Instability in High Mass Stars

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Abstract

This paper will examine the causes of instability in high-mass stars, those above $20M_{\odot}$, which are only found in rare instances in nature. The three leading hypotheses for the lack of high mass stars are as follows:

I The star begins to use the fusion fuel so rapidly that the star's fuel is extinguished before the star can fully collapse.

II The radiation pressure from the hot fusion burning of the core is greater in value than the gravitational pressure and the star begins to blow off its outer envelope of hydrogen until the gravitational pressure is reduced enough to lower the radiation pressure to a level that can be countered by the gravitation.

III Lastly, there is the possibility that the angular momentum of the initial cloud may be so great as to prevent the infall of all of the mass before the fusion process is finished in the core. The angular momentum of the cloud could also be so great as to cause the star to blow off the outer envelope layers until the angular momentum of the outer envelope is able to be controlled by the star's gravity.

These possibilities were evaluated through the use of computer modeling of low mass stars and the extrapolation of those models to the high-mass regime.

1 Introduction

This research is designed to provide a better understanding of the reasons for the instabilities in high-mass stars. Historically, stellar research has focussed on stars like the sun, which have long lifetimes and are able to maintain hydrostatic equilibrium easily. However, lately, there has been some interest in the lack of very high mass, $100M_{\odot}$ stars in the universe. One reason for this renewed interest is the Pistol Star, which will be discussed later in reference to the results of this paper. In addition, high-mass stars appear to develop in a very different environment from low-mass stars [3]. Most importantly, high-mass stars develop in an environment where the initial cloud of interstellar gas has a much higher angular momentum than that found in association with low-mass stars. While this research will not directly cover the effects of angular momentum or the initial cloud environment, a very useful reference paper [3] explains all of these different conditions and the effect on stellar stability and functioning.

In order to explore this question, we started with computational modeling to simulate low mass stars. When the results of this initial research concurred with previously published research[1] on stellar evolution and functioning, an attempt was made to extrapolate these results to the high mass regime. The analysis of these results is done by trying to match two sets of numerical integration, one from the outside in, and one from the inside out. If these two results match, then the solution is determined to be a stable state solution for that mass of star. Given the stable model solutions, one can then look at observational results and determine whether any of the large stars, within five to ten percent of the modeled star's mass, have values similar to those for the model. Given these matches of the modeled solutions to the actual solution, it is probable that the model is extendable to other stars throughout a mass range around the central value.

2 Historical Background

The vast majority of previous investigations into stellar models have dealt with the low-mass regime. A large reason for the lack of work in the study of high-mass stars has been due to the lack of adequate computational power. While the equations of stellar evolution and stability are non-trivial for the sun and other light stars, these equations are nearly impossible to solve without modern computers and computational techniques for high-mass stars. In addition, the use of modern computational equipment allows for the varying of multiple variable conditions and this enables easier determination of stable state solutions for a certain mass by varying radius and luminosity until the stable solution is found.

In addition to the technical limitations on the study of high-mass stars, there has been less interest in the study of high-mass stars than low-mass stars. This is probably due to the greater desire to understand the sun and similar stars, as any condition that would affect solar stability could be catastrophic for the earth. However, another reason for the lack of reference materials on high-mass stars is probably due to the lack of observational data on high-mass stars. For the sun, there is a large amount of observational data to confirm or deny the validity of the models produced. For high-mass stars, there is very little observational data to test whether the models are valid.

