and the baryon density operator (which counts the total number of baryons in a given volume Ω) is given by

$$\hat{\rho}_B = \frac{1}{\Omega}\sum_{\mathbf{k}\lambda} (A^{\dagger}_{\mathbf{k}\lambda} A_{\mathbf{k}\lambda} - B^{\dagger}_{\mathbf{k}\lambda} B_{\mathbf{k}\lambda}) \tag{19}$$

Note that ρ_B also equals $\psi^\dagger\psi$ and $2k_F^2/3\pi^2$ from Fermi statistics where $k_F = 1.42 f^{-1}$. $\delta\mathcal{H}$ is called the *zero-point energy*; it represents the energy difference between a filled negative energy Fermi sea of baryons with mass M^* and a filled negative Fermi sea of baryons of mass M. We will neglect this term for the purposes of our paper but the reader is encouraged to look at [5] and [6] for further details.

The remaining terms in Eq. 18 are called $\mathcal{H}_{MFT}$ all together. Since $\mathcal{H}_{MFT}$ and ρ_B are diagonal operators, this mean-field problem can be solved exactly once the meson fields are specified. All the eigenstates are known. We will take these diagonal operators, and look at their expectation values for statistical mechanical interpretations of nuclear matter.

2.4 Nuclear Matter

Once we know the hamiltonian for the system, we can find the energy levels. The ground state of nuclear matter in the MFT is obtained by filling levels up to k_F with a spin-isospin degeneracy of $\gamma = 4$ ($p\uparrow,\ p\downarrow,\ n\uparrow,\ n\downarrow$). Then the expectation value of the hamiltonian (Eq. 18) gives us the energy density $\varepsilon = E/\Omega$. Using Eqs. 12, and 13 to eliminate the fields ϕ_0 and V_0 and substituting these in Eq. 18 we obtain the equation of state for nuclear matter in MFT:

$$\varepsilon = \frac{g_v^2}{2m_v^2}\rho_B^2 + \frac{m_s^2}{2g_s^2}(M - M^*)^2 + \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k(\mathbf{k}^2 + M^{*2})^{1/2} \tag{20}$$

$$\rho_B = \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k = \frac{\gamma}{(2\pi)^3}k_F^3 \tag{21}$$

The first two terms in Eq. 20 arise from the mass terms for the vector and scalar fields. The final term is the relativistic energy of a Fermi gas of baryons of mass M^*. The effective mass M^* can be determined by minimizing the energy density of Eq. 20 with respect to M^*, since we have an isolated system at fixed B (baryon number) and Ω (volume). This leads to so-called *self-consistency* equation (SC-equation)

$$M^* = M - \frac{g_s^2}{m_s^2}\frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\frac{M^*}{(\mathbf{k}^2 + M^{*2})^{1/2}} \tag{22}$$

This integral can be solved numerically, yielding a transcendental self-consistency equation for the effective mass. The solution of the self-consistency equation for M^* yields an effective mass that is decreasing function of the density and temperature (see [5]).

We have managed to solve the field equations in a reliable manner that helps us understand NN interactions using RMFT[4] . However, to apply these equations to an environment such as the early universe, we need a more detailed understanding, especially one that depends on temperature.

[4]RMFT finds a deeper justification in terms of density functional theory. See [16]

3 Statistical Mechanics in QHD-I

We need a description of the fields ϕ_0 and V_0 that depends on density and temperature since the values for these quantities are well established by standard cosmology. The equation of state follows directly from the thermodynamic potential. However, its derivation is rather long and will be omitted here (see [5] for a detailed derivation). The results are

$$V_0 = \frac{g_v}{m_v^2}\rho_B \tag{23}$$

$$\varepsilon(\rho_B, T) = \frac{g_v^2}{2m_v^2}\rho_B^2 + \frac{m_s^2}{2g_s^2}(M - M^*)^2 + \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\sqrt{\mathbf{k}^2 + M^{*2}}(n_k + \bar{n}_k) \tag{24}$$

$$\rho_B = \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k(n_k - \bar{n}_k) \tag{25}$$

The self-consistency equation becomes

$$\phi_0 = \frac{g_s^2}{m_s^2}\frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\frac{M^*}{(\mathbf{k}^2 + M^{*2})^{1/2}}(n_k + \bar{n}_k) \tag{26}$$

in which n_k and $\bar{n}_k$ are the usual thermal distribution functions given by

$$n_k = \frac{1}{e^{\beta(E_k^* - \mu^*)} + 1} \quad \text{and} \quad \bar{n}_k = \frac{1}{e^{\beta(E_k^* + \mu^*)} + 1} \tag{27}$$

Here $E_k^* \equiv (\mathbf{k}^2 + M^{*2})^{1/2}$ and $\mu^* = \mu - g_v V_0$. Note that we have used equal and opposite chemical potentials for particles and antiparticles, which follows from the fact that the total baryon number is a conserved quantity. So we have derived all the properties of the meson fields. To understand how these fields affect the early universe all we have to do is to solve the SC-equation at a given temperature.

4 Cosmology

We spent a considerable amount of time trying to obtain a clear picture of the cosmological time slice in which we are interested. As mentioned earlier, this is what we call the "nuclear regime". Initially, we were aiming to work within the interval of

T = 300 MeV to 10 MeV. However, as our picture became clearer we narrowed the range down to T = 200 MeV - 100 MeV for reasons that will be explained later.

We need to know the time dependence of density and temperature of the universe in order to precisely locate our regime and understand its composition and thermodynamics better. We extracted most of our data from [9] and [1]-[4]. Some of the numbers in these references disagree, but overall the cosmological picture is the same in all modern texts. The quark-hadron transition is generally quoted to lie somewhere between T = 300 MeV and 200 MeV. We took 200 MeV as our starting point, since we are interested in interactions among nucleons and not quarks. In this stage the universe is assumed to be in thermal equilibrium. This is the case if the total chemical potential μ is zero. Later, using RMFT we will show that the chemical potential is indeed very small. So our assumption of thermal equilibrium is reasonable.

