Neutrino Oscillations through Extra Dimensions and Symmetries

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Science with Honors in
Physics from the College of William and Mary in Virginia,

by

Patrick Meade

Accepted for ______________________________
(Honors, High Honors or Highest Honors)

____
Advisor: Dr. Christopher D. Carone

____
Dr. Carl E. Carlson

____
Dr. Nahum Zobin

____
Dr. Keith Griffieon

Williamsburg, Virginia
May 2002

Abstract

In recent experiments it has been shown that the mixing angles θ_{12} and θ_{23} for neutrinos, which parameterize the probability for a neutrino to oscillate, both appear to be nearly maximal. This evidence has led to many papers concerning possible theoretical models that explain both the neutrino masses and mixing angles observed. One approach to this problem has been the seesaw mechanism, which suppresses neutrino masses, in combination with a flavor symmetry. Another possible approach is the exploitation of possible large extra dimensions. The existence of extra dimensions is also able to suppress neutrino masses and induce large oscillation probabilities. In this paper it will be examined whether or not combining the techniques of flavor symmetries and extra dimensions results in physically viable models.

Acknowledgements

I would like to thank my advisor Chris Carone for giving me the opportunity to do particle theory research as an undergraduate. I would also like to thank him for always answering my questions no matter how obvious they might have seemed to him. I would also like to thank Leiba Rodman for giving me the opportunity to do mathematics research as an undergraduate. Because of Chris and Leiba I can say that I have had the opportunity to realize the kind of work I want to pursue in my professional life, as well as learning a great many things. I would like to thank Nahum Zobin for interesting discussions on many aspects of physics from the mathematical point of view. Last but not least I would like to thank my family in all of its forms for giving me the time and space necessary to pursue my work and accomplish whatever I desired.

Contents

1 Introduction **1**

2 Neutrino Oscillation **2**

3 Seesaw Mechanism and Neutrino Oscillations in 4-d **5**

3.1 Neutrino Mass Bounds 6

3.2 Standard Model Mass Terms 6

3.3 Neutrino Mass and the Seesaw Mechanism 7

4 Flavor Physics **9**

4.1 $U(2)$ Flavor Physics 9

5 Extra Dimensions and Kaluza-Klein Excitations **11**

5.1 Kaluza-Klein Excitations 12

6 Neutrino Oscillations in Extra Dimensions **14**

6.1 Dienes-Dudas-Gherghetta Model 15

7 Extra Dimensional Flavor Restricted Couplings and Neutrino Oscillations **17**

7.1 Cross Check of the Analytic Solutions Using Numerical Methods 18

7.2 Prototype Matrices 20

7.3 Two Neutrino Model 23

7.4 Three Neutrino Model Model 25

8 Conclusions **27**

A $U(2)$-like Flavor Symmetries and Approximate Bimaximal Neutrino Mixing **30**

1 Introduction

Neutrinos are massless in the Standard Model (SM). There is only a left handed neutrino transforming in a weak $SU(2)$ doublet and there can be no mass term generated by this state alone. Neutrinos are also one of the most difficult particles to detect, since they are not charged and only experience the weak force. The difficulty inherent in observing neutrinos has allowed existing theory, with massless neutrinos, to stand for many years. Modern experiments are starting to probe this conventional wisdom and give us reason to re-examine our understanding of neutrino physics. This thesis will concentrate on the mechanisms of generating neutrino mass and oscillation in the context of four space-time dimensions and extra dimensions. In four dimensions there are well-established tools such as the seesaw mechanism and the use of flavor symmetries that explain neutrino masses and oscillations patterns. With the introduction of extra dimensions we can also explain neutrino masses and oscillations without relying on the seesaw mechanism. We will investigate if it is possible to combine flavor symmetries in the context of extra dimensions to induce phenomenologically viable results.

Neutrinos are generated by nuclear reactions in the sun (solar neutrinos) and by cosmic-ray interactions with the atmosphere (atmospheric neutrinos). In the case of cosmic rays interacting with the atmosphere, charged pions are created as a byproduct of the event. The pions then decay in the following way:

$$\pi^+ \;\rightarrow\; \mu^+\nu_\mu,\; \mu^+ \;\rightarrow\; e^+\nu_e\,\bar{\nu}_\mu, \tag{1}$$

and,

$$\pi^- \;\rightarrow\; \mu^-\bar{\nu}_\mu,\; \mu^- \;\rightarrow\; e^-\bar{\nu}_e\,\nu_\mu. \tag{2}$$

Therefore we should observe twice as many ν_μ's as ν_e's. The ratio of the number of ν_μ's to the number of ν_e's has been observed to be about .61 of the theoretical value of 2 [1]. Muon neutrinos appear to be missing from experiments. This suggests that some new physics may be affecting the neutrinos since the neutrino production mechanism in cosmic rays is well-understood.

In the sun there are various nuclear reactions that create neutrinos. The theoretical Solar Model predicts the number of neutrinos that should pass through the earth. As in the case of atmospheric neutrinos, experiments that observe solar neutrino fluxes find fewer neutrinos than are expected. Cosmic rays and the sun are two separate processes generating neutrinos that both yield a deficit of neutrinos. These two different sources of missing neutrinos suggest that new physics pertaining to neutrinos is relevant.

This thesis is structured in the following way. In Sec. 2 we will discuss neutrino oscillation. This is the accepted answer for how the lack of neutrinos in experiments versus theory can be reconciled. We will discuss neutrino mass in Sec. 3, since neutrino mass and oscillation are intertwined. In Sec. 4 we will then discuss how flavor symmetries can explain the differences between generations in the SM. We will then proceed to discuss extra dimensions and their effect on four dimensional physics in Sec. 5. In Sec. 6 we will review current models of neutrino oscillation arising from extra dimensions. We will then examine in Sec. 7 how flavor physics and extra dimensions can be combined to create phenomenologically viable models of neutrino oscillation. In Sec. 8 we will discuss directions that future research could take and possible pitfalls of extra dimensional models.

2 Neutrino Oscillation

The most popular mechanism that yields solutions to both the solar and atmospheric neutrino problems is purely quantum mechanical. The idea is that neutrinos oscillate as they time evolve. The way this happens is that there are two bases out of the infinite number of bases that span the Hilbert space in which the neutrinos evolve. One basis will be called a flavor basis which is defined by interactions, and the other a mass basis which represents eigenstates of the Hamiltonian operator.

$$| \, \nu_\alpha \, \rangle \qquad \text{flavor basis} \qquad | \, \nu_j \, \rangle \qquad \text{mass eigenstates} \tag{3}$$

Since both bases span the space, one basis is connected to the other by a unitary transformation,

$$\nu_\alpha = \sum_j U_{\alpha\, j} \; \nu_j. \tag{4}$$

For two neutrino flavors, we can write $U_{\alpha\, j}$ in terms of a mixing angle θ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{5}$$

Three flavors of neutrinos requires three angles, $\theta_{12}, \theta_{13}, \theta_{23}$ and a phase. To simplify the 3 by 3 case, we ignore the phase. The unitary matrix can then be represented as the product of three rotation matrices $U = R_{23} R_{13} R_{12}$ (e.g. Euler rotations in 3 dimensions), and may be parameterized,

$$c_{ij} = \cos\theta_{ij},$$

$$s_{ij} = \sin\theta_{ij},$$

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}. \tag{6}$$

From Eq. (4) and the orthonormality of basis vectors we have

$$\langle \, \nu_\alpha \mid \nu_j \, \rangle = U_{\alpha\, j}. \tag{7}$$

We can always express the time dependent neutrino state Ψ with the time evolution operator

$$| \, \Psi(t) \, \rangle = \exp\left(-iHt\right) | \, \Psi(0) \, \rangle. \tag{8}$$