3 Nuclear Burning

In a star, the initial energy source is gravitational collapse, and the main continuing source is nuclear fusion. To provide a simple overview of the interaction between gravity and fusion, the gravitational pressure heats the elements enough that some can overcome, by quantum tunneling, the electrostatic repulsion separating them. The primary route for this in light stars is the proton-proton chain. This reaction occurs in three distinct steps.

$$H^1 + H^1 \rightarrow D^2 + e^+ + \nu \tag{1}$$

$$D^2 + H^1 \rightarrow He^3 + \gamma \tag{2}$$

$$He^3 + He^3 \rightarrow He^4 + H^1 + H^1 \tag{3}$$

In terms of time, the rate limiting step is the first part of the reaction, which has a timescale of 1.4×10^{10} years. This is due to the nuclear transmutation probability, the likelihood that two nuclei will come into contact with each other. The nuclear transmutation factor includes in its calculation the binding energy, which must be overcome. It is this factor that does not favor this reaction. However, the second step, the combining of two deuterons to form a helium nucleus is very fast, on the order of ten seconds. In reviewing the reference works, [1] it has been found that for every He atom formed in the proton-proton chain, 26.2 MeV of energy are liberated. This energy expresses itself in the form of energetic photons, γ, which are responsible for the radiation pressure on the star and the electromagnetic spectrum generated.

4 Equations of Hydrostatic Equilibrium

Of the equations that are utilized in the initial modeling of the star, two of the most important are the equations of hydrostatic equilibrium. These two equations relate the radiation pressure of the star and the gravitational pressure of the star. As was noted above, the radiation pressure is caused by the nuclear burning within the star. In order for the nuclear burning to occur, the density and temperature at the center of the star must be high enough to allow the proton reactions, and thus fusion, to occur.

However, the radiation pressure acts to push the core of the star apart, lowering the fusion rate and preventing stellar collapse. Thus, an equilibrium condition develops, where the radiation pressure and gravitational pressure balance each other to keep the star from expanding or collapsing.

4.1 The Equation of Radiative Pressure

$$\frac{dP}{dr} = -\rho \frac{GM_r}{r^2} \tag{4}$$

This equation details the dependence of the radiative pressure on the mass of the star. The value M_r is the mass value at a particular level in the shell. In addition, ρ is the mass density of the stellar plasma and can be allowed to vary depending on the distance of the plasma shell from the core. Although this equation is given in terms of the actual value r, in our research we utilize a value x, which is a scaled dimensionless variable. Scaled dimensionless variables will be discussed in a section below.

4.2 The Equation of Stellar Mass

$$\frac{dM_r}{dr} = 4\pi r^2 \rho \tag{5}$$

This equation is really just an application of Gauss' Law for a spherically symmetric object, which we assume the star to be. While there will be a slight deviation between this result and the actual value, due to the angular momentum induced bulging, we are assuming a zero angular momentum state, for which this equation is fully valid. As a side note, there is no change to the variable definitions from those introduced in section 4.1.

5 The Equation of Thermal Equilibrium

$$\frac{dL_r}{dr} = \epsilon \rho 4\pi r^2 \tag{6}$$

The thermal equilibrium equation states that the power taken in by a layer of the star will be equal to the power released by the layer of the star which is simply the power produced in that layer by nuclear fusion. In addition, ϵ is the value of nuclear power produced per unit mass of material and depends on many factors, including the temperature, isotope constitution of the star, and density [1].

$$\epsilon = 2.5 \times 10^6 \rho X^2 (\frac{10^6}{T})^{2/3} e^{-33.8(\frac{10^6}{T})^{1/3}} \tag{7}$$

In this equation, X is the hydrogen fraction, which is set to 0.90 in our stellar models, ρ is the mass density, and T is the star's temperature. By slight rearrangement, one can see that the dependence on temperature is the greatest dependence as this equation includes the term:

$$e^{\frac{33.8}{3}\frac{T}{10^6}} (\frac{T}{10^6})^{3/2} \tag{8}$$

Because of this high temperature dependence, ϵ has a lower dependence on the composition fraction and density, and therefore will generally scale with the mass, because a higher mass will provide a higher burning temperature.

Therefore, as we have seen from the previous section, the equation for Thermal Equilibrium can be rewritten as:

$$\frac{dL_r}{dr} = \epsilon \frac{dM_r}{dr} \tag{9}$$

From this rearrangement of the equation, we can see that in fact the thermal emission for the layer,*the luminosity* is merely the nuclear energy coefficient multiplied by the mass in the shell.