At this point in time the medium essentially consists of a hot gas of photons (γ), neutrinos ($\nu_l, \bar{\nu}_l$), pions ($\pi^+,\ \pi^0,\ \pi^-$), electrons ($e^\pm$), muons ($\mu^\pm$), protons (p) and neutrons (n). The taus and antibaryons have already been annihilated because they are heavier, resulting in a very low baryon density. We include all 3 species of neutrinos as well as their antiparticles since the most recent data indicates an upper limit of 18.2 MeV for the tau neutrino [17]. Since the kinetic energy of the particles (200 MeV) is at least a considerable fraction of the rest masses, we consider this entire system to be a relativistic gas. The neutron to proton ratio at statistical equilibrium is given by

$$\left(\frac{n}{p}\right)_{eq} = e^{-Q/T} \tag{28}$$

in which $Q = E_p - E_n \to 1.293$ MeV (as energy$\to$ mass) is the energy difference between the neutron and the proton. For T = 200 MeV this ratio is 0.9935. So our initial assumption of equal number of protons and neutrons holds at this stage. This is a good approximation down to T = 100 MeV.

So far all we have mentioned can be found in a standard modern cosmology text.

However, to this hot mixture of particles we will add the scalar field $\phi_0(\rho_B, T)$ and vector field $V_0(\rho_B, T)$ that we introduced in Section 2 as the keystones of MFT in QHD. These fields will introduce some new effects into this well-drawn segment of cosmic history. However, as the reader will see later, only the scalar field interaction plays an important role since $V_0 \to 0$ as $\rho_B \to 0$.

Recall that all of contents of this soup are heavily dependent on the temperature, including the scalar and vector fields whose temperature dependences have been laid out in detail in Section 3. As the temperature of the universe decreases, some important changes occur. At $T = 130$ MeV the oppositely charged pions will annihilate into 2 photons leaving only the neutral pion, which in turn also decays into 2 photons. The neutrinos decouple from the photon background at $T = 3$ MeV and $t = 1$ sec (muon and electron neutrinos, respectively). Around $t = 14$ sec the electron-positron pairs will also annihilate producing 2 photons. All this energy generated in the annihilations goes into increasing the temperature of the photon background, which is why it is 1.4 times hotter than the neutrino background today. The baryon-antibaryon annihilation actually occurs in the quark-antiquark stage, which leaves us with a very diffuse gas of nuclear matter in the interval that we are interested. It should be noted that all these events occur too late (or too early) for us to be concerned with them except at $T = 200$ MeV.

Since the universe is expanding, the matter and energy densities are decreasing at a decelerating rate. We are especially interested in the changes of baryon density, which will affect the scalar and vector fields as can be seen in Section 3. So it was crucial for us to determine what the baryon density was for $T = 200$ MeV. Statistical mechanics of the early universe behaves quite well for photons, electrons and neutrinos. Given the temperature, one can easily calculate the number density n_γ of photons from blackbody radiation.

$$n_\gamma = \frac{2.404}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3 \tag{29}$$

This by itself does not say much. However in Appendix A we prove that $(RT)^3 = constant$, which implies that $\rho_B \propto R^{-3} \propto T^3$. Since any kind of density is inversely proportional to R^3 (volume), we see that baryon density $\rho_B \propto T^3 \propto n_\gamma$. Therefore, we can calculate the ratio of baryon number density to that photon number density and obtain the baryon density at T = 200 MeV. Although values for baryon to photon ratio range from 3×10^{-10} to 10^{-8} we will simply suffice with 10^{-9} here. From this, obtaining the baryon density is a simple step. For T = 200 MeV our calculations yield

$$\rho_B^{200MeV} = 2.52 \times 10^{29} cm^{-3}$$

Basically, this says we have one baryon per $10^9 fm^3$. This result is not surprising since most baryons were converted into energy during the earlier quark-antiquark annihilation epoch. We are indeed dealing with a hot diffuse distribution for baryons here, which puts us in lower left corner of the nuclear phase diagram (see [5]). As the universe expands, this number density will decrease even further. This is one extreme end of the QHD spectrum that few people have investigated, if any.

After determining the baryon density, our calculations indicate that the De Broglie wavelength for baryons was 3 orders of magnitude smaller than the actual spacing between the baryons, which implies that the use of classical statistical mechanics is appropriate in this regime. With that, we have established all the thermodynamical properties of the medium in this time period. We are dealing with a hot mixture of relativistic photons, neutrinos, muons, pions and a diffuse ensemble of baryons. The statistical mechanics is relativistic but classical (which implies that pressure = Energy/ 3 for completely relativistic particles). So now we can look at each individual contribution from different particles and compare them. All of this is done analytically in [1]. We just took the general results and made them dimensionless quantities normalized with respect to nucleon mass 939 MeV. Most of the energy and pressure contribution comes from neutrinos (3-fold degeneracy). These results are well known and are not interesting by themselves but they will be once we include the scalar field effects. And there still remains the question of chemical potential, QHD can answer for us.