A set of complete mass eigenstates can be inserted into Eq. (8). Thus a flavor state at any later time t is

$$| \, \Psi(t) \, \rangle = \sum_j \exp{(-i \, M_j \, \tau)} \, | \, \nu_j \, \rangle \langle \, \nu_j \, | \, \Psi(0) \, \rangle. \tag{9}$$

In this expression we can rewrite the Lorentz invariant phase factor, which is in terms of the mass of the eigenstate M_j, and the time in the frame of the neutrino τ, in terms of the lab frame

$$\exp{(-i \, M_j \, \tau)} = \exp{(-i \, (\, E_j \, t - p_j \, L \,))}. \tag{10}$$

The neutrinos that we see will be ultra-relativistic; thus we can make the approximation of $t \approx L$ for $c = 1$. Assuming that the neutrinos are of definite energy E as in [1], we have $p_j = \sqrt{E^2 - M_j^2} \approx E - M_j^2/2E$. Combining these assumptions we can rewrite the phase factor Eq. (10) as

$$\exp{(-i(M_j^2/2E)L)}. \tag{11}$$

Using Eqs. (11) and (9) we can rewrite the time expansion of a neutrino wave function that started in a specific flavor state ν_l as

$$| \, \nu_l(L) \, \rangle = \sum_j \exp{(-i \, M_j^2 \, L/2E)} \, U_{l \, j} | \, \nu_j \, \rangle. \tag{12}$$

This allows us to write out an expression for the amplitude of a neutrino going from one flavor state to another

$$A(\nu_l \rightarrow \nu_{l'}) = \sum_j U_{lj} \, U^*_{l'j} \, \exp{(-i \, \frac{M_j^2}{2} \frac{L}{E})}. \tag{13}$$

We can then write the probability for a neutrino of a specific flavor l to oscillate to a neutrino of flavor l' as

$$P(\nu_l \rightarrow \nu_{l'}) = |A(\nu_l \rightarrow \nu_{l'})|^2. \tag{14}$$

For the case of two neutrino mixing we arrive at the famous probability formula when the factors of $\hbar$ and c are inserted

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2[1.27 \Delta M_{21}^2(\text{eV}^2) L(\text{km})/E(\text{GeV})] \tag{15}$$

and

$$\Delta M_{ij}^2 \equiv M_i^2 - M_j^2. \tag{16}$$

One interesting consequence of the amplitude of oscillation expression Eq. (13) is that if all the masses vanish, the unitarity of U guarantees that

$$A(\nu_l \to \nu_{l'}) = \sum_j U_{lj}\, U_{l'j}^* = \delta_{l\,l'}. \tag{17}$$

This shows that massless neutrinos cannot oscillate. However, if neutrinos oscillate in the way we have outlined, we must introduce a way to generate their masses.

The evidence for neutrino oscillation is mounting with every recent experimental result, from SuperKamiokande to SNO. These experiments are not only showing that neutrinos oscillate, but that certain oscillations are favored. The most recent data is consistent with the preferred Large Mixing Angle (LMA) solution to the solar neutrino problem [2, 3, 4].

$$\begin{gathered} \sin^2 2\theta_{23} \geq .88 \\ .25 \leq \tan^2\theta_{12} \leq .63 \\ \sin^2 2\theta_{13} \leq .1 - .3 \end{gathered} \tag{18}$$

This data shows us that two mixing angles appear to be very large while θ_{13} is relatively small. This case represents nearly bimaximal neutrino mixing.

3 Seesaw Mechanism and Neutrino Oscillations in 4-d

If neutrinos have mass there must be an additional field introduced to give rise to a mass term. Before we discuss how to generate this mass term we will review the bounds on neutrino mass, and how mass terms are generated in the standard model.

3.1 Neutrino Mass Bounds

Solutions to the solar and atmospheric neutrino problems and subsequent experiments give bounds for the squared mass differences of neutrinos Eq. (16). For the solar neutrino solutions the squared mass difference is [1]

$$2 \times 10^{-5}\text{eV}^2 < \Delta m_{12}^2 < 1 \times 10^{-4}\text{eV}^2.$$

For the atmospheric neutrino solutions the squared mass difference is

$$5 \times 10^{-4}\text{eV}^2 < \Delta m_{23}^2 < 6 \times 10^{-3}\text{eV}^2.$$

Direct searches [1] looking at tritium beta decay can also place an upper bound on the ν_e mass at

$$m_{\nu_e} < 3 \text{ eV}$$

Neutrinos are known to be extremely abundant in the universe. Therefore due to this abundance there is a cosmological bound on their mass. Experiments that also focus on the non-observation of neutrinoless double beta decay also put a bound on neutrino masses, specifically that the electron neutrino has to be less than $\mathcal{O}(\text{eV})$. Taking into account the various bounds on neutrino masses we find that neutrino masses all have to be no more than $\mathcal{O}(\text{eV})$, but well below the weak scale,$\sim$ 246 GeV.

3.2 Standard Model Mass Terms

Fermion masses are generated by Dirac mass terms of the following form,

$$m\bar{\psi}_L\psi_R + h.c. \tag{19}$$

while terms such as the following vanish,

$$\bar{\psi}_L\psi_L = \bar{\psi}_R\psi_R = 0. \tag{20}$$

The SM left-handed fermions occur in $SU(2)$ doublets whereas the right-handed fermions transform as singlets, except there is no right-handed neutrino:

$$\begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \ u_{iR}, \ d_{iR}, \begin{pmatrix} e_i \\ \nu_i \end{pmatrix}_L, \ e_{iR}. \tag{21}$$

The particle labels represent the spinors and i represents the generation index. The problem with mass terms of Eq. (19) is that they break gauge invariance. Therefore the fermions of the SM can not have mass terms such as in Eq. (19). The actual mass term in the SM is generated by the introduction of the Higgs field. The Higgs scalar ϕ transforms as a $SU(2)$ doublet. Therefore the Yukawa interaction term $f\phi\bar{l}_L l_R$ is gauge invariant, where f is the Yukawa coupling. When the gauge symmetry is spontaneously broken by ϕ obtaining a vacuum expectation value (VEV) the fermion term obtains a mass

$$f\phi\bar{l}_L l_R \rightarrow f\langle\phi\rangle\bar{l}_L l_R, \tag{22}$$

where

$$f\langle\phi\rangle$$

plays the role of the mass m. Neutrino mass can not be generated in this manner in the SM since there is no right-handed neutrino.

3.3 Neutrino Mass and the Seesaw Mechanism

To generate neutrino mass we must first introduce a right-handed neutrino into our theory beyond the SM, which allows us to write a mass term. The mass terms will obtain a mass of the order of the VEV of the Higgs. For the case of electroweak symmetry breaking, this VEV gives the neutrino a mass that is the order of the weak scale, which is much too high. Therefore beyond just introducing a right-handed neutrino, we must introduce it in such a way that the observed neutrinos are light enough. This is accomplished by the seesaw mechanism.

