# Using Information to Reformulate Quantum Mechanics

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by

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**Abstract**

Anton Zeilinger's recent work provides an intuitively satisfying account of the foundations of quantum mechanics motivated by the Copenhagen Interpretation. The current project is to write a rigorous pedagogical article to illuminate the emerging concept of quantum information in light of Zeilinger's proposal. Accepting Zeilinger's view would amount to a reformulation of familiar concepts in quantum mechanics in terms of information. The goal of the paper is not only to clarify and promote Zeilinger's view, but also to advocate teaching quantum mechanics in this reorganized way. The paper is meant to be accessible even at an undergraduate level of physics education. It explicitly illustrate that a spin one-half particle and a Mach-Zehnder interferometer carry one bit of information. Finally, it describes how these results affect our understanding of wave-particle duality.

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# 1 Introduction

The essential measure of success for a physical theory is that it ought to consistently make predictions that match experimental outcomes. In this sense, Quantum Theory is extremely successful. Although the wave function formulation of non-relativistic quantum mechanics developed by Schrödinger is the most widely used formulation, there are many others. Each may make use of different assumptions, but they all make exactly the same predictions of experimental results (similar to the many treatments of classical mechanics, such as the Newtonian, Lagrangian, and Hamiltonian formulations) [1]. These different mathematical formulations of quantum mechanics are operationally identical; that is, for each formulation physicists interpret certain parts of the mathematics in such a way as to make correct predictions of the probabilities of future measurements. Because of these operational interpretations, quantum mechanics works unquestionably well. Nevertheless, these mathematical formulations house intuitively baffling implications. Daniel Styer and his colleges note: "Quantum mechanics seems strange to our classical eyes, so we employ mathematics as our sure guide when intuition fails us. The various formulations of quantum mechanics can repackage that strangeness, but they cannot eliminate it[1]."

There is a second more conceptual level of interpretation of a physical theory. This has to do with the implications of the physical theory to our general worldview. Operationally, this sort of interpretation is irrelevant, even though most formulations were developed with specific conceptual interpretations in mind. This level of interpretation addresses another criterion for assessing the adequacy of a physical theory: the theory needs not only to make predictions that agree with the phenomena, but also ought to provide a satisfactory explanation of these phenomena. Whereas, operationally, the bizarre nature of the quantum world is a brute fact, it is this second level of interpretation that can suggest a method of dealing with the strangeness of quantum physics.

There is a host of these interpretations from which to choose. The Copenhagen Interpretation is historically the most important, since it contains the philosophical foundations from which the earliest formulations of the theory developed. Since this interpretation did not have just one leading spokesperson, it is difficult to capture its essence definitively. However, one important characterization of it seems to be its ontological minimalism. By ontological, I mean referring to what exists. In scientific contexts, ontological entities are often said to exist in reality independently of observers. For example, it may be argued that it is the objectivity of science that gives insight into the nature of what exists in reality. The Copenhagen Interpretation holds that the physical theory need only express what we know about the results of experiments, since this is all we observe. It need not make any claims as to the deeper reality of the physical systems, since this may be beyond our knowledge. Thus, quantum physics makes epistemological, not ontological claims. This sort of scientific understanding is crucial in quantum physics where the observer plays such a key role in the experiment.

Classical physics, on the other hand, retains the luxury of drawing conclusions that are proudly said to be objective, or independent of the particular observer. It is easy and safe to extrapolate these conclusions into ontological claims about the reality of the physical world. In this case, the scientific theory does not assume the additional burden of explaining the phenomena in ontological terms. It need only justify how we acquire scientific knowledge (a task for all of science), and ontological explanation comes along for free.

What then of the mysterious implications of quantum physics which seem to lack this natural ontological explanation? According to an epistemological doctrine, all that can be said is that we know quantum systems act in these certain ways under certain experimental conditions. These sorts of statements are fundamental to the theory. If these statements make the behavior of quantum systems seem counterintuitive, then it is fundamentally counterintuitive.

This point of view is attractive to many scientists, since it emphasizes the operational success or testability of a physical theory rather than its ontological significance, which by the very nature of ontological claims (claims about what exists independently) may not be testable in principle. However, for those who find intuitive explanation indispensable, this sort of interpretation may remain unsatisfactory. This discontent has motivated many to propose other interpretations of quantum mechanics that postulate ontological entities, such as other minds, other worlds, consistent histories and hidden variables, to name a few[2].

Anton Zeilinger hopes to alleviate the need for the postulation of these extraneous ontological entities. He aims to show that when brute quantum strangeness is reduced to its simplest form, it is actually intuitively appealing. He then uses this reorganization to provide a richer explanation of quantum phenomena while maintaining the ontological minimalism of the Copenhagen Interpretation. He does this by proposing a fundamental conceptual principle of quantum mechanics: "An elementary system carries one bit of information." This principle connects the classical notion of the bit of information (the smallest measure of classical information) with the quantum concept of an elementary system (the simplest quantum system.) Quantum physics, in general, only predicts the probabilities of measurement results. Only when the probability of a result is 100% does the probability appear like a proper classical prediction. The information content for such a measure is one bit. Zeilinger's principle states that for an elementary system (such as an electron with respect to its spin) there is exactly one measurement for which such a classical-looking prediction can be made (measurement along a certain axis). Zeilinger claims that this fact accounts for all seemingly counterintuitive phenomena, including two topics this paper will address: wave-particle duality and the randomness of quantum measurements.

To see how Zeilinger adds intuitive appeal to the skeleton of the Copenhagen

Interpretation, consider a specific issue arising from the interpretational debate: that of the reality of the wave function. Bohr, one of the founders of the Copenhagen Interpretation, held that the wave function is a state of the mind describing our knowledge of the system and that the collapse of the wave function is a lifting of our veil of ignorance. Again, this is a purely epistemological interpretation. However, others held that the wave function ought to be interpreted as a description of the 'real state of nature'. There are numerous ways to interpret this phrase. It could mean, for example, that the wave function 'exists,' that it is 'physical,' or that it is part of 'reality.' Each of these possibilities is an ontological claim and carries its own debatable philosophical baggage.

It may seem reasonable to try to attribute some kind of 'reality' to the wave function, given that claims about objective reality within the realm of classical physics flow freely. One way to think about these ontological claims is in terms of "elements of reality," a term coined by Einstein, Podolsky and Rosen in their famous attack on the completeness of quantum physics (the details of which are not relevant here) [3]. According to EPR, an element of reality is a property that can be predicted with 100% certainty without disturbing the system.

It is easy to describe classical properties as elements of reality. Disturbance of a classical system by observation is so slight that it is generally not a problem. Furthermore, classical systems are strongly deterministic such that complete knowledge of initial conditions in principle yields complete knowledge of the system's properties for future times. Thus, attributing elements of reality to classical physical systems seems natural. Indeed, EPR's motivation for developing this concept probably came from a reversed line of reasoning. It seems as though they asked themselves: What are the attributes of the things that are certainly real, like classical systems? The description of element of reality nicely extracts the essence of our natural attitude towards classical scientific investigation as well as everyday observation.

Attributing elements of reality even to classical systems, while harmless to a knowledge-based theory, is not strictly necessary. A persistent bent towards a pure epistemological theory seems like the only reason not to attribute them to classical systems given the conceptual clarity and natural desire to do so. Nevertheless, the above definition of ‘element of reality’ is an operational one that characterizes a classical and well-behaved particle. Thus, the epistemological purist can avoid talk of ontology altogether by strictly using this operational definition instead of mentioning ‘element of reality’. With this in mind, I’ll keep the ‘element of reality’ terminology for simplicity.

We can now use Zeilinger’s principle to attribute this intuitive description of nature to quantum systems. A bit of information *is* an element of reality: it is a certain measurement result (or property) known without disturbing the system since the information is based on past experiences of other identically prepared systems. Thus, the spin of an electron along the correct axis is the element of reality for that system. The wave function certainly describes our state of knowledge of the system, but may also describe a real state of nature. The wave function explicitly describes this real state when it is in an eigenstate of the operator representing the measurement to which the element of reality corresponds.

It seems that elements of reality are discrete. This makes sense given their connection to information. That the world always answers our yes or no questions with a yes or no is our evidence that there is a smallest amount of information: the bit. That an elementary system carries only one bit of information accounts for other bizarre quantum phenomena. For example, no other measurement of the spin of the electron can be predicted with certainty because if it could, then the system would carry more than one bit of information. This and all the conceptually strange results are packaged into one conceptual principle. This principle has intuitive appeal, given its connection to classical physics.

Finally, there is no need to postulate extraneous ontological entities. The only new concept needed is information which is carried in physical systems. It is not a purely ontological concept since it is connected to the knowledge of the observer. However, a bit of information, for all practical purposes has all of the elements that are required for a classical object to have ontological reality. Thus, although those repulsed by new ontological concepts *can* maintain the strict mentality of the Copenhagen Interpretation, those who need a firm grasp of what is real in their physical theories can safely incorporate harmless 'elements of reality' into their worldview. This seems to eliminate the need for those nasty 'other worlds' and such that seem like unfair prices to pay for a little intuitive satisfaction.

## 1.1 Overview

First, I will describe Zeilinger's concept of information, showing how it differs from the more traditional measure of information due to Shannon. This will provide the framework for a careful treatment of Zeilinger's principle which connects the classical bit to the quantum bit (or qubit). Then I will explicitly show that his principle holds for two examples of elementary systems. The first example considered is a quanton (an object small enough to exhibit quantum behavior[1]) of spin one half. The prototypical example of this is the electron. The other elementary system considered is a Mach-Zehnder Interferometer with respect to the measurements which can reveal path information or quanton interference. This analysis shows that a qubit can be represented by a device called a Bloch Sphere which provides a novel way to envision wave-particle duality. I will conclude by reemphasizing how Zeilinger's proposal provides new insight to the classical/quantum divide[2].

[1] Mario Bunge first coined the term *quanton* which fits nicely into the particle zoo already containing protons, mesons, leptons, etc.[4]

[2] In addition to the references mentioned in the endnotes, I have also found the following sources formative in the development of my thesis:

## 2 Information

Although the term "information" comfortably passes through everyday conversations as if it referred to a concrete entity, the concept eludes a precise definition. Nevertheless, our ease in using the term suggests an intuitive understanding of it, and most dictionary definitions agree that it is some sort of knowledge from observation. There are different aspects of information to study such as the content of information (what certain symbols mean), the pragmatic value of information (how useful these symbols are for the receiver's purposes) and the amount of information contained in a system. Physics is concerned with this last aspect of measuring information. Here, 'system' refers to both the physical entity as well as certain measurements that may be made on that entity. I could make many measurements on a coin, for example, its mass, position, etc. However, for simplicity, I want to talk about measurements with only two possible outcomes, like how the coin lands when we flip it.