5 QHD-I at T=200-150 MeV

One of the reasons why we chose T = 200 MeV is because $M^*/M = 0.5$ in this case. This can be seen from the SC-equation plot given in [5] and [7]. Determining the value of the chemical potential is basically a 3-step iterative process. First, we define a dimensionless variable $\chi = M^*/M$. That way SC-equation given by Eq. 26 simply becomes a dimensionless function of χ.

$$1 - \chi - C_s^2 \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k \frac{\chi}{\sqrt{k^2 + \chi^2}} (n_k + \bar{n}_k) = 0 \tag{30}$$

in which the dependence on the chemical potential is hidden in the distribution functions:

$$n_k = \frac{1}{e^{\beta(\sqrt{k^2+\chi^2} - \mu^*)} + 1} \quad \text{and} \quad \bar{n}_k = \frac{1}{e^{\beta(\sqrt{k^2+\chi^2} + \mu^*)} + 1} \tag{31}$$

Then, we pick a random chemical potential, an educated initial guess, that we substitute into Eq. 30. The integrals can only be evaluated numerically so this is an iterative process. We are looking for the root of the SC-equation. Once we get the root we substitute that in Eq. 26 and finally compare the answer with the actual baryon density in Eq. 29. Then we adjust the initial guess to get a closer result to the known baryon density. After a few attempts, we finally obtained the right chemical potential, which also resulted in $\chi = 0.50028$. This is very close to the $\mu = 0$ value which is 0.5, which also implies that the chemical potential must be small. For the value of χ and baryon density given above, the chemical potential in dimensionless units is $\mu = 2.68 \times 10^{-9}$ (T = 0.213 in the same units). This is indeed a very small number so we can feel confident about the initial assumptions we made using $\mu \approx 0$.

Now that we know what the chemical potential is, we can finally calculate the energy density and pressure contributions due to the scalar field. For energy density we use Eq. 24 and the expression for pressure can be found in [5]

$$p = -\frac{1}{2}\frac{(1-\chi)^2}{C_s^s} + \frac{2}{3\pi}\int_0^{k_F} d^3k \frac{k^2}{\sqrt{k^2+\chi^2}} \tag{32}$$

Here we have dropped the term proportional to ρ_B^2 because it is very small compared to the other two. Numerical evaluations of the second term in Eq. 32 showed that it, too, is negligible compared to the first term. *This results in a negative pressure a little larger than the photon pressure.* This negative pressure from the scalar field totally cancels out the photon pressure. When we look at the SC-equation plots given in [5] and [7]. We can see that as T increases, χ decreases; therefore the overall pressure grows. Similar but the opposite occurs at the other end of the spectrum when we decrease T. This can be seen from the first law of thermodynamics which states that -dE/dV = p at constant baryon number. Now, at constant volume and baryon number the system will minimize its energy with respect to ϕ_0 as given by [5]. This ground state energy is simply a function of volume in T = 0 case in which $E = (m_s^2\phi_0^2/2)V$. Therefore, the pressure is just $p = -m_s^2\phi_0^2/2$ hence it is negative. What this means is that the scalar field likes to eliminate extra energy and it does this via a negative pressure. To follow up on this, we also reproduced similar results for T = 150 MeV to see how the scalar field behaves. Obviously we cannot go higher than 200 MeV since we would run into quark-hadron phase transition. And there really is no point in going much lower than 150 MeV since the scalar field effects are already becoming significantly weaker at that temperature as one can see from the self-consistency plot from [5] and [7]. The numerical results for thermodynamic properties of the medium at 200 MeV and 150 MeV are posted in Table 1 below. As we can deduce from the numbers given in Table 1 the scalar field has a significant contribution to the medium of that era. Whether these effects are detectable or change the cosmology of the epoch in a way that could be discovered by scientists is beyond the scope of this paper. The reader should not be misled into assuming that the detection of these effects is unimportant. Rather, we leave that problem to the expert cosmologists. It is possible that important effects such as a large negative scalar pressure might have left some sort of a trace for the experts to pick up [5].

Table 1: The early universe in dimensionless numbers. Each quantity is in units of nucleon mass M = 939 MeV.

	T = 200 MeV	T = 150 MeV
$\chi = \frac{M^*}{M}$	.50028	.98082
T	.21299	.15974
μ	2.6827×10^{-9}	6.666×10^{-9}
ε_ν	3.5546×10^{-3}	1.1247×10^{-3}
ε_e	2.3698×10^{-3}	$.7489 \times 10^{-3}$
ε_γ	1.3541×10^{-3}	$.4280 \times 10^{-3}$
ε_{sc}	4.6747×10^{-4}	$.6884 \times 10^{-6}$
ε_π	1.9155×10^{-3}	$.5249 \times 10^{-3}$
ε_μ	2.3199×10^{-3}	$.7214 \times 10^{-3}$
p_ν	1.1849×10^{-3}	$.3749 \times 10^{-3}$
p_e	7.8992×10^{-4}	2.499×10^{-4}
p_γ	4.5138×10^{-4}	1.428×10^{-4}
p_{sc}	-4.6747×10^{-4}	$-.6884 \times 10^{-6}$
p_π	5.9323×10^{-4}	1.7260×10^{-4}
p_μ	7.4604×10^{-4}	2.2641×10^{-4}

[5] We do plan to investigate the effects of this modification on the evolution of the standard Robertson-Walker metric in general relativity in this epoch

6 Cross sections in the Early Universe

As mentioned earlier in this paper, we aim to study the changes in the cross sections brought about by a shift in nucleon mass caused by the scalar field ϕ. We will first look at the scattering cross sections for the weak interactions below:

$$\nu_l + n \leftrightarrow p + l^-$$

We will also compute the cross section for *deuteron photodisintegration* given by the following

$$d + \gamma \rightarrow n + p$$

Once we get an expression for the cross sections for these events, we will compare these results with the ones that we have computed using the shifted mass. As the reader will see later, the effects are noticeable but it is unlikely that they would change the overall cosmology.