First we must consider the effects of introducing the right-handed neutrino N_R. We make this neutrino carry no charge under any SM groups:

$$SU(3) \times SU(2) \times U(1). \tag{23}$$

This particle can have a direct mass term

$$M\bar{N_R^C}N_R, \tag{24}$$

where C is the charge conjugation operator, which sends particle to antiparticle. This type of mass term is called a Majorana mass and violates lepton number, unlike Eq. (19). This mass can be as large as we want since gauge invariance is not broken. The right-handed neutrino can also have a Dirac mass

$$\langle\phi\rangle\bar{\psi}_L N_R = m\bar{\nu}_L N_R \tag{25}$$

where ϕ is the Higgs field. If we were to write down our interaction terms in terms of a basis of ν_L and N_R, when we diagonalize the matrix we obtain our observable masses

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \rightarrow \begin{pmatrix} m^2/M & 0 \\ 0 & M \end{pmatrix}. \tag{26}$$

When we set M to be very high, since we have never seen right-handed neutrinos, this naturally gives us the very suppressed mass of the left-handed neutrinos we see. This is a simple two dimensional version of the seesaw mechanism, but the principle remains the same for larger numbers of neutrinos. A more formal way of writing out the seesaw mechanism only in terms of the Dirac and Majorana mass matrices is

$$M_{LL} \approx M_{LR}M_{RR}^{-1}M_{LR}^{\dagger}. \tag{27}$$

The idea behind the seesaw from a Feynman diagram point of view is that we have Dirac type neutrinos that exchange a heavy right-handed neutrino and we integrate out the heavy neutrino. There are two vertices which are at the Dirac mass scale and a propagator of the heavy mass scale, yielding m^2/M as in Eq. (26). The mass m

is naturally the order of the weak scale, and M can be of order the Grand Unified Theory (GUT) scale. We then obtain, for $m \sim 10^2$ GeV and $M \sim 10^{14}$ GeV, light neutrinos with mass $\sim 10^{-1}$ eV. Putting M at the GUT scale is natural to do since in either $SU(5)$ or $SO(10)$ GUTs we can introduce the right-handed neutrinos as singlets.

4 Flavor Physics

The SM is arranged into three nearly identical generations. The only difference among the generations is that each is successively heavier than the previous one. For the leptons the generations are arranged as

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}. \tag{28}$$

The SM tells us how each of these doublets transform, but does not tell us anything about the relation of one generation to the others. This is because all the SM groups act within each generation.

In the SM the Yukawa couplings of Section 3.2 are free parameters that are adjustable to give the specific masses observed. What would be more predictive is a theory that related the generations and predicted the observed masses. The way to accomplish this is to have a symmetry G_f that acts horizontally across the generations, which restricts the forms of the Yukawa couplings. When the symmetry is broken specific patterns of masses and mixing angles for the fermions are fixed. T

4.1 $U(2)$ Flavor Physics

his basic idea of explaining the origins between the different "flavors"/generations is called flavor physics. One successful model of flavor physics is $G_f = U(2)$. In this model [5, 6, 7] the matter fields are placed in $2 \oplus 1$ representations. This leaves the heaviest generation as an invariant, thus separating this model from others such as $SU(3)$[8]. Writing the generations of matter fields in terms of tensors we have $F^a \oplus F^3$ where a is a $U(2)$ index and F is Q_L, U_R^C, D_R^C, L_L, or E_R^C which are the fields for left-handed and right-handed quarks and leptons. The following flavon fields (our flavor carrying fields) are introduced ϕ_a, S_{ab} and A_{ab} where ϕ is a $U(2)$ doublet, and $S\,(A)$ is a symmetric (anti-symmetric) $U(2)$ triplet (singlet). We can write out the Yukawa couplings in terms of these flavons as follows:

$$\frac{1}{M_f}\begin{pmatrix} S_{ab} \oplus A_{ab} & \phi_a \\ \phi_a & 1 \end{pmatrix}. \tag{29}$$

The $U(2)$ symmetry is then broken in the following way to obtain fermion masses and mixing angles

$$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} \text{nothing.} \tag{30}$$

The symmetry is broken by our flavon fields obtaining a VEV, and in this way we can parameterize the breaking in terms of a small dimensionless quantity $\langle\, flavon\,\rangle/M_f$. M_f represents the highest scale at which our theory is well defined. The following pattern of VEVs is consistent with Eq. (30):

$$\frac{\langle\phi\rangle}{M_f} = \begin{pmatrix} 0 \\ \epsilon \end{pmatrix}, \quad \frac{\langle S\rangle}{M_f} = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon \end{pmatrix}, \quad \frac{\langle A\rangle}{M_f} = \begin{pmatrix} 0 & \epsilon' \\ -\epsilon' & 0 \end{pmatrix}. \tag{31}$$

Combining the VEVs of Eq. (31) with the decomposition of the Yukawa textures into flavons, we can obtain the form of our Yukawa textures. As an example, for the down type quarks, this texture is

$$Y_D \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}. \tag{32}$$

With $\epsilon \approx .02$ and $\epsilon' \approx .004$ it has been shown that $U(2)$ can reproduce all of the charged fermion masses and mixing angles. One can not underestimate the power of such a model in its predictive ability. Instead of 12 small, free parameters in the SM we can obtain them all from the choice of 2 small parameters.

The $U(2)$ model has also been extended into predicting neutrino masses and mixing angles [9]. The basic idea is to introduce a set of 3 right-handed neutrinos that transform under a $2 \oplus 1$ representation under $U(2)$. The Dirac and Majorana mass matrices are then constructed in the same way as any other Yukawa texture, and then seesawed using Eq. (27). It has been successfully shown that with simple extensions of $U(2)$ [10] nearly bimaximal mixing can be achieved, this paper is included in Appendix 8.

5 Extra Dimensions and Kaluza-Klein Excitations

The concept of extra dimensions has been around for many years. Kaluza-Klein theory was originally intended as a unification of gravity and electromagnetism by the introduction of a fifth dimension. Albeit the theory was not successful, many tools from it have been re-applied in modern particle physics. String theory also requires the existence of many extra spatial dimensions. Therefore investigating the phenomenology of extra dimensions at lower energy scales has become a major interest over the past decade.

The extension of our four-dimensional world by adding extra dimensions has come in many shapes and sizes. There have been extensions using flat extra spatial dimensions [11, 12], as well as, using warped extra dimensions [13, 14]. It has also been suggested that these extra dimensions can be actually quite large even $\mathcal{O}$(mm) [15]. Large extra dimension serve to lower the Planck scale. By using Gauss's law we have $M_{pl} = 2 \times 10^{18}$ GeV related to the fundamental (4+n)-dimensional scale and the volume of the extra dimension, V_n, by

$$M_{pl}^2 = M^{n+2} V_n. \tag{33}$$

This allows us to escape the desert that we normally have between the weak scale and the Planck scale.

5.1 Kaluza-Klein Excitations

The introduction of extra dimensions not only lowers the Planck Scale by the geometry of the extra dimensions; it also effects other physics such as lowering the GUT scale [11, 12] and, as we will see, suppressing neutrino masses [16, 17]. The theories examined in this paper deal only with flat extra dimensions.