The most well-established way to measure information is due to Shannon and is the standard measure used in most information theories today. It measures the amount of uncertainty we have about the result of a measurement and thus the amount of information gained or produced by performing the measurement. For example, one bit of information is gained by answering the question, "Did the coin land on heads or tails?" Psychologically, a bit of Shannon information characterizes the maximum amount of surprise we feel when we find out the result of a measurement of a simple system that has only two possible outcomes. (Much more complicated systems can provide much more information or surprise than just the flip of a coin can.) If we

B. Englert, Fringe Visibility and Which-Way Information: An Inequality. Phys. Rev. Let. **77**, 2154 (1996).

U. Fano, Description of States in Quantum Mechanics by Density Matrix and Operator Techniques. Rev. of Mod. Phys. **29**, 74-93 (1957).

Schwindt, P., Kwiat, P., and Englert, B. Quantitative Wave-Particle Duality and Non-erasing Quantum Erasure. Phys. Rev. A **60**, 4285 (1999).

von Baeyer, H. In the Beginning was the Bit. New Scientist Magazine, Feb (2001).

flipped a double headed coin, however, the outcome would be neither interesting nor surprising. On Shannon's view, no information has been gained.

Alternatively, Zeilinger proposes that information measure not the uncertainty, but the amount of positive knowledge that we already have about future measurement results. If I have a fair coin, then I do not know anything about how it will land when I flip it, so I have no information about the measurement. If I have an unfair coin which lands on heads more often than on tails, I feel like I have more information than before, but I still don't know for sure how the coin will land. So I have more than zero bits of information, but less than one bit. However, if I have a double headed coin, then I will always know the answer to the question with certainty, and so I have one bit of information. For both Shannon and Zeilinger, information is a function of the probabilities of measurement results. However, Zeilinger's flips the interpretation of the measure of information upside down, as shown in Figure 1. Furthermore, although Zeilinger's measure looks like a mere flip of Shannon's, it is mathematically and conceptually different. Zeilinger claims that this new interpretation of information is the optimal way to describe quantum systems, whereas some implicit assumptions of Shannon's view are inappropriate to account for certain quantum properties.

## 2.1 Shannon Information

In his 1949 paper, "A Mathematical Theory of Communication," [5] Shannon expands upon the work of H. Nyquist and R.V.L. Hartley[6]. Nyquist worked on how to transport messages efficiently, but did not refer to these messages as "information." Hartley first developed a measure of information to describe the number of different messages that could be produced by a set of $n$ possible symbols. Intuitively, a string of $N$ symbols ought to contain $N$ times the amount of information that one symbol contains. To account for this, Hartley defined the amount of information $H_H$ as the

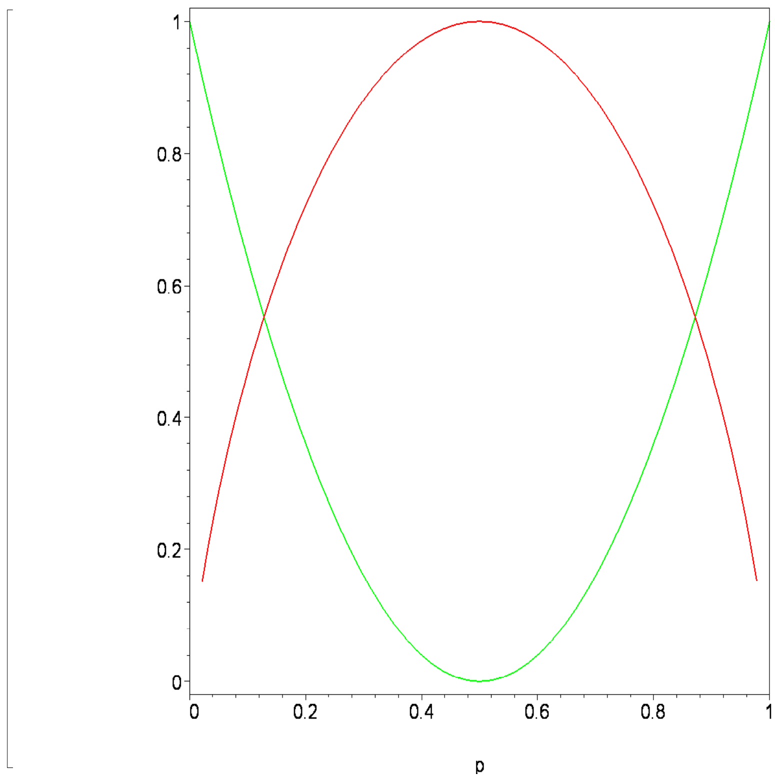


Figure 1: Information content of a two state measurement vs probability $P^{+}$, where $P^{-} = 1 - P^{+}$. This graph shows a conceptual difference between Shannon's and Zeilinger's understandings of information. On Shannon's view, we gain one bit of information from a two state system when the probabilities for each possible measurement are 50% each, and we gain no information if the probabilities are 0 and 1. However, for Zeilinger, if the probabilities are 50/50, then we have no information about the system, but we do have one bit when the probabilities are 1 and 0. (The program used to generate this graph does not support the extreme endpoints of the logarithmic function. )

logarithm of the number of distinguishable messages that can be produced:

$$H_H(n^N) = \log n^N = N \log n. \tag{1}$$

The choice of the base of the logarithm specifies the units used to measure information. In this paper, the systems considered will always be two state systems. Thus, the logarithm will always be to the base 2 and the unit will be the bit (short for binary unit.) This choice allows for the interpretation of information as the number of yes or no questions necessary to determine the exact sequence of the N symbols.

Hartley assumed that all of the symbols had an equal chance of being used. Shannon expanded the concept of information by explicitly including the probabilities of each possible outcome. Thus, we have probabilities $p_1$, $p_2$, ..., $p_n$ for each of $n$ different possible symbols or of $n$ distinguishable outcomes of an experiment. Shannon's measure of information gained from an experiment is

$$H = -\sum_{i=1}^{n} p_i \log p_i. \tag{2}$$

This reduces back to Hartley's measure when all probabilities are equal ($p_i = \frac{1}{n}$) and there is only one symbol produced ($N = 1$).

To calculate the information gained by tossing a fair coin it is necessary to take into account the possibility of the coin landing on heads ($P^+ = \frac{1}{2}$) and tails ($P^- = \frac{1}{2}$). (I will use the notation $P^+$ and $P^-$ instead of $p_1$ and $p_2$ when there are only two possible measurement results.) This yields $\left(-\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\frac{1}{2}\right) = 1$ bit of information. This is expected since it only takes one yes or no question to figure out the outcome of a coin toss. However, to calculate the amount of information gained by tossing three coins in a row, all eight possible outcomes must be taken into account (HHH, HHT, HTH, THH, TTH, THT, HTT, TTT). Each outcome has a 1 in 8 chance of occurring. Thus the information gained is $\left(-\sum_1^8 \frac{1}{8}\log\frac{1}{8}\right) = \log 8 = 3$ bits. This matches our intuition that tossing three coins gives three times as much information as tossing one coin. Thus, when N represents the number of repetitions of the experiment (or the length of a string of symbols as in Hartley's information measure) Shannon also expresses his measure of information as:

$$H = -N\sum_{i=1}^{n} p_i \log p_i, \tag{3}$$

which sums only over the possible outcomes of a single experiment.

A similar example is an eight-sided die. There are eight possible outcomes with equal chances of occurring, so according to Equation 3, there are three bits of information gained from tossing the die. It contains the same amount of information as tossing three coins and requires three yes or no questions to figure out which number was rolled.

All of the possible outcomes in the above examples have an equal chance of occurring, but in general each distinguishable outcome may have a unique probability. By explicitly including these probabilities into the measure of information, Shannon connected the idea of information to uncertainty or missing information. If I have an urn with 99 black marbles and one white marble and I draw one at random, I have a pretty good idea which color it will be; I have very little missing information and thus will gain little information from actually drawing a marble. However, if there are 50 black and 50 white marbles, I am maximally uncertain as to which color I will draw. On Shannon's understanding of information, the more uncertain I am about a measurement, the more information I gain from actually doing the measurement.

For the urn with 99 black marbles and one white marble, the probability of drawing a black marble is, of course, 99%. Operationally, this probability is meaningful only when we repeat the measuring process very many times. 99% of the marbles I draw should be black, but this may not be the case if I only repeat the drawing, say, 100 times.

For an urn filled with N total marbles with n different colors, if I draw one marble after another until all the marbles have been drawn, the exact number of possible sequences, W, is given by the equation

$$W = \frac{N!}{(Np_1)!\,(Np_2)!\ldots\,(Np_n)!}, \tag{4}$$

where $p_1$, $p_2$, ... ,$p_n$ are the probabilities for drawing each different color marble. So $Np_1$ is the number of, say, blue marbles originally in the urn, $Np_2$ the number of red marbles, and so on. However, since these probabilities have meaning only for large N, the Stirling approximation can be used to find that the number of possible combinations is $W \simeq 2^H$. Thus, the information is still interpreted as $\log W$ [7].

With the emergence of quantum information, many have naturally attempted to use Shannon information to describe the information contained in quantum systems. However, Zeilinger shows that some distinctions between classical and quantum systems prove fatal for such an attempt. For example, classical and quantum probabilities are not on equal footing. As mentioned at the beginning of the section, physics is concerned with measuring the information contained in a system, and a given system refers to specific measurements. On Shannon's view, the amount of information contained in a system is equal to the amount of uncertainty that will be removed upon making the relevant measurements. However, knowing everything there is to know about a system (like its mass, position, etc) includes knowing with certainty the outcomes of the measurements in question. In this situation all probability is removed from the system. For example, knowing everything there is to know about a coin being tossed includes knowing with certainty how the coin will land. The Shannon uncertainty quantifies how much knowledge about the measurements of interest is missing. However, even if I know everything there is to know about, say, an electron, I in general do not know with certainty the outcome of measuring the spin along some axis. The probabilities associated with quantum measurements are irreducible. God does not play dice in the classical world, but He does in the quantum world.

This difference is crucial for bit counting. For a system of three coins (or one coin tossed three times), the toss of each coin accounts for one of the three bits of information contained in the system. If Shannon's view were applied to quantum systems, any measurement of, say, the spin of an electron could produce one bit of Shannon information if there is an equal chance of finding spin up and spin down. Furthermore, for an electron initially prepared spin up along the $z$ direction, two consecutive measures of the spin along the $x$, then $y$ axes can produce two bits of information. There is nothing to distinguish these two bits from the two bits of information produced by two flips of a coin. However, quantum mechanics says that after the $y$ direction measurement, the spin is in a superposition of states with respect to the $x$ axis. While this gives knowledge about the spin in the $y$ direction, it also takes away knowledge about spin in the $x$ direction. Shannon's view of information does not account for this loss of knowledge. Zeilinger claims that this unfortunate consequence is a result of some implicit assumptions in Shannon's theory: that new observations always increase knowledge and that the order of experiments does not affect the information content of a system[7].