6.1 Weak Interaction Cross Sections

In this section we perform a thorough computation of the scattering cross section of the neutrino in the following reaction

$$\nu_e + n \leftrightarrow p + e^-$$

We employ a point-particle interaction and use *Fermi's golden rule* which gives us the following expression for the transition probability W_{fi}

$$W_{fi} = 2\pi |\langle f|H_w|i\rangle|^2 \delta(E_f - E_i) dn_f \tag{33}$$

in which the delta function implies overall energy conservation and dn_f is the density of final states. The interaction hamiltonian is given by

$$\mathcal{H}_w = \frac{-G}{\sqrt{2}} \left[\bar{\psi}_e \gamma_\mu (1+\gamma_5)\psi_{\nu_e}\right] \left[\bar{\psi}_p \gamma_\mu (1+\gamma_5)\psi_n\right] \tag{34}$$

in which $G = 10^-5/m_p^2$ is called Fermi's constant. $\bar{\psi}_i$ and ψ_i are fermion creation and annihilation operators, respectively and in our metric $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$. Finally, the reader should not forget that $\mathcal{H}_w$ is the hamiltonian density per unit volume. The total hamiltonian will be obtained by integrating over 3-space.

Before we resume with the calculation of the matrix element, let us relate the transition rate to the differential cross section by

$$d\sigma = \frac{2\pi|\langle f|H_w|i\rangle|^2\delta(E_f - E_i)dn_f}{I_{inc}} \tag{35}$$

in which I_{inc} is called the incident flux and defined by

$$I_{inc} = \frac{1}{\Omega}\frac{\sqrt{(k_\nu \cdot k_n)^2}}{E_\nu E_n} \tag{36}$$

Here k_i^μ is fermion 4-momentum and Ω is the volume of a box with periodic boundary conditions, which will be explained further below. The density of states in Eq. 35 is as follows

$$dn_f = \frac{\Omega}{(2\pi)^3}\int d^3k_e \tag{37}$$

For the particle creation and annihilation operators used in Eq. 34 we use the standard plane wave form given below

$$\psi(x) = \frac{1}{\sqrt{\Omega}}\sum_{\mathbf{k}\lambda}\left[a_{\mathbf{k}\lambda}u(\mathbf{k}\lambda)e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.\right] \tag{38}$$

in which h.c. is the hermitian conjugate with which we will not deal because there are no antiparticles being created or annihilated in this particular reaction. The wave function given above is a product of creation/annihilation operators with 4-component Dirac spinors $u(\mathbf{k}\lambda)$ carried on by a plane wave and summed over all momenta and helicities. A little algebra shows that the matrix element in Eq. 35 can be written as follows

$$\langle f|H_w|i\rangle = \frac{-G}{\sqrt{2}}\frac{1}{\Omega}\delta_{\mathbf{k}_\nu+\mathbf{k}_n,\mathbf{k}_p+\mathbf{k}_e}\ \bar{u}_e\gamma_\mu(1+\gamma_5)u_\nu\bar{u}_p\gamma_\mu(1+\gamma_5)u_n \tag{39}$$

The Kronecker delta function implies conservation of momentum, which is a principle that we must uphold in every physical situation. The next step is to determine the square norm of this matrix element. Since the square of a Kronecker delta function is a delta function we can pull it out, as well as G^2 and Ω^2. So we must focus on the norm square of the remaining terms in Eq. 39. And we also sum over the final momenta $\mathbf{k}_e$ and $\mathbf{k}_p$ which makes makes the delta function disappear (gives us 1) so the remaining terms give us

$$d\sigma = \pi G^2 \left|\bar{u}_e \gamma_\mu (1+\gamma_5) u_\nu \, \bar{u}_p \gamma_\mu (1+\gamma_5) u_n\right|^2 \delta(E_f - E_i) \frac{d^3 k_e}{(2\pi)^3} \frac{k_\nu E_n}{\sqrt{(k_\nu \cdot k_n)^2}} \tag{40}$$

For now we ignore the spins and sum over the helicities. We take the average of the sum of initial helicities and multiply it with the sum of final helicities. This would normally introduce a factor of $\frac{1}{4}$ since each particle can be left handed or right handed. However neutrinos can only be left-handed therefore we have $\overline{\sum_i}\sum_f \longrightarrow \sum_{\lambda_\nu}\frac{1}{2}\sum_{\lambda_n}\sum_{\lambda_e}\sum_{\lambda_p}$, which introduces a factor of $\frac{1}{2}$ into Eq. 40. So all that is left is to calculate the norm square of the matrix element in Eq. 40. After a few clever manipulations and some delicate algebra, we sum over the fermion spins by making use of positive energy projection operators defined in [15] the final result reads

$$|m.e.|^2 = 4Tr\left(\gamma_\mu(1+\gamma_5)\frac{\gamma_\mu k_\nu^\mu}{2E_\nu}\gamma_\lambda \frac{\gamma_\mu k_e^\mu}{2E_e}\right) Tr\left(\gamma_\mu(1+\gamma_5)\frac{\gamma_\mu k_n^\mu}{2E_n}\gamma_\lambda \frac{\gamma_\mu k_p^\mu}{2E_p}\right) \tag{41}$$

We also rewrite the energy conserving delta function as a 4 dimensional delta function conserving overall 4 momentum. Then, all that is left is to calculate these traces. There are several readily established relations on the traces of Dirac gamma matrices that we take from [12], [13] and [15]. The final answer is a Lorentz invariant quantity whose dimensions are length squared.

$$d\sigma = \frac{G^2}{\pi^2}\delta^4(k_\nu + k_n - k_e - k_p)\frac{d^3k_e}{2E_e}\frac{d^3k_p}{2E_p}\,\frac{4(k_\nu \cdot k_n)(k_e \cdot k_p)}{|k_\nu \cdot k_n|} \tag{42}$$

where the last fraction equals the product of the traces in Eq. 41. Performing the d^3k_p and the dk_e integrals eliminates the delta function because of conservation of energy and momentum. Next we do the inner products assuming massless electrons since 200 MeV $>>$ $m_e c^2 = .511$ MeV . The result is a Lorentz invariant quantity.