In adding δ extra space-time dimensions of radius R, we now have to extend our fields from the usual four dimensional coordinates $\mathbf{x} \equiv (x_0, x_1, x_2, x_3)$ to also include the extra dimensional coordinates $\mathbf{y} \equiv (y_1, y_2, \ldots, y_\delta)$. The full coordinates will be denoted $x = (\mathbf{x}, \mathbf{y})$ For the sake of simplicity we will only consider $\delta = 1$; however, the extension to larger δ is straightforward. When compactifying the extra dimension on a circle it is required that our complex field $\Phi(x)$ be periodic under

$$y \rightarrow y + 2\pi R. \tag{34}$$

With this assumption we can expand $\Phi(x)$ in terms of the Fourier modes of the extra dimension

$$\Phi(x) = \sum_{n=-\infty}^{\infty} \Phi^{(n)}(\mathbf{x}) \exp(iny/R). \tag{35}$$

The fields that depend on only the four dimensional coordinates $\Phi^{(n)}(x)$ are called the Kaluza-Klein (KK) modes. The mass of each KK modes is

$$m_n^2 \equiv m_0^2 + \frac{n^2}{R}. \tag{36}$$

The term m_0 is the mass of the zero-mode. These modes can then be expanded with the decomposition of our complex field as

$$\Phi(x) = \Phi_+(x) + i\Phi_-(x),$$

with

$$\Phi_+(x) = \sum_{n=0}^{\infty} [\Phi^{(n)}(\mathbf{x}) + \Phi^{(-n)}(\mathbf{x})] \cos(ny/R),$$

and

$$\Phi_-(x) = \sum_{n=1}^{\infty} [\Phi^{(n)}(\mathbf{x}) - \Phi^{(-n)}(\mathbf{x})] \sin(ny/R). \quad (37)$$

Only the Φ_+ terms would survive if Φ were real. We can make a stricter restriction on the fields if we choose to compactify on a Z_2 orbifold instead of a circle. In this case we identify

$$y \to -y, \quad (38)$$

and the odd and even modes tranforms as,

$$\Phi_-(\mathbf{x}, -y) = -\Phi_-(\mathbf{x}, y)$$

$$\Phi_+(\mathbf{x}, -y) = \Phi_+(\mathbf{x}, y). \quad (39)$$

This type of orbifold compactification allows us to easily distinguish between Φ_+ and Φ_-. Orbifolds also allow us to construct viable theories with chiral fermions, while circles do not. The next logical question is now that there are infinitely many KK modes with the introduction of the extra dimensions, why aren't these modes observed and what does the infinite tower of modes do to the SM. We do not want the SM fermions to have KK excitations that could be observed, thus the we place them at fixed points on the orbifold. Orbifold fixing of the SM particles allows only particles that are pure singlets under the SM to propagate into the bulk, which is what we call the extra dimensions. Concepts such as compactifying on an orbifold and fixing particles on the orbifold sound ad hoc, but there is a reason why they become natural to do. Orbifolds and orbifold fixed points are natural principles in string theory; if the motivation is to imbed the idea of extra dimensions into string theory these techniques follow logically.

6 Neutrino Oscillations in Extra Dimensions

Once extra dimensions are introduced we lose a very important tool in neutrino oscillation physics, namely the usual seesaw mechanism. The main idea from the seesaw was that we could generate the small neutrino masses by integrating out heavier particles. This gave us the relationship that $m_\nu \sim m^2/M$, where M is some high scale . We assume the mass m to be at the scale of electroweak breaking, i.e., around a TeV. Then to obtain the small neutrino masses we had to place M at the order of the GUT scale. In extra dimensions the Planck scale is suppressed by the volume factor of Eq. (33). If the size of the extra dimensions is large, for instance $\mathcal{O}$(mm), the Planck scale is $\mathcal{O}$(TeV). When we lose the higher energy scale we become powerless to suppress the neutrino masses by the usual m/M factor. Therefore, we require a new mechanism to suppress the neutrino masses in the presence of extra dimensions.

The way neutrino mass is suppressed with extra dimensions starts with introducing a neutrino into the bulk instead of introducing a right-handed neutrino in four-dimensions as we did before. The coupling between the extra neutrinos, living only in the bulk, and the ordinary left-handed neutrinos that are confined to the brane, our four dimensional world, is volume suppressed by the extra dimensions. This suppressed coupling then generates the small masses of the brane neutrinos when you calculate the eigenvalues of a neutrino mass matrix that includes both brane and bulk neutrinos. This "new" seesaw has been formulated in a few different ways, such as in the choice of basis and lepton number violation. It has also been shown and will be demonstrated in this thesis, that not only does the "new" seesaw suppress neutrino masses it can also generate large neutrino oscillation probabilities.

6.1 Dienes-Dudas-Gherghetta Model

In the Dienes-Dudas-Gherghetta model, only one neutrino is introduced into the bulk which consists of only one extra dimension. On the brane, three left handed neutrinos are introduced corresponding to their flavor eigenstates ν_i $(i = 1, 2, 3)$. Each of these three neutrinos has its own unique Majorana mass m_i on the brane. These left-handed neutrinos are flavor diagonal, which means that there is no mixing among the brane neutrinos if we were restricted to only these three neutrinos. The role that the Majorana mass terms play on the brane is simply to distinguish the three flavors. In this model there is no interpretation of the scale or origin of these masses; they are just taken as input parameters.

The bulk neutrino is introduced as a five-dimensional Dirac fermion Ψ, which can be decomposed into two, two component Weyl spinors $\Psi = (\psi, \bar{\chi})^T$. The Dirac fermion in this model contains no flavor indices, which means it is flavor neutral for all processes. When the fifth dimension is taken to be compactified on a Z_2 orbifold, it is natural to take one the Weyl spinors to be even under the Z_2 action and the other to be odd. In the case of this model, ψ is even under the Z_2 action, i.e., $y \to -y$ while χ is odd. The left-handed neutrinos are restricted to the brane at the orbifold fixed point $y = 0$, where χ vanishes on the brane. The brane/bulk coupling is then only between ν_i and ψ. In the Dienes model this brane/bulk coupling is taken to be flavor-universal, i.e., one coupling for all three brane neutrinos. In simple extensions[18] there can be also a coupling for each flavor. From these assumptions the Lagrangian has the following form:

$$\mathcal{L}_{brane} = \int d^4x \sum_{i=1}^{3} \{ \bar{\nu}_i i \bar{\sigma}^\mu D_\mu \nu_i + m_i(\nu_i \nu_i + h.c.) \}$$

$$\mathcal{L}_{bulk} = \int d^4x \, dy \ \ M_s \{ \bar{\psi} i \bar{\sigma}^\mu \partial_\mu \psi + \bar{\chi} i \bar{\sigma}^\mu \partial_\mu \chi - i \psi \partial_y \bar{\chi} + i \bar{\chi} \partial_y \psi \}$$

$$\mathcal{L}_{coupling} = \int d^4x \sum_{i=1}^{3} (\hat{m} \nu_i \psi \,|_{y=0} + h.c.). \tag{40}$$

In the above Lagrangian M_s is taken to be the mass scale of the higher fundamental theory. A basis is then chosen such that the mass matrix for the bulk will be diagonal. This entails making linear combinations of the spinors of the five dimensional field Ψ, thus for $n > 0$ we will have $N^{(n)} \equiv (\psi^{(n)} + \chi^{(n)})/\sqrt{2}$ and $M^{(n)} \equiv (\psi^{(n)} - \chi^{(n)})/\sqrt{2}$. The full Lagrangian (40) is then written in terms of the five dimensional field Ψ expanded in terms of its Kaluza-Klein excitations:

$$\psi(x,y) = \frac{1}{\sqrt{2\pi\, R}} \sum_{n=0}^{\infty} \psi^{(n)}(x) \cos(ny/R)$$

$$\chi(x,y) = \frac{1}{\sqrt{2\pi\, R}} \sum_{n=1}^{\infty} \chi^{(n)}(x) \sin(ny/R). \tag{41}$$