### 2.2 Zeilinger's Measure of Information

Zeilinger addresses this issue by noting that knowing the probabilities of the measurement results for each mutually exclusive measurement exhausts all there is to know about *all* relevant measurements. Unlike a coin, where there is only one measurement of interest (how the coin lands when tossed), there are an infinite number of measurements to be concerned about for the spin of an electron: the measurements of spin along all axes. Knowing the probabilities of finding spin up and spin down for three orthogonal directions of the electron accounts for all the knowledge of all the measurements of spin in the same way that knowing three spatial components accounts for knowing the exact position of a point in three dimensional space. Thus, the total information of the system ought to sum over the information for each of these measurements. Zeilinger, instead of saying that the electron 'contains' this information, says that the electron 'carries' this information. This terminology avoids the implication that a definite outcome exists prior to measurement, which may be said about a classical measurement when other properties of that system are known. What does exist before a measurement is information that is carried by the *whole system*. Since it exists before the measurement, it must be independent of which measurement the scientist chooses to perform.

Zeilinger's measure of information quantifies the certainty of an experimental result before the experiment takes place. This avoids the difficulty of the loss of knowledge in quantum systems. Information is again a function of the probabilities of the possible measurement results, like Shannon's information. This is convenient since the mathematical formalism of quantum mechanics predicts just these probabilities. Zeilinger, like Shannon, interprets probabilities as a statement about measurement results of large and hypothetical ensembles of identical experiments. The only time when quantum mechanics appears to predict definite measurement results is when the probability is equal to 100%. In this situation, Zeilinger's information shows a sign of overlap with Shannon information since for Shannon information randomness can be be eliminated.

Zeilinger's equation for the information for one measurement of an elementary system is

$$I(P^+, P^-) = 2\left[\left(P^+ - \frac{1}{2}\right)^2 + \left(P^- - \frac{1}{2}\right)^2\right] \tag{5}$$

where 2 is the normalization constant for a two-state system and $P^+$ and $P^-$ represent the probabilities for the two possible outcomes of the experiment (such as the probabilities for finding spin up and spin down in the electron example.) It is easily seen that the probabilities for a completely random result ($\frac{1}{2}$ and $\frac{1}{2}$) yield a measure of zero bits of information, whereas the probabilities of a certain result (0 and 1) yield one bit of information, as expected. While mathematically very different, Figure 2 shows the similarity between Shannon's and Zeilinger's measures of information for

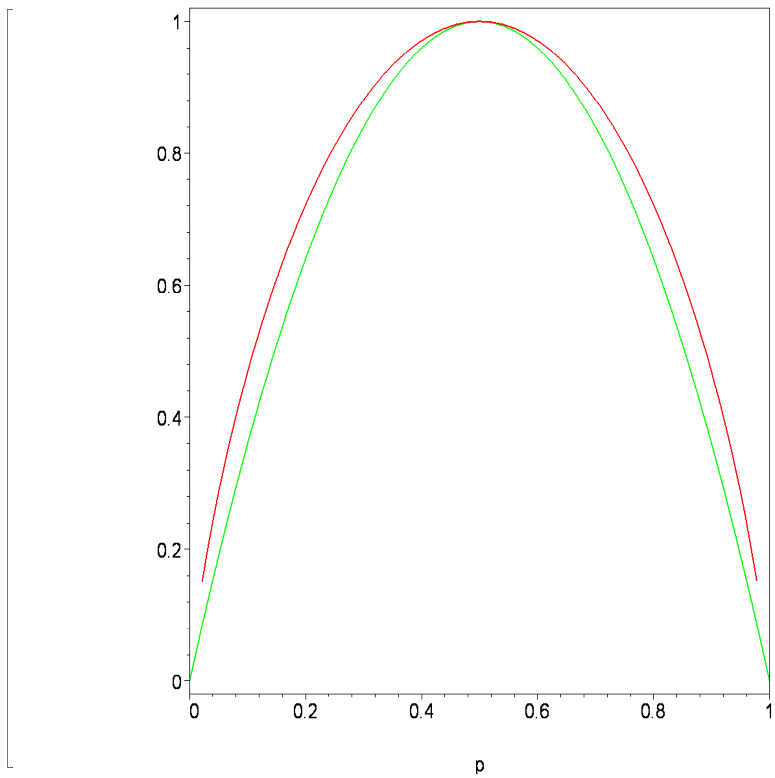


Figure 2: Information content (measured in bits) of a two state measurement vs probability $P^+$, where $P^- = 1 - P^+$. Shannon's logarithmic measure of information is slightly greater that Zeilinger's measure. This graph ignores the flip of Zeilinger's information content that is shown in Figure 1.

a two state system with probabilities $P^+$ and $P^- = 1 - P^+$, ignoring the flip shown in Figure 1.

Zeilinger defines the total information contained in a system so that it is independent of the choice of measurements. For an electron's spin, for example, the total information can be calculated by summing over the individual measures of information for any set of three perpendicular directions. In general, the sum over the information for all mutually exclusive measurements gives the total information for a system. The knowledge required to calculate this information is the probabilities for the outcomes of mutually exclusive measurements. When certain knowledge of the outcome of any one measurement excludes any possibility of making an informed prediction of the outcome of some other measurement, the two experiments are said to be mutually exclusive. For an elementary system, the total information is given by:

$$I_{total} = \sum_{j=1}^{3} I_j(P_j^+, P_j^-), \tag{6}$$

where $j$ sums over the measures of information for the three mutually exclusive experiments.

## 3 Zeilinger's Fundamental Principle of Quantum Mechanics

Zeilinger notes that using his analysis, the measure of information for an elementary system is always one. This motivates his fundamental principle:

$$An\, elementary\, system\, carries\, one\, bit\, of\, information.[8] \tag{7}$$

An elementary system is also known as a qubit to indicate that it carries the smallest unit of quantum information. A classical bit must be either a 0 or a 1, but the information content of a qubit can be in a superposition of these two states[3] mathematically described as $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$, where $|\psi\rangle$ is the superposition state and $|a|^2$ and $|b|^2$ are the probabilities for finding it in the pure state $|0\rangle$ or $|1\rangle$, respectively. Because of this superposition property, qubits have the advantage of handling tasks such as factoring numbers in exponentially less time than classical bits[9]. Physicists comfortably speak about qubits, use them in calculations, and are currently trying to create qubits stable enough to use in a quantum computer (see Appendix C).

Zeilinger's achievement is to connect the realm of quantum information to classical information by showing the relationship between the physical system of the qubit and the bit. Although the qubit is a complex and mysterious container of information, somewhere in it is just a classical bit. That is, there is one and only one measurement

[3]The debate over the meaning of superposition is as intense as the debate over the reality of the wave function. However, whatever 'superposition' means, it still has an undisputed operational definition.

in the quantum system where the result can be predicted with certainty. This special measurement may change with time or may change after some other experiment is performed, but there is always one bit carried in the elementary system. An elementary system is so simple that there is no other hidden information in the system that can help us predict any other measurement results with certainty. It carries the absolute least amount of information that nature will allow. If we ask a yes or no question, we always get a yes or no answer: we never experience ourselves in a superposition of yes and no. Although this seems obvious starting with a classical perspective, it seems strange starting with a quantum perspective where systems are always in superpositions of certain states. Thus, the fact that information is quantized can help not only to understand some strange aspects of quantum mechanics, but also help to understand why the classical world is the way it is.

# 4 Spin One-Half Quanton

The clearest example of an elementary system is a spin one-half quanton, such as an electron. Feynman repeatedly emphasizes that the case of the spin one-half particle is mathematically identical to the cases of all two state systems[4]. Thus, the analysis in this section can be used as a guide for other elementary quantum systems.

## 4.1 Information Content of the System

Here, using Zeilinger's interpretation of information, I will show explicitly that the spin of an electron carries exactly one bit of information. The simplest case occurs

[4] In the **Feynman Lectures on Physics**[10], Feynman states on page 10-17, "So if we can solve the electron problem *in general*, we have solved *all* two-state problems," and "In the next chapter we will look some more into the mathematical techniques for handling the important case of a spin one-half particle – and, therefore, for handling two-state systems in general." Also, on page 11-9: "We only wished to make the point that *all* systems of two states can be made analogous a spin one-half object precessing in a magnetic field."

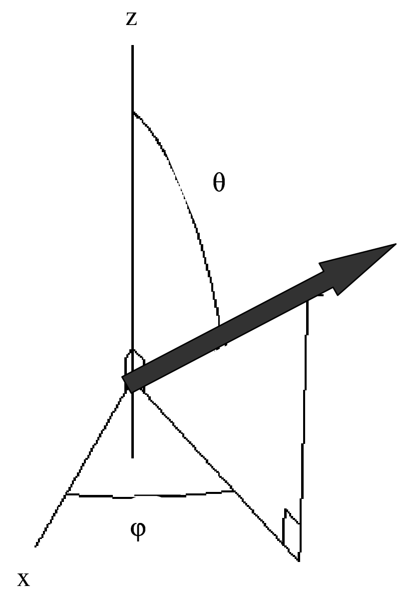


Figure 3: The vector, indicating the direction of the prepared spin, is labeled by two parameters, $\theta$ and $\phi$.

when an ensemble of electrons is prepared to be spin up along some direction[5] (call it the $z$ direction). Then the probability of measuring and finding that electron with spin up in the $z$ direction again is 100%, so that the information with respect to this measurement is $I_z = 1$ bit. Furthermore, for all measurements of spin in orthogonal directions (including the $x$ and $y$ directions), the results will be completely random. The information content for measurements in these directions are: $I_x = I_y = 0$ bits. Thus, the electron carries one bit of total information.

More generally, the electron carries one bit of information regardless of the direction in which its spin has been prepared. Feynman computed the probability that an electron will have its spin up or down along a certain axis, given that the electron was prepared with its spin up along some arbitrary direction labelled by the polar coordinates $\theta$ and $\phi$, as in Figure 3[10]. The probability depends only on the angle $\Omega$, which is the angle between the initial direction and the axis of interest. These

[5]This can be done by directing a bunch of electrons through a Stern Gerlach apparatus set up at the appropriate angle and discarding the spin-down electrons.