Finally we include the nucleon form factor f_{SN} that takes into account the internal structure of the nucleus that we have left out so far. The cross section becomes

$$d\sigma = \frac{G^2}{\pi^2} f_{SN}^2 \, k^2 d\Omega \left[1 + \frac{k}{\sqrt{k^2 + M^2}}\right] \tag{43}$$

in which $k \equiv |k_\nu| = |k_e|$ in the center-of-momentum (CM) system and M denotes the nucleon mass as before. The form factor f_{SN} equals $[1 + q^2/(855MeV)^2]^{-2}$, in which q^2 is the 4-momentum transfer. In the case of massless leptons in the CM system, there is no energy transfer. Therefore $q^2 = 2k^2(1 - \cos\theta)$. Finally, we integrate over the solid angle and obtain the final result for the scattering cross section given below

$$\sigma(M) = \frac{2G^2k^2}{\pi} \left[1 + \frac{k}{\sqrt{k^2 + M^2}}\right] \int f_{SN}^2 \sin\theta d\theta \tag{44}$$

Note that the expression for the cross section is a function of nucleon mass M, which means that when nucleon mass shifts under the scalar field the cross section will change and become [6]

$$\sigma(M^*) = \frac{2G^2k^2}{\pi} \left[1 + \frac{k}{\sqrt{k^2 + M^{*2}}}\right] \int f_{SN}^2 \sin\theta d\theta \tag{45}$$

To see what these cross sections look like, we plotted them against normalized momentum k/M. The "pointed" curve is the "shifted" cross section, the solid curve is the unaffected cross section.

Finally we look at the respective ratio of the two cross sections

$$\frac{\sigma(M^*)}{\sigma(M)} = \frac{\left[1 + \frac{k}{\sqrt{k^2+M^{*2}}}\right]}{\left[1 + \frac{k}{\sqrt{k^2+M^2}}\right]} \tag{46}$$

This is the solid curve in Figure 2 below and there is a noticeable peak at low momentum. *So the shift in mass does effect the total cross section.* The formalism for the reverse reaction (see above) of this type is the same except for a factor of $\frac{1}{2}$ that we obtain when we are summing over the helicities. So although the cross sections might change, the overall ratio remains the same and since the ratio really is in what we are interested we need not worry about the extra factors.

[6] We assume that f_{SN} is unmodified in the medium

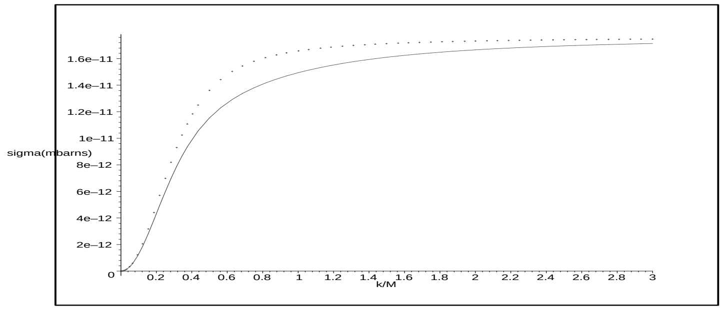

Figure 1: Scattering cross sections for semileptonic process involving an electron. The "pointed" curve is the shifted cross section.

We can perform the same type of calculation by switching leptons i.e. we look at the following reaction

$$\nu_\mu + n \leftrightarrow \mu^- + p$$

Since the muon is about 206.8 times heavier than the electron, we keep the mass terms in the relativistic energies, which changes Eq. 44 as follows

$$\sigma(M) = \frac{2G^2k^2}{\pi}\left[1 + \frac{k}{\sqrt{(k^2+M_\mu^2)(k^2+M^2)}}\right]\int f_{SN}^2 \sin\theta d\theta \tag{47}$$

And the ratio of the cross sections becomes

$$\frac{\sigma_\mu(M^*)}{\sigma_\mu(M)} = \frac{\left[1 + \frac{k}{\sqrt{(k^2+M_\mu^2)(k^2+M^{*2})}}\right]}{\left[1 + \frac{k}{\sqrt{(k^2+M_\mu^2)(k^2+M^2)}}\right]} \tag{48}$$

This curve is displayed in points in Figure 2. As we can see the extra mass term introduced by the muon has a very small effect in the overall ratio. Once again, the reverse muon reaction above yields an identical ratio the factors of $\frac{1}{2}$ eliminate each other .

As we can see from Figure 2, there is again a noticeable change in the cross sections with the introduction of a scalar field. These effects dissipate as the temperature of the universe drops below 100 MeV.

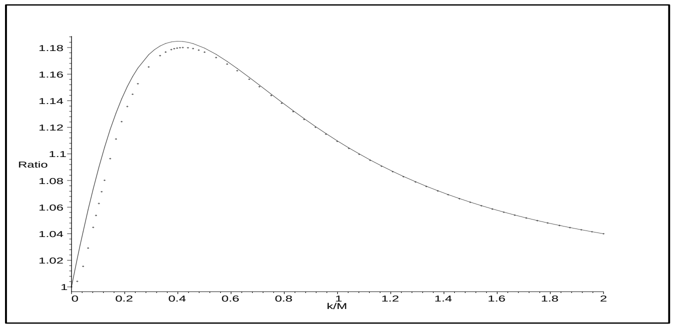

Figure 2: The ratio of cross sections. The solid curve is for the electron reaction and the pointed curve is for the muon reaction.

It is hard to extrapolate the consequences of such a change. Once again, detection of such a change is left to the experimental astrophysicists. Detecting this would be more challenging of a task than detecting the effects of the negative pressure mentioned earlier. Many events take place in the Early Universe without our having the means to detect them in our present level of advancement. In this paper, we base our work on our claim that these fields presumably existed and had some effects in the physics of the early universe.

6.2 Deuteron Photodisintegration

In this section we investigate what happens in the case of a nuclear interaction. We pick the simplest nuclear reaction that we know has occurred in the early universe, namely, deuteron photodisintegration.

$$\gamma + {}^2H \rightarrow n + p$$

To compute the cross section, we will make use of Fermi's Golden rule once more with point couplings. However, we will use non-relativistic quantum mechanics because deuteron wave functions can be extremely complicated in field theory.