The five dimensional Lagrangian (40) is then compactified by integrating over the fifth dimension and the result obtained is the following:

$$\mathcal{L} = \int d^4x \; \{ \sum_{i=1}^{3} \{ \; \bar{\nu}_i i \bar{\sigma}^\mu D_\mu \nu_i + \bar{\psi}^{(0)} i \bar{\sigma}^\mu \partial_\mu \psi^{(0)} + \sum_{n=1}^{\infty} \; \bar{N}^{(n)} i \bar{\sigma}^\mu \partial_\mu N^{(n)} + \bar{M}^{(n)} i \bar{\sigma}^\mu \partial_\mu M^{(n)}$$

$$+ \{ \sum_{i=1}^{3} m_i \nu_i \nu_i + \frac{1}{2} \sum_{i=1}^{\infty} [(\frac{n}{R}) N^{(n)} N^{(n)} - (\frac{n}{R}) M^{(n)} M^{(n)})]$$

$$+ \, m \sum_{i=1}^{3} \nu_i \; (\psi^{(0)} + \sum_{n=1}^{\infty} N^{(n)} + \sum_{n=1}^{\infty} M^{(n)} + \; h.c.\}\} \tag{42}$$

The new coupling $m \equiv \hat{m}/\sqrt{2\pi M_s R}$ is the volume suppressed brane/bulk coupling that results from the rescaling of the Kaluza-Klein modes of ψ and χ. When we choose a basis as follows

$$\mathcal{N}^T \equiv (\nu_1, \nu_2, \nu_3, \psi^{(0)}, N^{(1)}, M^{(1)}, N^{(2)}, M^{(2)}, \ldots), \tag{43}$$

the mass terms in the compactified Lagrangian (42) take the form of $\frac{1}{2}(\mathcal{N}^T \mathcal{M} \mathcal{N} + h.c.)$

in which

$$\mathcal{M} = \begin{pmatrix} m_1 & 0 & 0 & m & m & m & m & m & \dots \\ 0 & m_2 & 0 & m & m & m & m & m & \dots \\ 0 & 0 & m_3 & m & m & m & m & m & \dots \\ m & m & m & 0 & 0 & 0 & 0 & 0 & \dots \\ m & m & m & 0 & 1/R & 0 & 0 & 0 & \dots \\ m & m & m & 0 & 0 & -1/R & 0 & 0 & \dots \\ m & m & m & 0 & 0 & 0 & 2/R & 0 & \dots \\ m & m & m & 0 & 0 & 0 & 0 & -2/R & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (44)$$

Even though the brane neutrinos do not directly mix with themselves, they will oscillate due to the KK modes. The important thing to note is the uniform coupling m represents a flavor-neutral theory. Lam [18] extends this model to have flavor-dependent couplings, i.e., a different coupling for each brane neutrino; however, this extension does not give an explanation of the origin or strength of the couplings.

7 Extra Dimensional Flavor Restricted Couplings and Neutrino Oscillations

The models of neutrino masses describe in Section 4.1 use symmetries to restrict the form of the Yukawa couplings. When a specific flavor symmetry was used to restrict the mass matrices in combination with the seesaw mechanism to suppress the neutrino masses, it gave rise to experimentally allowed patterns of neutrino oscillations and squared mass ratios. The mass matrices became infinite when extra dimensions were introduced, but a flavor symmetry could restrict their form in the same way as it could in four dimensions.

7.1 Cross Check of the Analytic Solutions Using Numerical Methods

Before analyzing new models a technique had to be developed that would enable us to look at probabilities of neutrino oscillation for arbitrarily complicated infinite dimensional mass matrices. We used a series of computer programs written in MAPLE to numerically diagonalize and compute Eqs. (13) and (14). Computations are done for finite matrices, and we look for convergence in the oscillation probabilities as the matrix grows to large finite sizes. This was necessary since analytically diagonalizing these matrices is formidable, if not impossible in most cases. In order to check that the computer code was working properly we tested it on a model proposed by Dienes [19] for which an analytical solution was possible.

A specific example was chosen from [19] in a test of the computer code. In this example the mass matrix was of the form of Eq. (44) except for an overall rescaling by R, the compatification radius, making all parameters dimensionless. The parameters were chosen to be $m = 0.01$, $m_{1,2} = 1 \mp \delta m/2$, $\delta m = 5 \times 10^{-2}$, and $m_3 = 5$. Probabilities of ν_2 survival were plotted as a function of τ which is defined as

$$\tau \equiv \frac{L}{2E}. \tag{45}$$

The analytic result came from Fig. 1 in the Dienes paper [19] and is shown in our Fig. 1. The results from the computer code were plotted in Fig. 2. As can be seen from Fig. 1 and Fig. 2, the computer code reproduced the analytic expressions for probabilities solved in the Dienes paper [19]. The convergence to the Dienes analytic expression occurred in the first ten Kaluza-Klein modes, i.e. a matrix of dimension 20 by 20. This enabled us to use the computer code to find the probabilities using any form of matrix. The only possible problem that presents itself was the convergence of the numerical solutions for other models. The mass matrices are infinite dimensional in extra-dimensional theories. The numerical code uses large finite matrices to approximate the eigenvectors of the infinite dimensional matrices.

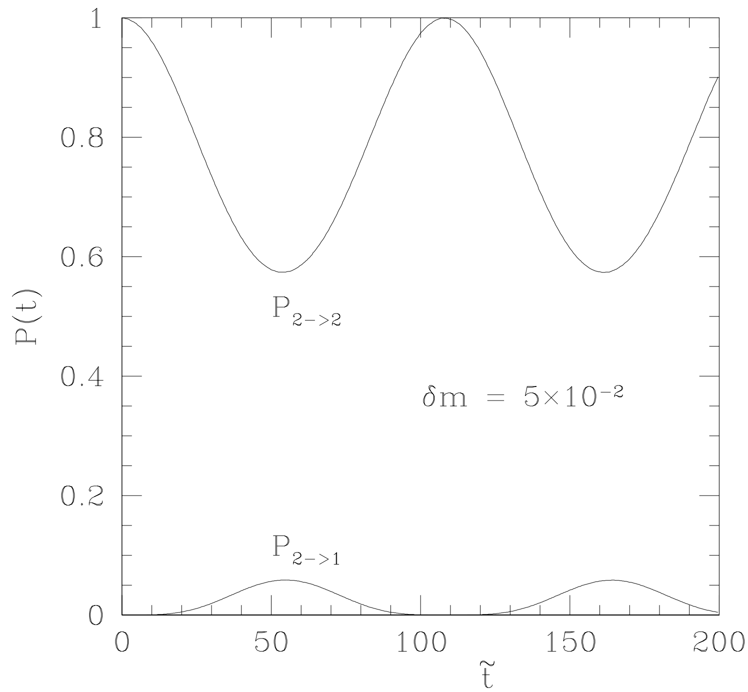

Figure 1: The analytic solution of the probabilities from [19] with the values of $m = 0.01$, $m_{1,2} = 1 \mp \delta m/2$, $\delta m = 5 \times 10^{-2}$, and $m_3 = 5$.

Models with convergent solutions as a function of the size of the matrix will be the only ones presented in this thesis. Since the most massive particles in our theory should not effect the low momentum physics, convergent solutions are the only physically sensible possibilities.