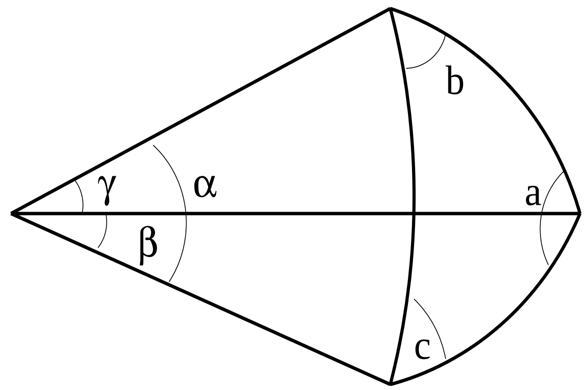


Figure 4: The law of cosines for spherical trigonometry relates the angles of the wedge of a sphere.

probabilities are:

$$P^+ = \cos^2\left(\frac{\Omega}{2}\right) \qquad \text{and} \qquad P^- = \sin^2\left(\frac{\Omega}{2}\right). \tag{8}$$

According to Figure 3, $\Omega_z$, which is the angle between the axis of the prepared spin and the $z$ axis, is just $\theta$, so the probabilities to find the spin up and spin down along the $z$ axis are:

$$P_z^+ = \cos^2\left(\frac{\theta}{2}\right) \qquad \text{and} \qquad P_z^- = \sin^2\left(\frac{\theta}{2}\right). \tag{9}$$

To find $\Omega$ with respect to the other two axes, we invoke the law of cosines for spherical trigonometry:

$$\cos\gamma = \cos\alpha\,\cos\beta + \sin\alpha\,\sin\beta\,\cos c, \tag{10}$$

where $\gamma$, $\alpha$, $\beta$ and $c$ are indicated in Figure 4. To find $\Omega_x$, the angle between the prepared spin and the $x$ axis, we use $\gamma = \Omega_x$, $\alpha = \phi, \beta = 90^\circ - \theta$ and $c = 90^\circ$. Then,

$$\cos\Omega_x = \cos\phi\sin\theta. \tag{11}$$

Thus, according to Equation 8, the probabilities for finding spin up and down along the $x$ axis are:

$$P_x^+ = \frac{1}{2} + \frac{1}{2}\cos\phi\sin\theta \qquad \text{and} \qquad P_x^- = \frac{1}{2} - \frac{1}{2}\cos\phi\sin\theta. \tag{12}$$

For the $y$ axis, we use $\gamma = \Omega_y$, $\alpha = 90^\circ - \phi, \beta = 90^\circ - \theta,$ and $c = 90^\circ$. Therefore,

$$\cos\Omega_y = \sin\phi\sin\theta, \tag{13}$$

so the probabilities are:

$$P_y^+ = \frac{1}{2} + \frac{1}{2}\sin\phi\sin\theta \qquad \text{and} \qquad P_y^- = \frac{1}{2} - \frac{1}{2}\sin\phi\sin\theta. \tag{14}$$

Thus, from Equation 5, the information content for the measurements of spin along the $x, y$, and $z$ axes are:

$$I_x = \cos^2\phi\sin^2\theta, \qquad I_y = \sin^2\phi\sin^2\theta, \qquad \text{and} \qquad I_z = \cos^2\theta. \tag{15}$$

The summation of these measures yields one bit of information for the spin one half quanton:

$$I_{total} = \sum_{j=1}^{3} I_j\left(P_j^+, P_j^-\right) = 1. \tag{16}$$

### 4.2 The Qubit Interpretation

We can represent the information that a qubit carries by a unit vector fixed to and pointing away from the origin (see Figure 5). All possible positions of this vector trace out a sphere of unit radius. This is known as the Bloch Sphere[11]. The sphere lies in three-dimensional "information space." The two states of any measurement of the system correspond to two opposite points on the sphere. Any three orthogonal axes represent mutually complementary experiments, each with two possible outcomes. The projection of the unit vector onto any axis indicates the information content with respect to that axis. Three such projections onto any set of three mutually complementary axes is sufficient to calculate the total information of the system. Since there is only one bit, all other axes not aligned with the vector represent measurements with some element of necessary randomness. Axes perpendicular to the vector represent the measures that are maximally and necessarily random.

The three spatial dimensions of the electron correspond directly to the three dimensions of information space in the Bloch sphere. The direction in which the spin was prepared corresponds to the direction of the unit vector in the Bloch sphere. Thus, the electron is the clearest example of an elementary system as a qubit.

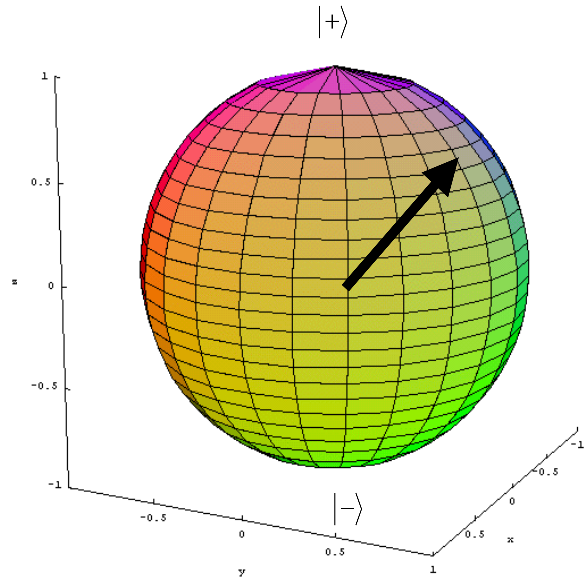


Figure 5: All possible positions of a unit vector trace out the Bloch Sphere in three-dimensional information space. The vector represents a bit of information. It is pointing in direction that indicates a certain measurement with 100% probability of some result.

## 5 Wave-Particle Duality

In 1806, Young demonstrated the interference of light with his double slit experiment. Since then, modifications of his experiment have been used to show that individual quantons display both wave behavior and particle behavior. It is important to note that wave and particle "behavior" (or wave and particle "nature") refers to the outcomes of specific experimental conditions. That is, certain experimental conditions show evidence of interference which is said to be the mark of wave behavior, and other experimental conditions reveal evidence of the path taken by the quanton travelled (often called 'Which Way knowledge') which is said to be the mark of particle behavior.

Wave and particle behavior are said to be complementary aspects of the same quantum system, because when the experiment reveals evidence of complete interference, there can be no Which Way knowledge, and vise versa. In 1979, Wooters and

Zurek reported that while the above statement is certainly true, it is also possible to observe partial interference evidence and have partial Which Way knowledge[12]. Analyzing this topic using Zeilinger's concept of information for a two state system shows a refined relationship between wave and particle behavior.

The double slit experiment, however, proves cumbersome for these purposes. This is because, although there are two distinct states for path information, interference patterns are observed on a continuous screen[6]. The double slit experiment may be easily replaced with a Mach-Zehnder Interferometer made of simple beam splitters. Output revealing interference of quantons is discrete in the Mach-Zehnder Interferometer allowing for the convenient analysis of wave particle duality in terms of information.

### 5.1 The Beam-Splitter

A beam-splitter will transmit part of a classical beam of light and reflect the rest of it. The physical composition of the beam splitter determines what fractions of the light are transmitted and reflected. Here, we discuss mainly 50/50 beam-splitters which split a classical beam of light into two beams of equal intensity. A symmetric beam splitter is one that has the same effect on light from either direction. Energy conservation ensures that the two output beams of a symmetric beam-splitter always have a relative phase difference of $\frac{\pi}{2}$ (see Appendix 1). In quantum mechanics, physicists say that the *probability wave* is split half and half, so that if a single quanton is shot at a beam splitter and there are two detectors set up to find out if it was transmitted or reflected, it will end up in each detector 50% of the time. Thus, the beam splitter is a primitive quantum device that offers a simple way to extract the

[6]It is possible to treat the double slit experiment as a two state system by setting up detectors only at certain places within the interference pattern, making it operationally equivalent to a Mach-Zehnder Interferometer treated below. However, it is much easier to deal with the Mach-Zehnder Interferometer itself, especially since we treat it as ideal.

inherent randomness from quantons into physical output[7].

## 5.2 The Mach-Zehnder Interferometer

A Mach-Zehnder Interferometer[8] is a construction of two identical symmetric beam-splitters and two perfectly reflecting mirrors as shown in Figure 6. Like the double slit experiment, this device demonstrates the interference of light or quantons. Instead of analyzing entire interference patterns from a projection screen, as in the double slit experiment, the Mach-Zehnder interferometer has only two possible output arms, making data analysis much easier to handle. Evidence of constructive and destructive interference can be detected from the data collected from just these two places.

### 5.2.1 Classical Light

In a Mach-Zehnder interferometer, a beam of laser light is split by the first beam-splitter into two paths. Both of these beams are reflected back by perfect mirrors to the second beam-splitter where each beam is again split into a reflected and a transmitted beam. The final two output beams of light have contributions from both paths of the interferometer, so that interference is possible. The beam that exits out of the "asymmetric" output arm is composed of one beam that is reflected twice and one that is transmitted twice. Both beams that exit from the "symmetric" output arm have been reflected once and transmitted once.

Consider an ideal Mach-Zehnder Interferometer constructed with two identical

[7] Zeilinger's group has recently employed a beam splitter to build a true random number generator. Classical computers using algorithms to generate "pseudo-random" numbers. However, since the beam-splitter is a quantum system, the production of the number is an irreducibly random process, as discussed in the introduction. A description of Zeilinger's device can be found at http://quantum.univie.ac.at/index.shtml.

[8] We assume that the interferometer is ideal: the optical devices are lossless and work perfectly and the internal paths are exactly the same. This takes us far from experimental practicalities. Experimentally, nobody worries about making the lengths of the paths exactly equal, for example. However, these simplifications are helpful for clarifying how the interferometer works.