The differential cross section is given by Eq. 35. The density of states dn_f is similar to the one before but this time we integrate over proton momentum, $dn_f = \Omega d^3k_f/(2\pi)^3$. A simple calculation also shows that $I_{inc} = (1 + v_{^2H})/\Omega$. However the rest of the problem is more different than it was in the lepton capturing case. The hamiltonian is the standard free-particle hamiltonian including the electromagnetic vector field in the minimal fashion along with a nucleon-nucleon potential $V(|\mathbf{x}_p - \mathbf{x}_n|)$. Taking these into account the interaction hamiltonian is given by

$$H' = -\frac{e}{2mc}\left[\mathbf{p}_p \cdot \mathbf{A}(\mathbf{x}_p) + \mathbf{A}(\mathbf{x}_p) \cdot \mathbf{p}_p\right] \tag{49}$$

where we write the vector field $\mathbf{A}$ as follows

$$\mathbf{A}(\mathbf{x}_p) = \sum_{\mathbf{k}} \sum_{\lambda=\pm 1} \frac{1}{(2\omega_k \Omega)^{1/2}} \left[\hat{e}_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda} e^{i\mathbf{k}\cdot\mathbf{x}} + h.c.\right] \tag{50}$$

where $a_{\mathbf{k}\lambda}$ is the photon annihilation operator. After we transform to center-of-mass $(\mathbf{R}, \mathbf{r})$ coordinates, we must choose what type of wave functions we will be using for the initials and final states in the matrix element $\langle f|H'|i\rangle$. We choose the following forms

$$|i\rangle = |\mathbf{k}\lambda\rangle \frac{1}{\sqrt{\Omega}} e^{i\mathbf{p}_i\cdot\mathbf{R}} \psi_i(\vec{r}) \tag{51}$$

$$|f\rangle = |0\rangle \frac{1}{\sqrt{\Omega}} e^{i\mathbf{p}_f\cdot\mathbf{R}} \psi_f(\mathbf{r}) \tag{52}$$

In the equations above, ψ_i is a bound state of the deuteron and ψ_f will be chosen to be a plane-wave state. The choice for the deuteron wave function is a more complicated and subtle step and will explained in more detail further on. The following expression gives us the differential cross section

$$d\sigma = \frac{\pi}{\omega_k \Omega}\left(\frac{e}{m}\right)^2 |\langle\psi_f|H'|\psi_i\rangle|^2 \delta(E_f - E_i - k) \frac{\Omega d^3k_p}{(2\pi)^3} \frac{\Omega}{1 + v_{^2H}} \tag{53}$$

in which the matrix element is

$$\langle\psi_f|H'|\psi_i\rangle = \int d\mathbf{r} \psi_f^*(\mathbf{r}) \mathbf{p}_p \psi_i(\mathbf{r}) e^{i\mathbf{k}\cdot\frac{\mathbf{r}}{2}} \cdot \hat{e}_{\mathbf{k}\lambda} \tag{54}$$

A little algebra yields

$$\langle\psi_f|H'|\psi_i\rangle = (E_f - E_i) m \langle\psi_f|\hat{e}_{\mathbf{k}\lambda} \cdot \mathbf{x}_p e^{i\mathbf{k}_\gamma\cdot\mathbf{x}_p}|\psi_i\rangle \tag{55}$$

where $\mathbf{k}_\gamma$ is the photon momentum and $\mathbf{x}_p$ is the position of the proton. Rewriting this in terms of new coordinates, we get $\mathbf{x}_p = \mathbf{R} + \mathbf{r}/2$ the matrix element becomes [7]

$$\langle \psi_f | H' | \psi_i \rangle = (E_f - E_i) m \hat{e}_{\mathbf{k}\lambda} \cdot \delta_{\mathbf{P}_{pn}, \mathbf{k}_\gamma + \mathbf{P}_{^2H}} \int d\mathbf{r} \psi^*_{pn}(\mathbf{r}) \frac{vr}{2} e^{i\mathbf{k}_\gamma \cdot \mathbf{r}} \psi_{^2H}(\mathbf{r}) \tag{56}$$

in which $\psi^*_{pn}(\mathbf{r})$ denotes the final proton-neutron and $\psi_{^2H}(\mathbf{r})$ denotes the initial bound deuteron wave functions. Some readers might recognize this as the electric dipole moment. The delta function assures overall momentum conservation. We let the final wave function be of a plane wave form $\psi_{pn}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}/\sqrt{\Omega}$ where $\mathbf{k} \equiv (\mathbf{k}_p - \mathbf{k}_n)/2$ is the relative momentum of the final state. With this choice of wave functions the volume Ω drops out of Eq. 53.

The choice of a wave function for the deuteron was more tricky and its foundations less sturdy. The problem with the deuteron is that we do not really know whether a bound state can exist for our scalar field. The free deuteron binding energy is 2.2 MeV. If the nucleon mass is halved, it is quite possible that the deuteron is no longer bound. In fact, this depends on the detailed short-distances internuclear interaction in this medium. We, here, assume that the deuteron remains bound in this scalar-field dominated early universe. This could be because the scalar mesons somehow make the deuteron potential well deeper. Whatever the cause may be we choose the following wave function for the bound state

$$\psi_{^2H} = N \frac{e^{-\bar{\gamma} r}}{r} \tag{57}$$

in which N is the normalization and $\bar{\gamma} \propto \gamma_B$ where $\gamma_B = \sqrt{2\mu\epsilon_B}$, μ is the reduced mass and $\epsilon_B = 2.2$ MeV . Since we do not exactly know the structure of the deuteron, we will use $\bar{\gamma}$ as a parameter. As the reader will see later, changing this parameter has very little effect on the overall cross section. At this stage we have

$$d\sigma = \frac{k_\gamma \alpha}{2\pi} d^3k_p \, \delta \left(2\sqrt{M^2 + k_p^2} - \sqrt{M_D^2 + k_\gamma^2} - k_\gamma \right) \frac{1}{1 + v_{^2H}}$$