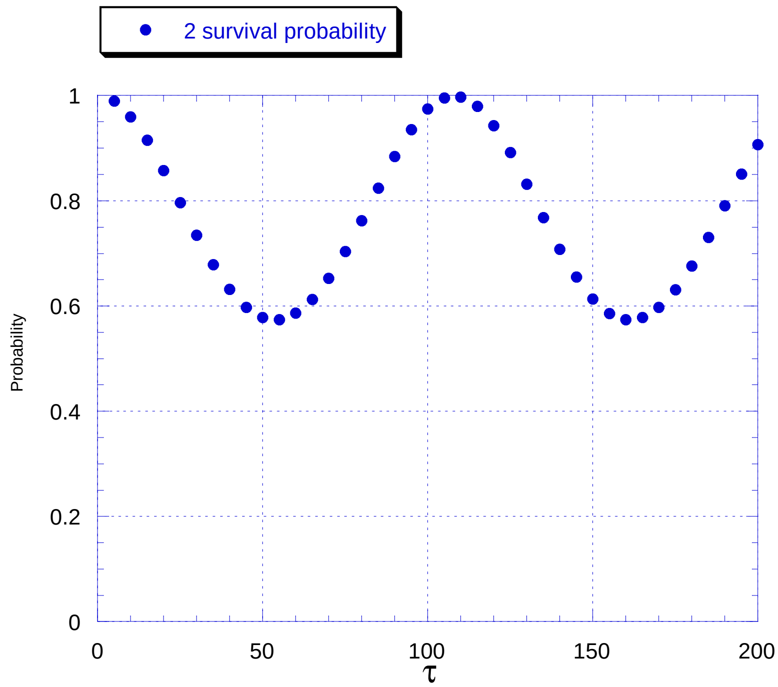

Figure 2: Numerical method result for calculation of the ν_2 survival probability with the values of $m = 0.01$, $m_{1,2} = 1 \mp \delta m/2$, $\delta m = 5 \times 10^{-2}$, and $m_3 = 5$.

7.2 Prototype Matrices

Using the DDG model [16] as a starting point and making the same extension as Lam [18] to include flavor dependent couplings , the mass matrix,

$$\mathcal{M} = \begin{pmatrix} m_1 & 0 & 0 & d_1 & d_1 & d_1 & d_1 & d_1 & \ldots \\ 0 & m_2 & 0 & d_2 & d_2 & d_2 & d_2 & d_2 & \ldots \\ 0 & 0 & m_3 & d_3 & d_3 & d_3 & d_3 & d_3 & \ldots \\ d_1 & d_2 & d_3 & 0 & 0 & 0 & 0 & 0 & \ldots \\ d_1 & d_2 & d_3 & 0 & 1 & 0 & 0 & 0 & \ldots \\ d_1 & d_2 & d_3 & 0 & 0 & -1 & 0 & 0 & \ldots \\ d_1 & d_2 & d_3 & 0 & 0 & 0 & 2 & 0 & \ldots \\ d_1 & d_2 & d_3 & 0 & 0 & 0 & 0 & -2 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{46}$$

was examined, where m_1, m_2, and m_3 are dimensionless as before when re-scaled by the compactification radius R. It can be written down from a Lagrangian with the same fields and parameters as Eq. (40) with one minor change. We make the Dirac mass terms flavor dependent and arrive at the uncompactified Lagrangian

$$\mathcal{L}_{brane} = \int d^4x \sum_{i=1}^{3} \{ \bar{\nu}_i i \bar{\sigma}^\mu D_\mu \nu_i + m_i(\nu_i \nu_i + h.c.) \}$$

$$\mathcal{L}_{bulk} = \int d^4x \, dy \;\; M_s \{ \bar{\psi} i \bar{\sigma}^\mu \partial_\mu \psi + \bar{\chi} i \bar{\sigma}^\mu \partial_\mu \chi - i\psi \partial_y \bar{\chi} + i \bar{\chi} \partial_y \psi \}$$

$$\mathcal{L}_{coupling} = \int d^4x \sum_{i=1}^{3} (\hat{d}_i \nu_i \psi \mid_{y=0} + h.c.). \tag{47}$$

Before further discussion of this model a new notation is introduced,

$$\mathcal{M} = \begin{pmatrix} M_{brane} & M_{coupling} \\ M_{coupling}^T & M_{bulk} \end{pmatrix}. \tag{48}$$

The infinite dimensional mass matrix $\mathcal{M}$ is block decomposed into: M_{brane} the three by three brane neutrino mass matrix, $M_{coupling}$ which extends from the zero KK mode to infinity and couples to each of the three brane neutrinos, and M_{bulk} which comprises the KK modes mass terms. From these definitions it is trivial to examine Eq. (46) and Eq. (48) to find the forms of M_{brane}, $M_{coupling}$, $M_{KK\ modes}$.

The first approach to applying $U(2)$ flavor physics to the infinite dimensional mass matrix was to let the brane neutrinos transform as a $2 \oplus 1$. The bulk neutrino N_R transforms as a singlet. This means that we can use the same form of mass matrix as Eq. (46) with d_i being restricted by symmetry. Thus this first extension changed the form of both M_{brane} and $M_{coupling}$ but left M_{bulk} unchanged. M_{brane} now assumes a usual $U(2)$ texture,

$$M_{brane} \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}. \tag{49}$$

$M_{coupling}$ has the specific form of $d_1 = \epsilon'$, $d_2 = \epsilon$, and $d_3 = 1$. This extension was not physically viable when the oscillation probability of ν_2 was examined with respect to these couplings. The way this was ascertained was from the atmospheric neutrino problem. We know that almost half of the ν_μ are missing. When we examine Fig. 3 there are no missing ν_μ for $d_1 = .004$, $d_2 = .02$, and $d_3 = 1$. This model was also analyzed with a variety of couplings to show that there exists values such that large oscillation can be found. The results shown in Fig. 3 demonstrate that only in a stronger coupling limit do oscillations arise in this model. The box in Fig. 3 represents SuperKamiokande data for ν_μ survival probability which looks at a range of τ from 0 to 30 and is extracted from [20].

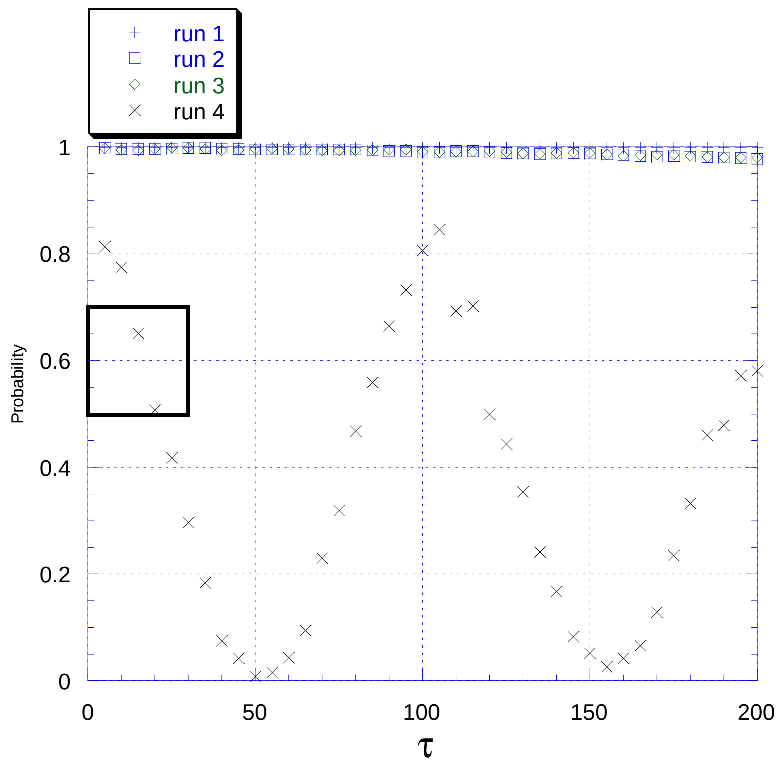

Figure 3: Numerical method result for calculation of the ν_2 survival probability with various couplings and $\epsilon = .02$ and $\epsilon' = .004$ fixed. Run 1 was with couplings of $d1 = 0.004$, $d2 = .02$ and $d3 = 1$. Run 2 was with couplings of $d1 = 0.04$, $d2 = .02$ and $d3 = .2$. Run 3 was with couplings of $d1 = 0.02$, $d2 = .02$ and $d3 = .02$. Run 4 was with couplings of $d1 = 0.2$, $d2 = .2$ and $d3 = .2$. The boxed region represents the allowed region of ν_μ probability as taken from [20].