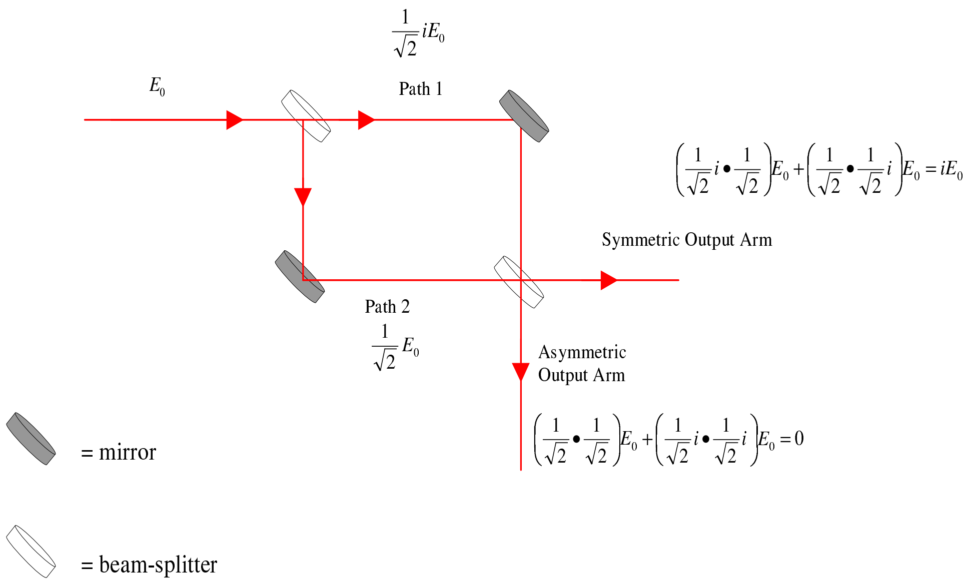


Figure 6: The Mach Zehnder Interferometer for classical light: Laser light is incident upon a symmetric 50/50 beam splitter which splits the light into two paths. Each beam is reflected back to a second beam splitter, which again splits each beam into a reflected and transmitted beam. Thus, both outgoing beams are composed of two beams each so that interference is possible. The beam exiting from the "symmetric" output arm is composed of two beams of light that have both been transmitted once and reflected once, whereas the beam exiting from the "asymmetric" output arm is composed of one beam that has been transmitted twice and one that has been reflected twice. The electric fields are written using the convention that a reflected beam picks up no phase shift and the electric field of a transmitted beam picks up a phase shift of $\frac{\pi}{2}$.

50/50 symmetric beam splitters. Assume, for simplicity, that the length of the two paths inside the interferometer are equal and that the phase shifts due to the two mirrors are equal ($\phi_{M1} = \phi_{M2}$). Also, let the phase of the initial light from the source be 0 so that the initial electric field is just $E_0$. Let $\phi_T$ and $\phi_R$ be the phase shifts due to transmission and reflection of a beam splitter respectively. Then, the electric field of the light transmitted through $B_{in}$ into path 1 is:

$$E_1 = \frac{1}{\sqrt{2}} e^{i\phi_T} E_0, \tag{17}$$

and the field for the light reflected into path 2 is:

$$E_2 = \frac{1}{\sqrt{2}} e^{i\phi_R} E_0. \tag{18}$$

The field for the beam leaving the symmetric output arm is a superposition of two waves:

$$E_S = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} e^{i\phi_R} E_0 \right) e^{i\phi_T} + \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} e^{i\phi_T} E_0 \right) e^{i\phi_R} = E_0 e^{i(\phi_T + \phi_R)}. \tag{19}$$

Thus, the intensity of this beam must be equal to $I_0$, the intensity of the initial beam:

$$I_S = |E_S|^2 = I_0. \tag{20}$$

The field for the asymmetric output arm is:

$$E_A = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} e^{i\phi_R} E_0 \right) e^{i\phi_R} + \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} e^{i\phi_T} E_0 \right) e^{i\phi_T} = \frac{1}{2} E_0 \left( e^{i2\phi_T} + e^{i2\phi_R} \right). \tag{21}$$

The intensity of this beam must vanish since all of the intensity exits out of the symmetric output arm:

$$I_A = |E_A|^2 = \frac{1}{4} E_0 \left( 2 + e^{i2(\phi_T - \phi_R)} + e^{-i2(\phi_T - \phi_R)} \right) = \frac{1}{4} E_0 \left[ 2 + 2\cos 2\left(\phi_T - \phi_R\right) \right] = 0, \tag{22}$$

so that $-1 = \cos\left(2\left(\phi_T - \phi_R\right)\right)$ and, thus $\phi_T - \phi_R = \frac{\pi}{2}$. (This is derived in general for any symmetric beam splitter in Appendix 1.)

We can now write down the electric fields in each part of the interferometer using the convention mentioned in Appendix 1, where $\phi_R = 0$ and $\phi_T = \frac{\pi}{2}$ (as in Figure

6). Half of the light is transmitted into Path 1 with an electric field of $\frac{1}{\sqrt{2}}iE_0$, and the other half is reflected into Path 2 with an electric field of $\frac{1}{\sqrt{2}}E_0$. The symmetric output arm is a combination of the reflected light from Path 1 and the transmitted light from Path 2:

$$E_S = \left(\frac{1}{\sqrt{2}}i \cdot \frac{1}{\sqrt{2}}\right) E_0 + \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}i\right) E_0 = iE_0. \tag{23}$$

The asymmetric output arm is composed of the reflected light from Path 2 and the transmitted light from Path 1:

$$E_A = \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\right) E_0 + \left(\frac{1}{\sqrt{2}}i \cdot \frac{1}{\sqrt{2}}i\right) E_0 = 0. \tag{24}$$

This confirms that when the two paths are exactly equal there is complete constructive interference in the symmetric output arm and complete destructive interference in the asymmetric output arm.

A unique feature of the Mach-Zehnder Interferometer is that the intensities of the two output waves (at $S$ and $A$) as a function of the phase difference between in the two paths of the interferometer are always offset by $\frac{\pi}{2}$. The total phase of one of the beams of light inside the interferometer can be altered by changing one of the path lengths. As the phase is slowly altered, the intensity of the symmetric beam decreases and the intensity of the asymmetric beam increases until all of the light exits out of the asymmetric output arm at a phase change of $\frac{\pi}{2}$. This reverses as the phase change continues to increase (as seen is Figure 7).

If the phase difference between paths 1 and 2 is held at zero and the reflection and transmission coefficients vary instead, the intensities of the output beams also change. In general, the intensity of light from the symmetric output arm is $4\,(RT)^2$, where $R$ and $T$ are the reflection and transmission coefficients, respectively. Since energy is conserved, the intensity for the asymmetric output arm is $1 - 4\,(RT)^2$. At the extreme, when the beam splitters are replaced by glass plates or perfect mirrors, all of the intensity exits out of the asymmetric output arm.

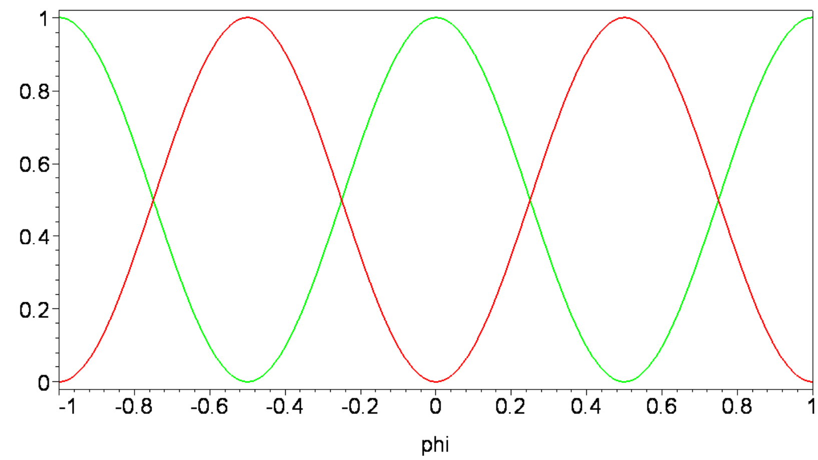


Figure 7: The intensities of the beams exiting out of the symmetric output arm (red) and the asymmetric output arm (green) as a function of the phase difference between the two paths inside the interferometer. The angle $\phi$ is plotted in units of $\pi$. The intensity is plotted in units of the initial beam's intensity, $I_0$.

### 5.2.2 Quantum Light

Since classical light is analogous to quantum probability waves, the analysis of the Mach-Zehnder interferometer may be extended to quantons, in particular, to photons which are quantized bits of light. If a quanton is sent into a Mach Zehnder Interferometer as described above with equal path lengths, it will exit out of the symmetric output arm 100% of the time. Dirac notation is a clear and convenient way to express this explicitly, showing wave functions as superpositions of states. For example, Feynman[10] writes the state describing a double slit experiment using Dirac notation, invoking what he calls the second general principle of quantum mechanics:

> When a particle can reach a given state by two possible routes, the total amplitude for the process is the sum of the amplitudes for the two routes considered separately. In our new notation (Dirac notation) we write that

$$|s\rangle_{both\,holes\,open} = |s\rangle_{through\,1} + |s\rangle_{through\,2}\,. \tag{25}$$

Likewise, the state of a quanton can be in a superposition of two possible paths through the interferometer that has 50/50 beam splitters:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right), \tag{26}$$

expressed in the basis states $|1\rangle$ and $|2\rangle$ which refer to paths 1 and 2 of the interferometer. (This state is normalized, unlike Feynman's.)

To calculate the probability that the initial state will evolve into some final state using Dirac notation, we take the square of the amplitude of the inner product of the initial state with the final state. We calculate the probability that the quanton will exit out of the symmetric or asymmetric output arm as a function of two parameters introduced in the system. The first is an adjustable phase shifter, such as the one described above. The second variable controls the amount of Which Way marking available in the system.

#### 5.2.3 Entanglement

We introduce full Which Way marking into a system with initially polarized light by inserting a polarization rotator into Path 2 of the interferometer. The polarization rotator turns the polarization by 90° with no loss of intensity. The state of the quanton can now be described by:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle\,|0^\circ\rangle + |2\rangle\,|90^\circ\rangle\right) \tag{27}$$

The polarizations of these photons are now said to be completely entangled with the Which Way information. This means that if the polarization is measured after the quanton passes through the entire system, then we know with certainty through which path of the interferometer the quanton travelled. This "Which Way" knowledge is

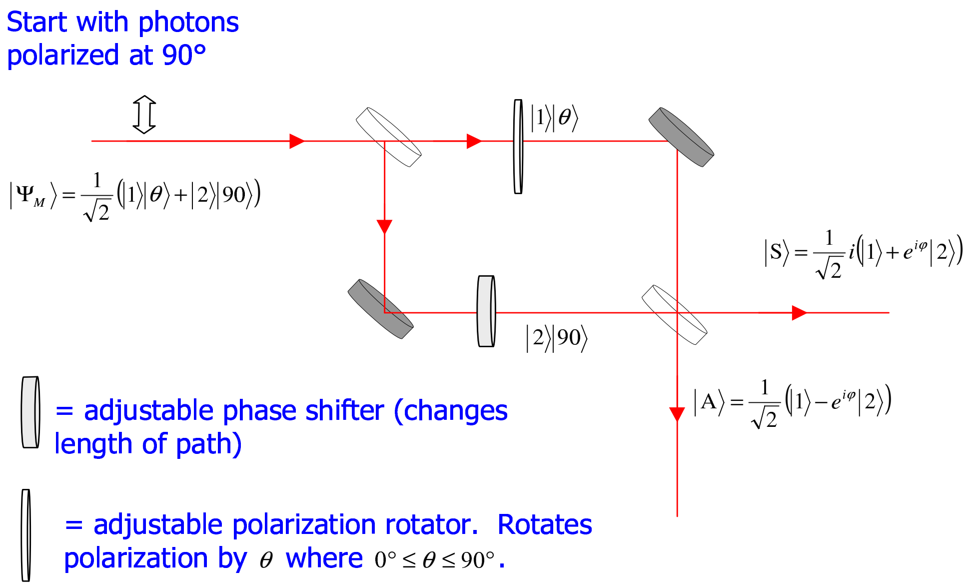


Figure 8: Photons enter the interferometer polarized at 90°. An adjustable polarization rotator rotates the photons in Path 1 by an angle $\theta$. An adjustable phase shift is inserted into Path 2.

usually said to correspond to particle nature whereas interference is said to correspond to wave nature. If Which Way information is known with certainty, then there will be no detectable interference effects. Likewise, evidence of perfect interference (such as photons exiting from the symmetric output arm 100% of the time) excludes Which Way information. Thus, measurements that reveal Which Way information and interference information are mutually exclusive. This explains the disappearance of interference patterns for a fully marked system. However, as long as the paths are marked, the polarization does not actually have to be measured. As long as measurement is possible, that is, there is the “threat” of measurement, the interference effects will be absent. This gives rise to the possibility of a quantum erasure, which can eliminate the possibility of measuring the Which Way information by erasing the polarization marking and thus regaining evidence of interference.