[7] The integral over $\mathbf{R}$ goes to 0

$$\frac{1}{2}\sum_{\lambda}\left|\hat{e}_{\mathbf{k}_\gamma\lambda}\cdot\nabla_{\mathbf{k}_\gamma}\int e^{i\left(\frac{1}{2}\mathbf{k}_p-\mathbf{k}_p\right)\cdot\mathbf{r}}\psi_{^2H}(\mathbf{r})d^3r\right|^2 \tag{58}$$

Finally we obtain

$$d\sigma=\frac{2\alpha\bar{\gamma}}{1+v_{^2H}}\sqrt{M^2+k_p^2}\,k_\gamma\,k_p\,d\Omega_p\,\frac{\frac{1}{2}\sum_\lambda|\hat{e}_{\mathbf{k}_\gamma\lambda}\cdot\mathbf{k}_p|^2}{\left[\left(\frac{1}{2}\mathbf{k}_p-\mathbf{k}_p\right)^2+\bar{\gamma}^2\right]^4} \tag{59}$$

using conservation of energy and momentum we manipulate this equation further, and finally obtain the expression for differential scattering cross section in center-of-momentum frame.

$$\frac{d\sigma}{d\Omega}=\frac{\frac{1}{2}\alpha\bar{\gamma}k_\gamma\left[\frac{k_\gamma^2+k_\gamma\sqrt{M_D^2+k_\gamma^2}}{2}\right]^{3/2}\sqrt{M_D^2+k_\gamma^2}\,\sin^2\theta_p}{\left[\frac{3}{4}k_\gamma^2+\frac{k_\gamma}{2}\sqrt{M_D^2+k_\gamma^2}-k_\gamma\left[\frac{k_\gamma^2+k_\gamma\sqrt{M_D^2+k_\gamma^2}}{2}\right]^{1/2}\cos\theta_p+\bar{\gamma}^2\right]^4} \tag{60}$$

We plotted the expression above at different values for photon momentum and $\bar{\gamma}$ against scattering angle θ. Here we only included the plots for $k_\gamma \equiv k = 0.3$ (282 MeV/c) and $\bar{\gamma} = \gamma_B/4$ (see Figure 3). The pointed curve is $d\sigma(M^*)/d\Omega$ and the solid curve is $d\sigma(M)/d\Omega$.

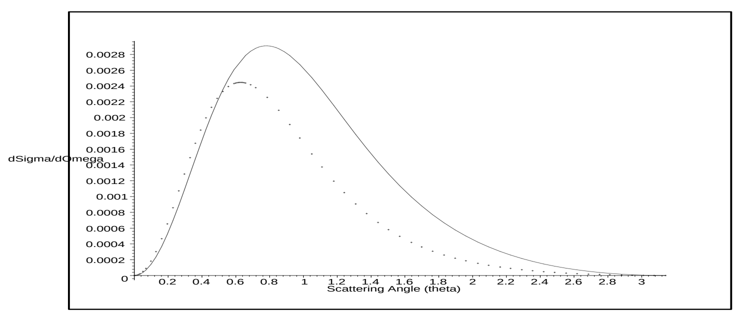

Figure 3: Differential scattering cross sections for deuteron photodisintegration at k = 282 MeV/c. The pointed curve is the shifted cross section and the solid curve is the unaffected cross section.

We plot the respective ratios in Figure 4.

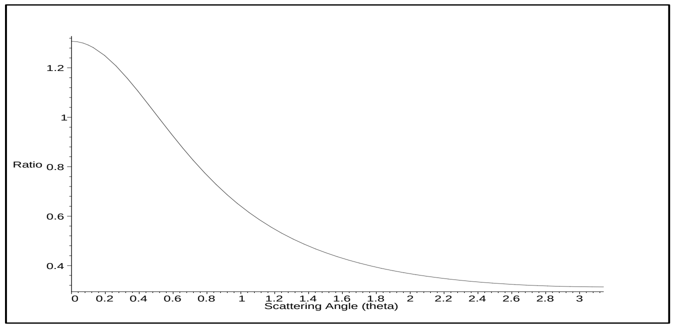

Figure 4: The ratio of the differential cross sections at k = 282 MeV/c.

Next we integrate over the solid angle to get the total cross section.

$$\sigma(M) = \pi\alpha\bar{\gamma}k_\gamma \left[\frac{k_\gamma^2 + k_\gamma\sqrt{M_D^2 + k_\gamma^2}}{2}\right]^{3/2} \sqrt{M_D^2 + k_\gamma^2}$$

$$\times \int_0^\pi \frac{\sin^3\theta d\theta}{\left[\frac{3}{4}k_\gamma^2 + \frac{k_\gamma}{2}\sqrt{M_D^2 + k_\gamma^2} - k_\gamma\left[\frac{k_\gamma^2 + k_\gamma\sqrt{M_D^2 + k_\gamma^2}}{2}\right]^{1/2}\cos\theta + \bar{\gamma}^2\right]^4} \qquad (61)$$

In Figure 5 we plot $\sigma(M^*)$ (pointed) and $\sigma(M)$ (solid) against photon momentum with $\bar{\gamma} = \gamma_B/4$. The black dots are actual experimental results.