7.3 Two Neutrino Model

To better understand the oscillation probability, a toy model was devised using only two neutrinos. Instead of introducing only one bulk neutrino Ψ_1, a second bulk neutrino Ψ_2 was also added. Letting these two bulk neutrinos transform as a $\bar{2}$ and the brane neutrinos transform as a 2 under $U(2)$, a general form of the mass matrix was constructed. Choosing the basis as

$$\mathcal{N}^T \equiv (\nu_1, \nu_2, \psi_1^{(0)}, \psi_2^{(0)}, N_1^{(1)}, M_1^{(1)}, N_2^{(1)}, M_2^{(1)}, N_1^{(2)}, M_1^{(2)}, N_2^{(2)}, M_2^{(2)}, \ldots), \tag{50}$$

with linear combinations of odd and even KK states for both flavors of bulk neutrino, the mass matrix $\mathcal{M}$ takes the form:

$$\begin{pmatrix} 0 & \epsilon & a & 0 & a & a & 0 & 0 & a & a & 0 & 0 & \ldots \\ \epsilon & 1 & 0 & a & 0 & 0 & a & a & 0 & 0 & a & a & \ldots \\ a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ a & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ a & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & a & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & a & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & \ldots \\ a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & \ldots \\ a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & \ldots \\ 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & \ldots \\ 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{51}$$

For M_{brane} a hierarchical pattern was assumed. For $M_{coupling}$ we can understand the form from terms in the Lagrangian. As we said the left-handed neutrinos transform as a 2, ν^b, and the bulk neutrinos transform as a $\bar{2}$, Ψ_b. In this case, a is a coupling of a 2 and $\bar{2}$ in a term of the Lagrangian that looks like $a\bar{\nu}_b\psi^{c^b}$, since only the ψ

component of the bulk neutrinos couples to the brane neutrinos. This example was meant only to test other patterns inside of $M_{coupling}$ with certain group theoretical ideas in mind.

For the first run a value of $a = .004$ was chosen, or in $U(2)$ language $a \sim \epsilon'$. The resulting probability was plotted in Fig. 4. This model shows that we can introduce large mixing with very minimal coupling, however the large oscillation probability is directly correlated with τ. The behavior of the probability could perhaps be sinusoidal if examined over a larger range of τ but large τ is unaccessible in experiments. What would be more interesting would be a larger oscillation probability at a lower value of τ such as in the SuperKamiokande data referred to earlier.

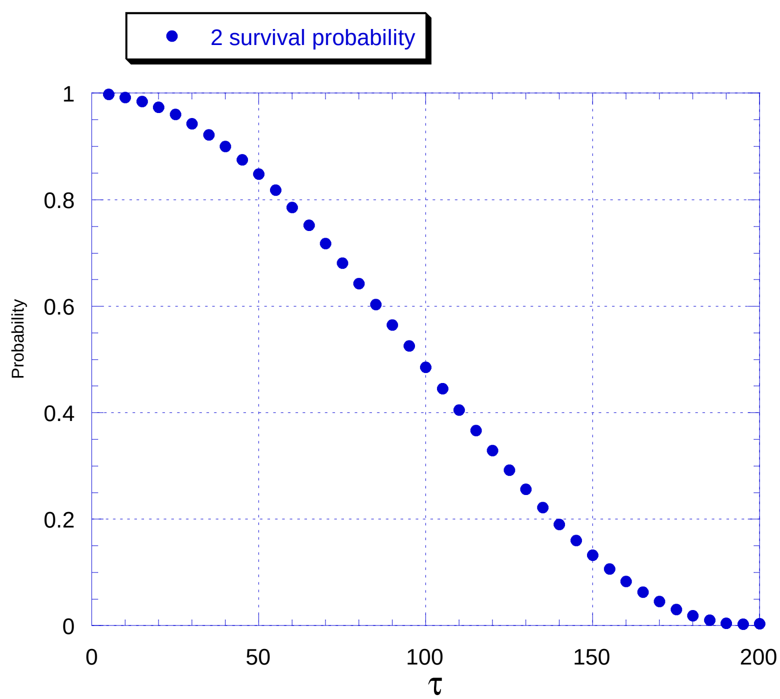

Figure 4: Numerical method result for calculation of the ν_2 survival probability with the value of the coupling $a = .004$.

For the second run, the value of $a = .02$ was chosen, or in $U(2)$ language $a \sim \epsilon$. This resulted in the probability shown in Fig. 5. This result seemed more physically interesting due to the maximal oscillation at a lower τ. This was a reasonable result

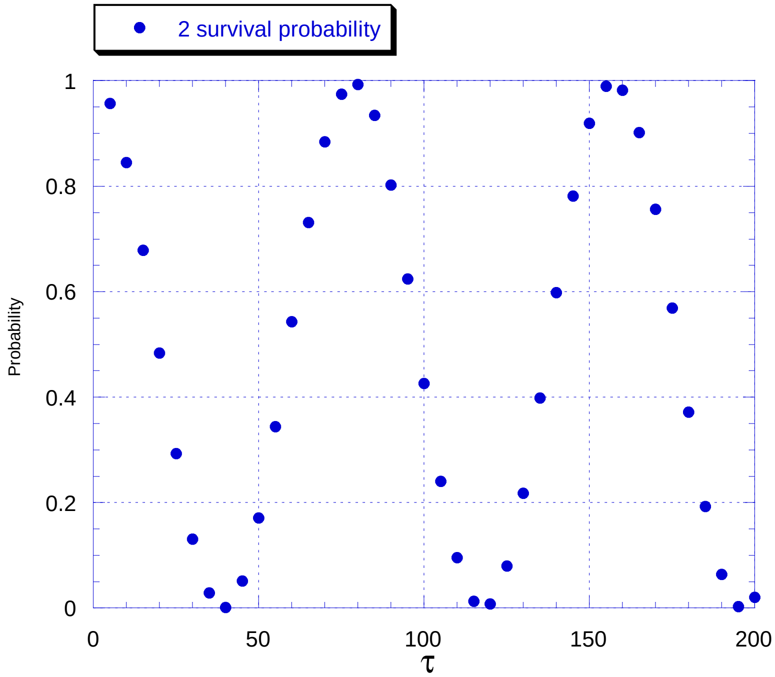

Figure 5: Numerical method result for calculation of the ν_2 survival probability with the value of the coupling $a = .02$.

and phenomenologically useful for looking at a ν survival probability but it does not comment upon what ν will oscillate to. This could be examined in more detail; but since there are three brane neutrinos it is prudent to move onto 3 neutrino models, so as to understand their interplay with the strength and pattern of the couplings.