In the marked Mach-Zehnder system, if the polarization rotator is allowed to be adjustable, it is possible to observe evidence of partial which way knowledge and partial interference that was first reported by Wooters and Zurek. Varying the angle

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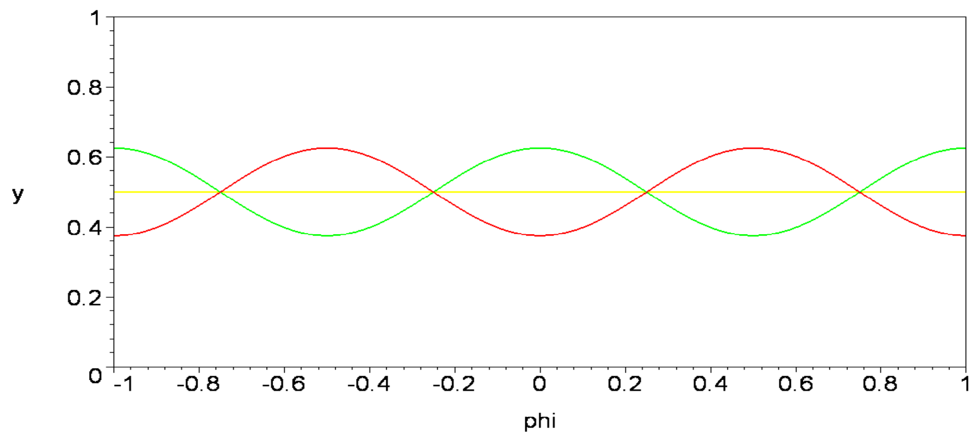


Figure 9: The intensities of the beams exiting out of the symmetric and asymmetric output arms as a function of the phase difference between the two paths inside the interferometer when the polarization is partially entangled with the path information. The angle $\phi$ is ploted in units of $\pi$. The intensity plotted in units of $I_0$. When the path information is fully entangled, this plot becomes a flat horizontal line at half the initial intensity.

of the polarization rotator can change the polarization from $0°$ to $90°$. A plot of intensity verses the phase shift, $\phi$, is shown in Figure 9 for partial entanglement. When there is full entanglement, there is no interference effects and this plot becomes a flat horizontal line at half of the initial intensity. This now gives two parameters to completely describe the system: a phase shift, $\phi$, and the angle of the polarization, $\theta$. The probability that an interference pattern will emerge can now be calculated in terms of these two parameters.

### 5.2.4 Information Content of the System

Using the example of the spin one-half electron, we can show that the information carried by a Mach-Zehnder Interferometer is equal to one. The initial state of the system is:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right). \tag{28}$$

This means that if detectors are set up in Path 1 and Path 2 and many quantons are sent into the interferometer, each detector would fire half of the time[9]. The states are orthogonal so that:

$$\langle 1|1\rangle = \langle 2|2\rangle = 1 \qquad \text{and} \qquad \langle 1|2\rangle = \langle 2|1\rangle = 0. \tag{29}$$

We start with only the adjustable phase shift of $\phi$ in Path 2. To find the probability that the quanton exits from a certain output arm, we need to write down the state that is a superposition of the two possible paths that the quanton could take to get to its destination (using the usual conventions for the phase shifts due to the beam splitters). The final state of exiting through the symmetric output arm for a given phase shift $\phi$ is given by the superposition:

$$|S\rangle = \frac{1}{\sqrt{2}} i\left(|1\rangle + e^{i\phi}|2\rangle\right). \tag{30}$$

The state of exiting from the asymmetric arm is given by:

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle\left(i^2\right) + e^{i\phi}|2\rangle\right) = \frac{1}{\sqrt{2}}\left(-|1\rangle + e^{i\phi}|2\rangle\right). \tag{31}$$

The probability of the initial state $|\psi\rangle$ evolving into the final states $|S\rangle$ and $|A\rangle$ are given by:

$$|\langle\psi|S\rangle|^2 = \left|\left(\frac{1}{\sqrt{2}}\left(\langle 1| + \langle 2|\right)\right)\left(\frac{1}{\sqrt{2}}\left(|1\rangle + e^{i\phi}|2\rangle\right)\right)\right|^2 = \frac{1}{4}\left|1 + e^{i\phi}\right| = \frac{1}{2}\left(1 + \cos\phi\right), \tag{32}$$

and

$$|\langle\psi|A\rangle|^2 = \left|\left(\frac{1}{\sqrt{2}}\left(\langle 1| + \langle 2|\right)\right)\left(\frac{1}{\sqrt{2}}\left(-|1\rangle + e^{i\phi}|2\rangle\right)\right)\right|^2 = \frac{1}{4}\left|-1 + e^{i\phi}\right| = \frac{1}{2}\left(1 - \cos\phi\right). \tag{33}$$

Thus, as the angle $\phi$ runs through a cycle of 0 to $2\pi$, the probability of the quanton exiting out of the symmetric output arm oscillates from 1 to 0 to 1 again, while the

[9]The probability that the state $|\psi\rangle$ will be found in the pure state $|1\rangle$ or $|2\rangle$ is given by $|\langle 1|\psi\rangle|^2 = \frac{1}{2} = |\langle 2|\psi\rangle|^2$.

probability for the asymmetric output arm oscillates from 0 to 1 to 0 as shown in Figure 7. This is clear evidence of "wave-like" superposition.

Now the polarization marker is introduced. The initial polarization is 90° and a half wave plate is inserted into path 1 such that the polarization is at an angle $\theta$, where $\theta$ can vary from 0° to 90°. The initial state can now be described by:

$$|\psi_M\rangle = \frac{1}{\sqrt{2}} \left( |1\rangle \, |\theta\rangle + |2\rangle \, |90\rangle \right) \tag{34}$$

The relation $\langle\theta|90\rangle = \sin\theta$ obeys the boundary conditions (the orthogonality conditions given above) for $\theta = 0°$ or 90°. The probability of finding this new marked state in the symmetric output arm is state $|S\rangle$ is given by:

$$\begin{aligned} P^S = |\langle\psi_M|S\rangle|^2 &= \left| \left( \frac{1}{\sqrt{2}} \left( \langle 1| \, \langle\theta| + \langle 2| \, \langle 90| \right) \right) \left( \frac{1}{\sqrt{2}} \left( |1\rangle + e^{1\phi} \, |2\rangle \right) \right) \right|^2 \\ &= \frac{1}{4} \left| \langle\theta| + \langle 90| \, e^{i\phi} \right|^2 = \frac{1}{2} \left( 1 + \sin\theta\cos\phi \right) \end{aligned} \tag{35}$$

Since probability must be conserved, the probability that the quanton exits out of the asymmetric output arm is just:

$$P^A = 1 - P^S = \frac{1}{2} \left( 1 - \sin\theta\cos\phi \right). \tag{36}$$

If $\theta$ is 90°, then the system is completely unmarked and the interference pattern is the same as the results above. If $\theta = 0°$, then the probability is independent of $\phi$ ($P = \frac{1}{2}$), and there is no interference.

Three complementary measurements of the Mach-Zehnder Interferometer are analogous to three orthogonal directions in the electron example. This analogy can be used to calculate the total information in the Mach-Zehnder Interferometer system. Given arbitrary values for the two parameters $\theta$ and $\phi$, three new sets of values can be written to correspond to the three general orthogonal directions in the electron example: $\theta_1 = 90°$, $\phi_1 = \phi + 90°$; $\theta_2 = \theta$, $\phi_2 = \phi - 90°$ and $\theta_3 = \theta - 90°$, $\phi_3 = \phi - 90°$. Inserting these values into the Mach-Zehnder probability equations yields the probabilities for three complementary measurements:

$$P_1^{S,A} = \frac{1}{2} \pm \frac{1}{2}\cos\phi, \qquad P_2^{S,A} = \frac{1}{2} \pm \frac{1}{2}\sin\theta\sin\phi, \qquad \text{and} \qquad P_3^{S,A} = \frac{1}{2} \pm \frac{1}{2}\cos\theta\sin\phi. \tag{37}$$

The information equation then yields:

$$I_1 = \cos^2\phi, \qquad I_2 = \sin^2\theta\sin^2\phi, \qquad \text{and} \qquad I_3 = \cos^2\theta\sin^2\phi. \tag{38}$$

The sum of these measures of information again yields 1:

$$I_{total} = \sum_{j=1}^{3} I_j(P_j^+, P_j^-) = 1. \tag{39}$$

Thus, both the spin one-half system and the Mach-Zehnder Interferometer system carry one bit, the smallest amount of information possible. From this it follows that knowing the result of one measurement with certainty forbids any informed prediction of the result of a complementary measurement.

### 5.3 The Qubit Interpretation of Wave-Particle Duality

Mutually exclusive experiments are distinct experiments that can be performed on one system (or many identically prepared systems). Because they measure distinct properties, they ask different questions about the system. In the familiar case of the spin one half electron, each mutually exclusive experiment asks a question about spin in a certain direction: "What is the spin in the $x$ direction,?" "...in the $y$ direction?" "...in the $z$ direction?" Since spin only has two states, there are only two answers to any of these questions.