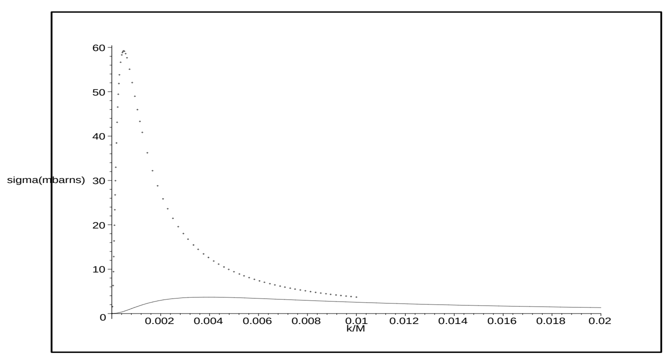

Figure 5: Photon cross sections in deuteron photodisintegration. Solid curve is the unaffected result and pointed curve is the shifted cross section. Note the high peak at very low momenta.

And in Figure 6 we plotted $\sigma(M^*)/\sigma(M)$ versus photon momentum k. As we can see, the two cross sections differ by a large factor at low momenta. However this ratio does not diverge at zero momentum.

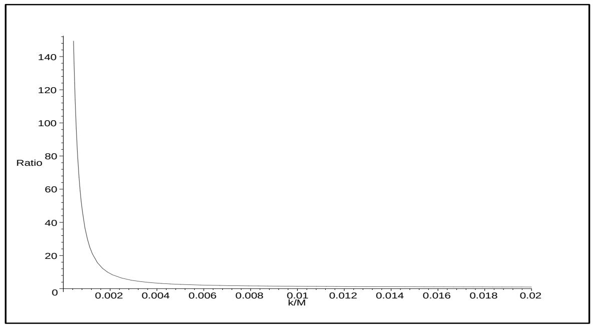

Figure 6: The ratio of the shifted cross section to the unaffected cross section. Note the increasing ratio at low momenta. However, this ratio goes back to 1 at k = 0 MeV/c.

Just as before, there are considerable changes due to the effects of the scalar field. But once again, we currently have no way of detecting such changes.

7 Conclusion

What started out as a curiosity has led to many interesting results. We constructed a model where scalar and vector field mesons govern the interaction between nuclei. Once we placed this model in a suitable regime in the early universe, we discovered that the scalar field has significant contribution between T = 200 MeV and T = 100 MeV. As we have shown earlier, the vector field was considerably weaker. We found out that the scalar field reduces the nucleon mass as T increases toward the quark-hadron transition. The scalar field also has a large contribution to total energy density of the universe. Even more surprisingly, there is a negative pressure generated by the scalar field which is large enough to eliminate the photon pressure at 200 MeV.

It seemed logical to assume that the shift in mass might give us interesting results on the reactions that took place in the early universe. We investigated the effects of such a change on semi-leptonic reactions and discovered that the scalar field does introduce changes in the cross sections. A similar type of phenomenon was observed when we recalculated the cross section for deuteron photodisintegration, provided the deuteron remained a bound state. All these effects are summarized in the plots shown above. As mentioned earlier, the consequences of such effects are tough to predict and require more advanced background in cosmology. Even if one managed to understand what happens to the early universe because of these changes it is very difficult to uncover these early relics of cosmic history.

A $(RT)^3$ is a constant

In other words, we prove that the entropy per comoving volume (entropy density s) is a constant. The total entropy in a volume R^3 is given by [1] and [9]

$$S = \frac{R^3}{T}\left(\rho_{eq}(T)c^2 + p_{eq}(T)\right) \tag{62}$$

in which ρ is equilibrium energy density and p is equilibrium pressure. Both these quantities are strictly functions of the temperature. To avoid cumbersome notation we rewrite Eq. 62 as follows

$$S = \frac{R^3}{T}(\rho + p) = \frac{V}{T}(\rho + p) \tag{63}$$

Now we use the first law of thermodynamics $dU = dQ - dW$ where $U = \rho V$ and $dW = pdV$. Therefore we get the known result for the differential of entropy

$$dS = \frac{dQ}{T} = \frac{1}{T}(dU + pdV) = \frac{1}{T}[Vd\rho + (\rho + p)dV] \tag{64}$$

which gives us the following partials

$$\frac{\partial S}{\partial V} = \frac{1}{T}(\rho + p) \quad and \quad \frac{\partial S}{\partial T} = \frac{V}{T}\frac{d\rho}{dT} \tag{65}$$

Therefore the second order partials have the following forms

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T} \tag{66}$$

which becomes

$$\frac{\partial}{\partial T}\left[\frac{1}{T}(\rho + p)\right] = \frac{\partial}{\partial V}\left[\frac{V}{T}\frac{d\rho}{dT}\right] \tag{67}$$

$$-\frac{1}{T^2}(\rho + p) + \frac{1}{T}\left(\frac{d\rho}{dT} + \frac{dp}{dT}\right) = \frac{1}{T}\frac{d\rho}{dT}$$

finally this gives us

$$\frac{dp}{dT} = \frac{1}{T}[\rho + p] \tag{68}$$

So using the equation above the change in pressure can be written as

$$dp = \frac{(\rho + p)}{T}dT \tag{69}$$

Now let us look at Eq. 63 again. Recall that the change in entropy was $dS = \frac{1}{T}[d(\rho V) + pdV]$ we can rewrite dS as

$$dS = \frac{1}{T}d[\rho V + pV] - \frac{V}{T}dp \tag{70}$$

Now we use the law of *conservation of energy* which says that dS=0 resulting in

$$d(\rho V + pV) = Vdp = V\frac{(\rho + p)}{T}dT \tag{71}$$

A little algebra gives us

$$\frac{1}{T}d[(\rho + p)V] - (\rho + p)V\frac{dT}{T^2} = 0 \tag{72}$$

which simply means

$$d\left[\frac{(\rho + p)V}{T}\right] = 0 \tag{73}$$

At this point it is hard to interpret the equation above. However recall that we are in a relativistic medium therefore both the energy density ρ and the pressure p are functions of T^4. Hence the expression in brackets in Eq. 73 is proportional to R^3T^3 so this says that

$$d[R^3T^3] = 0 \tag{74}$$

which was what we intended to prove as stated by [1] and [9].

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