7.4 Three Neutrino Model Model

In this section we will analyze a possible extension with three bulk neutrinos Ψ_1, Ψ_2 and Ψ_3 and three brane neutrinos ν_1, ν_2 and ν_3. The three bulk neutrinos will transform as a $\bar{2} \oplus 1$ under $U(2)$ and the three brane neutrinos will transform as a $2 \oplus 1$ under $U(2)$. Choosing the basis of

$$\mathcal{N}^T \equiv (\nu_1, \nu_2, \nu_3, \psi_1^{(0)}, \psi_2^{(0)}, \psi_3^{(0)}, N_1^{(1)}, M_1^{(1)}, N_2^{(1)}, M_2^{(1)}, N_3^{(1)}, M_3^{(1)}, \ldots), \tag{52}$$

we obtain

$$\begin{pmatrix}
0 & \epsilon' & 0 & a & 0 & 0 & a & a & 0 & 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0 & \dots \\
\epsilon' & \epsilon & \epsilon & 0 & a & 0 & 0 & 0 & a & a & 0 & 0 & 0 & 0 & a & a & 0 & 0 & \dots \\
0 & \epsilon & 1 & 0 & 0 & c & 0 & 0 & 0 & 0 & c & c & 0 & 0 & 0 & 0 & c & c & \dots \\
a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
a & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
a & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & \dots \\
a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & \dots \\
0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & \dots \\
0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & \dots \\
0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & \dots \\
0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix} . \quad (53)$$

The form of the mass matrix can be deduced from the same sort of arguments as in the two neutrino case. This model was then examined with a variety of choices of couplings a and b. In this three bulk and brane neutrino model, like in the original three left-handed neutrino and the one bulk neutrino model, only stronger couplings demonstrated large oscillation probability. In Fig. 6 one set of couplings is shown with the SuperKamiokande data for ν_μ survival probability boxed as taken from [20]. This analysis demonstrates that this type of model can reproduce allowed patterns

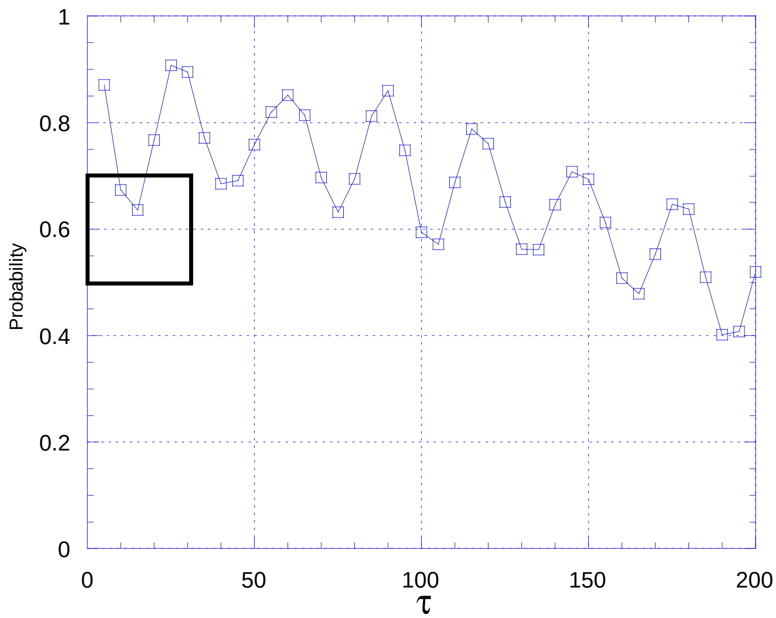

Figure 6: Numerical method result for calculation of the ν_2 survival probability with the values of the couplings: $a = .2$ and $c = .2$. The boxed region represents the allowed region of ν_μ probability as taken from [20].

in ν_μ oscillation probability. All matter in the SM comes in sets of three. Therefore models with three bulk neutrinos may be more natural compared with the minimal Dienes model.

8 Conclusions

This thesis has demonstrated two key points. First, that the use of numerical approximations to the infinite dimensional mass matrices, that arise in extra-dimensional models, converges and coincides with existing analytical solutions. Second, that the introduction of a flavor symmetry in the bulk is compatible with large oscillations in the brane neutrinos.

There are several issues open for one to examine. The first is an understanding of the probabilities involved in nearly bimaximal neutrino oscillation. A more careful study with respect to all experimental bounds on neutrino oscillation would have to be done to figure out what the exact probabilities are that need to be reproduced by a model. Another issue that needs to be examined further is the way flavor is broken in the bulk. The flavons that are introduced are higher dimensional operators that are suppressed by a flavor scale. In four dimensional scenarios the flavor scale is normally set at the GUT or Planck scale. As we have already discussed these scales have been lowered through volume suppression in extra dimensions. This means that our flavor scale will be suppressed as well and we will have flavor symmetry breaking at the TeV scale. This creates potentially dangerous problems with flavor changing processes and for any further investigation the model of flavor breaking would need to be specified. These problems associated with flavor symmetry breaking in the bulk have been addressed in [21] but are beyond the scope of this work.

With the numerical techniques developed one can carry out further analysis of new models with relative ease. The $U(2)$ flavor symmetry has been shown to work well in four dimensions, reproducing nearly bimaximal neutrino oscillations [10]. Hopefully with the numerical techniques developed and the early results demonstrated in this paper, $U(2)$ as flavor symmetry in extra-dimensions will be further investigated to see whether it can reproduce phenomenologically viable neutrino oscillation probabilities.

References

[1] Particle Data Group (D.E. Groom et al.), Eur. Phys. J. C **15**, 1 (2000).

[2] J. N. Bahcall, M. C. Gonzalez-Garcia and C. Pena-Garay, JHEP **0108**, 014 (2001).

[3] T. Toshito [SuperKamiokande Collaboration], hep-ex/9807308.

[4] S.F. King, hep-ph/0105261.

[5] R. Barbieri, G. Dvali, L.J. Hall, Phys. Lett. B **377** (1996) 76.

[6] R. Barbieri, L.J. Hall, A. Romanino, Phys. Lett. B **401** (1997) 47.

[7] R. Barbieri, L.J. Hall, S. Raby, A. Romanino, Nucl. Phys. B **493** (1997) 3.

[8] S.F. King, G.G. Ross, Phys. Lett. B **520** (2001) 243.

[9] C.D. Carone, L.J. Hall, Phys . Rev D **56** (1997) 4198.

[10] A. Aranda, C.D. Carone, P. Meade, Phys. Rev. D **65**, 013011 (2002)

[11] K.R. Dienes, E. Dudas, T. Gherghetta, Phys. Lett. B **436** (1998) 55.

[12] K.R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B **537** (1999) 47.

[13] L. Randall, R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370.

[14] L. Randall, R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690.

[15] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, Phys. Lett. B **429** (1998) 263.

[16] K.R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B **557** (1999) 25.

[17] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali and J. March-Russell, Phys. Rev. D **65**, 024032 (2002).

[18] C.S. Lam, J.N. Ng, Phys. Rev. D **64** (2001) 113006.

[19] K. R. Dienes and I. Sarcevic, Phys. Lett. B **500**, 133. (2001)

[20] N. Cosme, J. M. Frere, Y. Gouverneur, F. S. Ling, D. Monderen and V. Van Elewyck, Phys. Rev. D **63**, 113018 (2001).

[21] N. Arkani-Hamed, L. J. Hall, D. R. Smith and N. Weiner, Phys. Rev. D **61**, 116003 (2000).

A $U(2)$-like Flavor Symmetries and Approximate Bimaximal Neutrino Mixing

The following article was completed during the summer before my senior year. It has been published in Phys. Rev. D [10] and demonstrates the use of flavor symmetries to restrict neutrino oscillation probabilities in four dimensions.