Suppose I am provided with an unlimited number of electrons all of whose spins have been measured to be in the same direction. I am then asked in which direction their spin has been prepared. Since I have no idea which direction it may be, I just start measuring along different directions and using the measurement results to make educated guesses as to which direction I should try next. If I find the direction that always gives, say, spin up, then I have completed the task. However, there is another way to almost complete the task. If I know that measuring the spins along, say, the $x$ and the $y$ axes always yield 50% spin up and 50% spin down, then I know that the spin must have been prepared either spin up or spin down along the $z$ axis, but I do not know which.

We know this just because we understand well how an electron's spin works. But from Zeilinger's principle we could have described this in terms of information. A completely random result for an experiment means that there is no information about the system with respect to that measurement. But since we know that a two state quantum system carries one bit of information, the information must lie elsewhere in the system. Specifically it must lie in a measurement of an orthogonal direction, otherwise the first measurement would have revealed some tendency towards spin up or spin down. Completely random results for two different mutually exclusive measurements (say, measurements along the $x$ and $y$ axes) mean that the information must be located in the third mutually exclusive measurement. We don't know what the content of the information is (whether the spin is up or down) but we do know that the information is there and that there is an experiment that we can perform to answer the question, "Is the spin up or down along the $z$ axis?"

Thus, the information has some sort an objective character. It is there whether we know the measurement result or not. Thus, the information is not necessarily information that *we have.* In Zeilinger's words, it is information carried by the system.

In the double slit experiment for light of a given wavelength, I could set up a photon detector at a maximum peak of the interference fringes and one at the next minimum peak. Call this experimental setup A. If I then ask the question, "When a photon is detected in experimental setup A, which detector will measure it?" there are two possible answers. However, I will know with certainty that the detector set up at the maximum peak will always detect the photon when one is in fact detected (assuming the interference is perfect). I could set up another experiment by shifting both detectors over so that they were both exactly between a maximum and a minimum peak. Call this experimental setup B. Then when I ask the different question: "When a photon is detected in experimental setup B, which detector will measure it?" I don't know the answer until I perform the experiment and detect a photon in one of the detectors. I do know that the answer to the question will be completely random. Thus, these two measurements are mutually exclusive.

A better way to experimentally realize these two mutually exclusive measurements is with a Mach-Zehnder interferometer. Experimental setup A is now the ideal Mach-Zehnder interferometer where the lengths of the two paths are exactly equal. The question that this experiment asks is, "In experimental setup A, from which output arm does the photon leave?" I know with certainty that the answer will be the symmetric output arm. I also know that if one of the paths were half a wavelength longer than the other, then all the photons would exit out of the asymmetric output arm. However, if one of the paths were just a fourth of a wavelength longer than the other (the same as putting a $\frac{\pi}{2}$ phase shift into the ideal system), then half of the photons would exit out of the symmetric output arm and half out of the asymmetric. This is the new experimental setup B, and I can ask the question, "In experimental setup B, from which output arm does the photon leave?"

The third mutually exclusive experiment asks, "Which path did the photon take through the interferometer?" If the answer to this question is known, then the exit path of the photon and thus the answers to the first two questions will always be random. There are, however, many experimental ways to ask this question and to extract which way information (I used polarization markers, but it is also possible to use very gentle detectors set up in paths 1 and 2 to find Which Way information.) What I can conclude immediately, by analogy with the electron example, is that if I find that the answers to the first two questions are random, then there must be which path information available somewhere in the system. I don't even need to know which path the photon took to know that there is which path information available somewhere.

What can be concluded about wave-particle duality from these mutually exclusive measurements? First, it is important to note that the terminology is misleading. Classically, when two beams of orthogonally polarized light do not interfere, the beams themselves are not acting like particles, they are acting like waves that just are not mixing. Quantum mechanics says that quantons can always be described as deBroglie waves. This means that each quanton is always associated with a probabilistic distribution. However, when there is no interference in the interferometer, the probabilistic behavior that the quanton displays is the same behvior that a classical particle would display if it bounced off the beam splitter half of the time and traveled through the beam splitter half of the time. So it is safe to call them 'particles', even though the description of the quanton as a deBroglie wave never goes away.

On the other hand, quantons are, of course, quantized. For example, photons are discrete bundles of energy. There is no wavelike distribution of the photon when it hits a detector. Knowing that interference is occuring requires the observation of many of these discrete particle-like quantons in the detectors. Thus, exhibiting the 'wave nature' referred to in interferometry experiments does not exclude all particle aspects of the quanton, and vise versa. Keeping this in mind, better descriptions of 'particle nature' and 'wave nature' are 'Which Way information' and 'interference information,' even though the wave-particle description is more traditional[10].

In the experiment described, $\theta$ can be interpreted as the mixing angle, where $0°$ forbids mixing and $90°$ allows for total mixing. The phase difference between the two waves is described by the angle $\phi$. As above, the total system is described with the

[10] Below, I will write 'wave nature' and 'particle nature' to indicate the specific wave and particle properties relevant to interferometry experiments. These can always be replaced by their information equivalents.

probability formula derived above, $P^{+} = \frac{1}{2}(1 + \sin\theta\cos\phi)$. If I observe that if the measurement results for the first two experiments both turn out to be random (that is, if $\phi = 0$, then $P^{+,-} = \frac{1}{2}$ and if $\phi = \frac{\pi}{2}$, then $P^{+,-} = \frac{1}{2}$), then it follows from the probability formula that $\theta$ must be $0°$ and thus there is no mixing so there can be no interference information. But if $\theta = 0°$, then there is definite Which Way information available (or traditionally, the quantons display 'particle nature.')

The abstract representation of a qubit gives a novel way to envision what is traditionally referred to as wave-particle duality. This duality is normally described linearly, with wave nature at one extreme and particle nature at the other. The behavior of the quanton lies somewhere in between. The qubit representation by means of a Bloch Sphere takes into account an important continuous property of the wave, its phase, which can vary from 0 to 360°. Pure 'particle behavior' corresponds to the arrow pointing to one of the two poles of the sphere, whereas 'wave behavior' corresponds to the arrow aligned towards the equator (depending on its phase.) Any other alignment represents a combination of wave and particle behavior. The Bloch sphere clearly shows that these types of 'wave' and 'particle' behaviors are really just different manifestations of information, just as in the spin one-half example. By asking different questions (performing different experiments by choosing the angles $\theta$ and $\phi$), the experimenter chooses the type of information she would like to have.

## 6 Conclusions

A final perspective for discussing the fundamental nature of information comes from Heisenberg's description of the Copenhagen Interpretation[13]. He claims that quantum physics begins with a paradox: "It starts from the fact that we describe our experiments in the terms of classical physics and at the same time from the knowledge that these concepts do not fit nature accurately. The tension between these two starting points is the root of the statistical character of quantum theory." Things in our classical world do not seem to have the strange characteristics that we describe quantum systems to have. That is, classical objects always have definite properties and are never in superpositions of states. Also, because of quantum theory's irreducible statistical nature, quantum physics does not live up to classical standards. The conflict between classical and quantum physics is well maintained through debates concerning the so-called measurement problem.

This paradox is fundamental to the Copenhagen Interpretation. Its two starting points motivate two different fundamental questions of quantum mechanics. Wheeler asked the first question: "How come the quantum?" [14] and Zurek responded with the question "How come the classical?" [15] Zeilinger's contribution to this discussion is to show the connection between the two opposing views by introducing one fundamental concept, the bit of information. This becomes the starting point for both classical and quantum physics. Classical physics can be built from solid bits of information using tools at least similar to Shannon's classical information theory. A classical object can be exhaustively described by experimentally verifiable statements with definite truth values: i.e., bits of information. On the other hand, any non-classical aspect of quantum mechanics can be explained directly by Zeilinger's fundamental principle. That an elementary system carries only one bit of information explains why certain quantum measurements are irreducibly random and why quantum systems seem to exhibit counterintuitive wave-particle duality. Thus, the separation of the two distinct starting points, which is the crux of Heisenberg's paradox, has been eliminated, leaving just one starting point.

Zeilinger's principle, however, does not aim to deny all ambiguity. He has introduced a concept of information, which is readily and comfortably used in many contemporary domains, as fundamental. However, the assertion that a physical system carries a bit of information is itself enigmatic. The *carrying* of information seems to imply a physical characteristic of the system. Zeilinger indeed refers to some sort of property that remains with the system regardless of what measurement is performed on it. However, the concept of information only makes sense in relation to observations of experimental outcomes. Thus, Zeilinger's principle remains on the border of subjectivity and objectivity.

This addresses the same issues that Heisenberg's paradox emphasizes. Heisenberg asserted:

> There is no use in discussing what could be done if we were beings other than what we are. At this point we have to realize, as von Weizsacker has put it, that "Nature is earlier than man, but man is earlier than natural science." The first part of the sentence justifies classical physics, with its ideal of complete objectivity. The second part tells us why we cannot escape the paradox of quantum theory, namely, the necessity of using the classical concepts.

Likewise, Zeilinger clings to objectivity insofar as science has always been and remains objective: whereas science is certainly tailored to human understanding, from a scientific perspective the actual presence of a human observing the measurement result does not affect the way the experimental apparatus works. However, despite the classical world's congeniality towards identifying objectivity with ontology, the essence of science is embedded in empiricism, not ontology. Using information as fundamental asserts the correspondence between our scientific knowledge and our experimental observations as foundational. Thus, the only scientifically significant facts are those that would make a difference to our observations if they were otherwise. In fact, one definition of information is "a difference that makes a difference[16]." Ontological claims rarely fit this category. Whether or not the wave function exists in space and time makes no difference to the observations we make. However, the observations we do make dictate what we know. Thus, information is an ideal tool for expressing the entirety of our scientific knowledge.

Zeilinger concludes with Heisenberg that the implications of quantum theory do not justify the abandonment of classical concepts to describe reality, since the actual practice of science is necessarily described with classical language. Furthermore, the demand for theories with ontologically extraneous entities that transcend human experience for the purpose of eliminating any threat of subjectivity is unwarranted. Given Zeilinger's position, Heisenberg's paradox seems unnecessarily extreme. On Zeilinger's view, the tension between the opposing elements of the paradox are enveloped into a concept that is scientifically, linguistically and intuitively sensible.

# A Phase Difference Between Reflected and Transmitted Beams for Symmetric Beam Splitters

Here it is shown that the difference in phases of the transmitted and reflected beams of light $(\phi_T - \phi_R)$ is $\frac{\pi}{2}$ for a symmetric lossless beam splitter[17][18][19]. From conservation of energy, the intensity of the initial beam of light must be equal to the sum of the intensities of the two outgoing beams:

$$|E_0|^2 = |E_A|^2 + |E_S|^2 . \tag{40}$$

where $E_0$, $E_A$ and $E_S$ are the electric fields for the initial beam, the outgoing beam from the asymmetric arm and the outgoing beam from the symmetric arm, respectively. $E_A$ and $E_S$ are superpositions of the contributions from both paths of the interferometer, so:

$$|E_0|^2 = |E_{A1} + E_{A2}|^2 + |E_{S1} + E_{S2}|^2 , \tag{41}$$

where $E_{A1}$ and $E_{A2}$ are the electric fields of the beams transmitted from path 1 and reflected from path 2 towards the asymmetric output arm, respectively. Likewise, $E_{S1}$ and $E_{S2}$ are the electric fields reflected from path 1 and transmitted from path 2 towards the symmetric output arm. These electric fields may be written explicitly in terms of the real transmission and reflection amplitudes for each contributing beam $A_1$, $A_2$, $S_1$ and $S_2$ and the complex phases $\phi_{A_1}$, $\phi_{A_2}$, $\phi_{S_1}$ and $\phi_{S_1}$:

$$\begin{aligned} E_{A1} &= E_0 A_1 e^{i\phi_{A1}} \\ E_{A2} &= E_0 A_2 e^{i\phi_{A2}} \\ E_{S1} &= E_0 S_1 e^{i\phi_{S1}} \\ E_{S2} &= E_0 S_2 e^{i\phi_{S2}} . \end{aligned} \tag{42}$$

Combining these expressions with Equation 41 yields:

$$1 = A_1^2 + A_2^2 + S_1^2 + S_2^2 + 2A_1A_2 \cos(\phi_{A1} - \phi_{A2}) + 2S_1S_2 \cos(\phi_{S1} - \phi_{S2}) \tag{43}$$

Since each beam passes through two beam splitters, each amplitude is equal to the multiple of the two amplitudes of the individual beam splitters. Explicitly,

$$
\begin{aligned}
A_1 &= T_{\rightarrow} T_{\downarrow} \\
A_2 &= R_{\rightarrow} R_{\rightarrow} \\
S_1 &= T_{\rightarrow} R_{\downarrow} \\
S_2 &= R_{\rightarrow} T_{\rightarrow},
\end{aligned} \tag{44}
$$

where $T_{\rightarrow}$ indicates transmission from the left (as seen in Figure 6), $R_{\downarrow}$ indicates reflection from above, and so on. Since the two beam splitters are identical, there is no distinction made between, say, reflection coefficient for the the first beam splitter and the reflection coefficient for the second one. For symmetric beam splitters, the beam is affected in the same way from either side (so if the beam splitter is flipped, it will work the same.) Thus, $R_{\rightarrow} = R_{\downarrow}$ and $T_{\rightarrow} = T_{\downarrow}$, so that:

$$
\begin{aligned}
(A_1)^2 &= T^4 \\
(A_2)^2 &= R^4 \\
(S_1)^2 &= (RT)^2 \\
(S_2)^2 &= (RT)^2
\end{aligned} \tag{45}
$$

From conservation of energy, $1 = R^2 + T^2$. Squaring this equation, we find:

$$1 = T^4 + R^4 + 2(RT)^2 = (A_1)^2 + (A_2)^2 + (S_1)^2 + (S_2)^2 . \tag{46}$$

Letting $\alpha = (\phi_{A1} - \phi_{A2})$ and $\beta = (\phi_{S1} - \phi_{S2})$, equation 43 becomes:

$$0 = 2(TR)^2 (\cos\alpha + \cos\beta) . \tag{47}$$

Therefore,

$$\alpha = \beta + \pi. \tag{48}$$

The total phase of each beam is the addition of the phase due to the length of the path ($\frac{2\pi l_1}{\lambda}$ and $\frac{2\pi l_2}{\lambda}$ for paths 1 and 2, respectively, where $\lambda$ is the wavelength of the light and $l_1$ and $l_2$ are the lengths of paths 1 and 2, respectively), the phase picked up by the mirror ($\phi_{M1}$ or $\phi_{M2}$ for the mirror in path 1 or 2), and the phases due to transmission and reflection from the beam splitters ($\phi_T$ and $\phi_R$):

$$\begin{aligned}
\phi_{A1} &= \frac{2\pi l_1}{\lambda} + 2\phi_T + \phi_{M1} \\
\phi_{A2} &= \frac{2\pi l_2}{\lambda} + 2\phi_R + \phi_{M2} \\
\phi_{S1} &= \frac{2\pi l_1}{\lambda} + \phi_T + \phi_R + \phi_{M1} \\
\phi_{S2} &= \frac{2\pi l_2}{\lambda} + \phi_T + \phi_R + \phi_{M2}.
\end{aligned} \tag{49}$$

Combining these expressions with Equation 48 gives:

$$\begin{aligned}
\phi_{A1} - \phi_{A2} &= \phi_{S1} - \phi_{S2} + \pi \\
\frac{2\pi l_1}{\lambda} + 2\phi_T + \phi_{M1} - \left(\frac{2\pi l_2}{\lambda} + 2\phi_R + \phi_{M2}\right) &= \frac{2\pi l_1}{\lambda} + \phi_T + \phi_R + \phi_{M1} \\
&\quad - \left(\frac{2\pi l_2}{\lambda} + \phi_T + \phi_R + \phi_{M2}\right) \\
\phi_T - \phi_R &= \frac{\pi}{2}.
\end{aligned} \tag{50}$$

Thus, the two outgoing beams of a symmetric beam splitter are always out of phase by $\frac{\pi}{2}$. We can then use the convention that the reflected beam picks up no phase and that the transmitted beam picks up an added phase of $\frac{\pi}{2}$ or, equivalently, the electric field picks up a factor of $e^{i\frac{\pi}{2}} = i$.

## B Properties of the Mach-Zehnder Interferometer

In general, when the complex transmission and reflection coefficients are given by $Te^{i\phi_T}$ and $Re^{i\phi_R}$ with $T$, $R$, $\phi_T$ and $\phi_R$ real, then:

$$E_1 = E_0 T e^{i\phi_T} \tag{51}$$

$$E_2 = E_0 R e^{i\phi_R} \tag{52}$$

$$E_S = 2TRE_0 e^{i(\phi_T+\phi_R)} \tag{53}$$

$$E_A = T^2 E_0 e^{i2\phi_T} + R^2 E_0 e^{i2\phi_R}. \tag{54}$$

From conservation of energy, $|R^2| + |T^2| = 1$, and from Appendix 1, $\phi_T - \phi_R = \frac{\pi}{2}$. The intensities are then:

$$I_S = 4I_0 (TR)^2 \tag{55}$$

$$\begin{aligned} I_A &= I_0 \left(T^4 + R^4 + 2(TR)^2 \cos 2(\phi_T - \phi_R)\right) \\ &= I_0 \left[1 - 4(TR)^2\right] \end{aligned} \tag{56}$$

where $I_0$ is the intensity of the initial beam of light.

If, however, there is an added phase shift, $\phi^*$, to one of the paths of the interferometer (which is usually the case, since rarely are the two paths *exactly* the same), then the equations become:

$$E_S = \frac{1}{2} E_0 \exp i (\phi_T + \phi_R) (1 + \exp i\phi^*) \tag{57}$$

$$I_S = \frac{1}{2} I_0 (1 + \cos \phi^*) \tag{58}$$

$$E_A = \frac{1}{2} E_0 \exp i (2\phi_T + \phi^*) + \frac{1}{2} E_0 \exp 2\phi_R \tag{59}$$

$$\begin{aligned} I_A &= \frac{1}{2} I_0 (1 + \cos 2\phi_T - 2\phi_R + \phi^*) \\ &= \frac{1}{2} I_0 (1 + \cos \pi + \phi^*) = \frac{1}{2} I_0 (1 - \cos \phi^*). \end{aligned} \tag{60}$$

These are two intensity equations plotted in Figure 7.

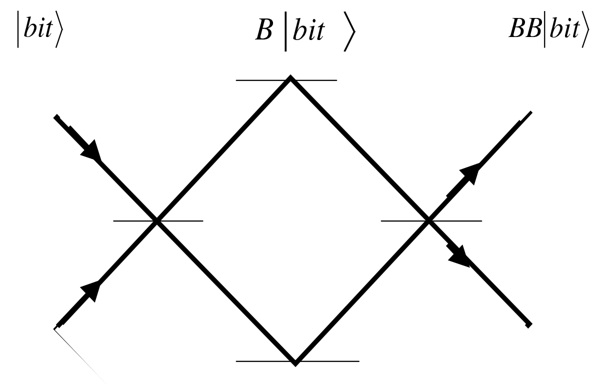


Figure 10: A ideal Mach Zehnder Interferometer can act as a not gate. The initial bit depends on which input arm the photon enters. If it enters through the upper arm, it will exit out of the lower arm, and vice versa. The individual beam splitter is said to be a square root of not gate.

## C The Mach Zehnder Interferometer as a Quantum Not Gate

A logical operator takes in one bit and outputs one bit (usually a 0 or a 1). For the ideal Mach Zehnder Interferometer as a quantum not gate, the one input bit is described by two modes or states. For example, for the Mach Zehnder Interferometer in Figure 10, the logical 0 can be encoded as the initial bit by $|01\rangle$ which means that a photon enters the interferometer from the lower input arm and the upper input arm is in the vacuum state. Likewise, the logical 1 can be encoded by $|10\rangle$ which means that a photon enters through the upper input arm and there is no photon in the lower arm. After the first beam splitter, the state of the system is:

$$B\,|01\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + i\,|10\rangle\right) \tag{61}$$

or

$$B\,|01\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + i\,|01\rangle\right) \tag{62}$$

where $B$ is the operator representing the beam splitter's action on a state.

After the second beam splitter, the state of the system is:

$$BB\,|01\rangle = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|01\rangle + i\,|10\rangle\right) + i\left(\frac{1}{\sqrt{2}}\left(|10\rangle + i\,|01\rangle\right)\right)\right]$$

$$= i\left|10\right\rangle \quad (63)$$

or

$$\begin{aligned} BB\left|10\right\rangle &= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(\left|10\right\rangle + i\left|01\right\rangle\right) + i\left(\frac{1}{\sqrt{2}}\left(\left|01\right\rangle + i\left|10\right\rangle\right)\right)\right] \\ &= i\left|01\right\rangle \end{aligned} \quad (64)$$

So, the interferometer changes the initial state to its opposite state, adding a phase shift of $i$. Thus, the interferometer itself works as a not gate. However, a single beam splitter acting on the initial state (leaving the state in a superposition of states, as in Equations 61 and 62) is said to work as a square root of not gate.

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