6 The Energy Transport Equation

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L_r}{4\pi r^2} \tag{10}$$

In this equation, T again represents the temperature, L_r is the luminosity, or net power flux out of the shell, c is the speed of light, $3.0 \times 10^{10} \frac{cm}{s}$ due to the use of cgs units in astrophysics, κ is the opacity of the layer of stellar plasma, ρ is the mass density, and a is a variation of the Stefan-Boltzmann constant, with $a = \frac{4\sigma}{c}$ [2]. Again going through the cgs units, the value for a is $7.56 \times 10^{-15} \frac{erg}{cm^3 K^4}$. The energy transport equation describes the radiative transfer of energy, or the transfer of photons through the stellar plasma. This equation shows the heating of a plasma layer by the interior layers. Essentially, the heating of the plasma is due to photoexcitation of the layer's constituent ions by the photons emitted by the inner layers. This equation also shows why the outer layers of the star are so much cooler than the inner layers. The stars are much cooler in their outer layer than inner layer because ρ of the outer envelope is only a small fraction of the initial value and the energy transport equation has a large dependence on ρ. Another, smaller contributor to the heating of the various elements is due to electron scattering. By combining both forms of scattering, there is a large effect deep in the core, where ρ is high, and the star is engaging in fusion. However, because of the dependence on L_r, a value that falls off rapidly as r is moved beyond 20% of the star's radius, the temperature will quickly drop and asymptotically approach 0, relative to the core temperature.

7 The Change in Variables

Throughout the research, we have worked in scaled, dimensionless variables rather than in the actual values. The main reason for this is that the scaled variables are simpler to use than the non-scaled variables. In addition, the scaled variables can be transformed, via logarithmic differentiation, into a set of dimensionless variables that will graphically depict the results of the stellar functioning. Following the initial work in these dimensionless variables, the important physical quantities, such as the central temperature and pressure can be determined. Once the central values have been determined, it is possible to extrapolate the actual temperature, pressure and luminosity values within the different shells.

7.1 The Initial Variable Transform

Utilizing the system developed by Martin Schwarzchild [1] we began by converting standard physical values, such as mass, luminosity, radial distance, and temperature into an initial set of dimensionless variables. The values for these transformations are below:

$$q = M_r/M \tag{11}$$

$$f = L_r/L \tag{12}$$

$$x = r/R \tag{13}$$

$$p = \frac{4\pi R^4 P}{GM^2} \tag{14}$$

$$t = \frac{kRT}{HGM\mu_s} \tag{15}$$

In these equations, μ_s is the composition variable, which will vary for the composition but for our case is given the numerical value of 0.59, for our mass fractions, with X = 0.90, Y = 0.09, Z = 0.01. X is the fraction of H, Y is the electron fraction, and Z is the neutron fraction.

$$\mu = \frac{X}{2} + \frac{4Y}{3} + 2Z = 0.45 + 0.12 + 0.02 = 0.59 \tag{16}$$

The values of R, M, L, T are the actual values for the given star. H is the mass of the hydrogen nucleus, $1.54X10^{-24}$ grams, k is the Boltzmann constant, again in cgs units. However, these transformations only cover the integration from the outside in. If any of the initial input values are not exactly correct, then the solutions will become increasing unstable as $r \rightarrow 0$. To fully utilize the dimensionless variables, we must utilize a second transformation and apply starting conditions to generate a set of variables that are utilized from the interior of the star outwards. These transformations are:

$$q^* = q/q_0 \tag{17}$$

$$f^* = f/f_0 \tag{18}$$

$$x^* = x/x_0 \tag{19}$$

$$p^* = p/p_0 \tag{20}$$

$$t^* = t/t_0 \tag{21}$$

In each of these equations the initial solution is found via the starting conditions below, or by combining two different initial conditions to eliminate that boundary condition, allowing an easier solution:

$$\frac{q_0}{t_0 x_0} = 1 \tag{22}$$

$$\frac{p_0 x_0^3}{t_0 q_0} = 1 \tag{23}$$

$$C\frac{p_0^2 f_0}{t_0^{9.5} x_0} = 1 \tag{24}$$

$$D\frac{p_0^2 t_0^{\nu-2} x_0^3}{f_0} = 1 \tag{25}$$

$$t_0 = t_c \tag{26}$$

In these equations, t_c is the central temperature and C and D are constants for a given star, but vary between stars. These equations are:

$$C = \frac{3}{4ac}\frac{k}{HG}^{7.5} 4\pi^{-3} \times \frac{\kappa_0 s}{\mu_s}^{7} .5 \times \frac{LR^{0.5}}{M^{5.5}} \tag{27}$$

$$D = \frac{HG^{\nu}}{k}\frac{1}{4\pi} \times \epsilon_{0s}{\mu_s}^{\nu} \times \frac{M^{\nu+2}}{LR^{\nu+3}} \tag{28}$$

In this equation ϵ_{0s} is the constant value for ϵ that is the value for the modeled star, based on given composition parameters and approximate temperature and density values. M, L and R are the actual values for the star, which are determined by astronomical observation or by fitting the solutions until a stable state is found. These equations and boundaries are utilized to refine the differential equations into a more soluble form. From here, it is easy to utilize the solution methodology mentioned below.

7.2 The Use of the Variables U & V

The main reason we use the variables U and V, defined later, are because they are scale independent and allow matching the inner solution to the outer solution. U and V represent the change in local values as compared to the average value or total values. This is more easily seen when U and V are written as in [1]:

$$U = \frac{4\pi r^3 \rho}{M_r} \tag{29}$$

$$V = \frac{\rho}{P}\frac{GM_r}{r} \tag{30}$$

From these equations, it is easy to see that U represents the quotient of the local density, ρ and the mean density, given by dividing the mass inside the sphere by the volume of the sphere. V is, by a somewhat more complex relationship, the energy caused by gravitational infall divided by the internal energy produced by the radiation pressure. Thus, these two equations determine whether the local area is of higher density than the surrounding plasma as well as to determine whether this layer is balanced or if the layer will be accelerating outward or inward.

$$U = \frac{x}{q}\frac{dq}{dx} \quad \rightarrow \quad \frac{px^3}{tq} \tag{31}$$

$$U = \frac{x^*}{q^*}\frac{dq^*}{dx^*} \quad \rightarrow \quad \frac{p^* {x^*}^3}{t^* q^*} \tag{32}$$

$$V = -x/pdp/dx \quad \rightarrow \quad \frac{q}{tx} \tag{33}$$

$$V = -x^*/p^*dp^*/dx^* \quad \rightarrow \quad \frac{q^*}{t^*x^*} \tag{34}$$

From these values for U and V, I was able to generate the plots seen in the figure below, Fig. 1. As one can see, it is very easy when utilizing U and V to see the trends in the stability as luminosity is varied. On the figure, the line originating in the lower right corner is that generated by the curve going from the inside out while the other curves are the results for calculating the outside in equations. In order to achieve a stable value for the star, the two curves must lie tangent to each other. These curves must be tangent because there must be a point of overlap, but not crossing, in order to achieve stable results for the stellar temperature and luminosity. If the curves do not lie tangent to each other, then there is not a single stable solution and the result will be an instability rather than a solution. As this figure illustrates, the only luminosity value that will provide a stable star with a mass of $100M_\odot$ is to have the luminosity equal to $6ML_\odot$, that is six million times the luminosity of the sun. However, trends are not the only values that can be gleamed from the U-V plot. By utilizing the matching points, it is possible to find the actual central temperature and pressure, as was indicated by the variable transformations above. When one has both the outside in and inside out values, they can be combined to give the actual central value for the variable.

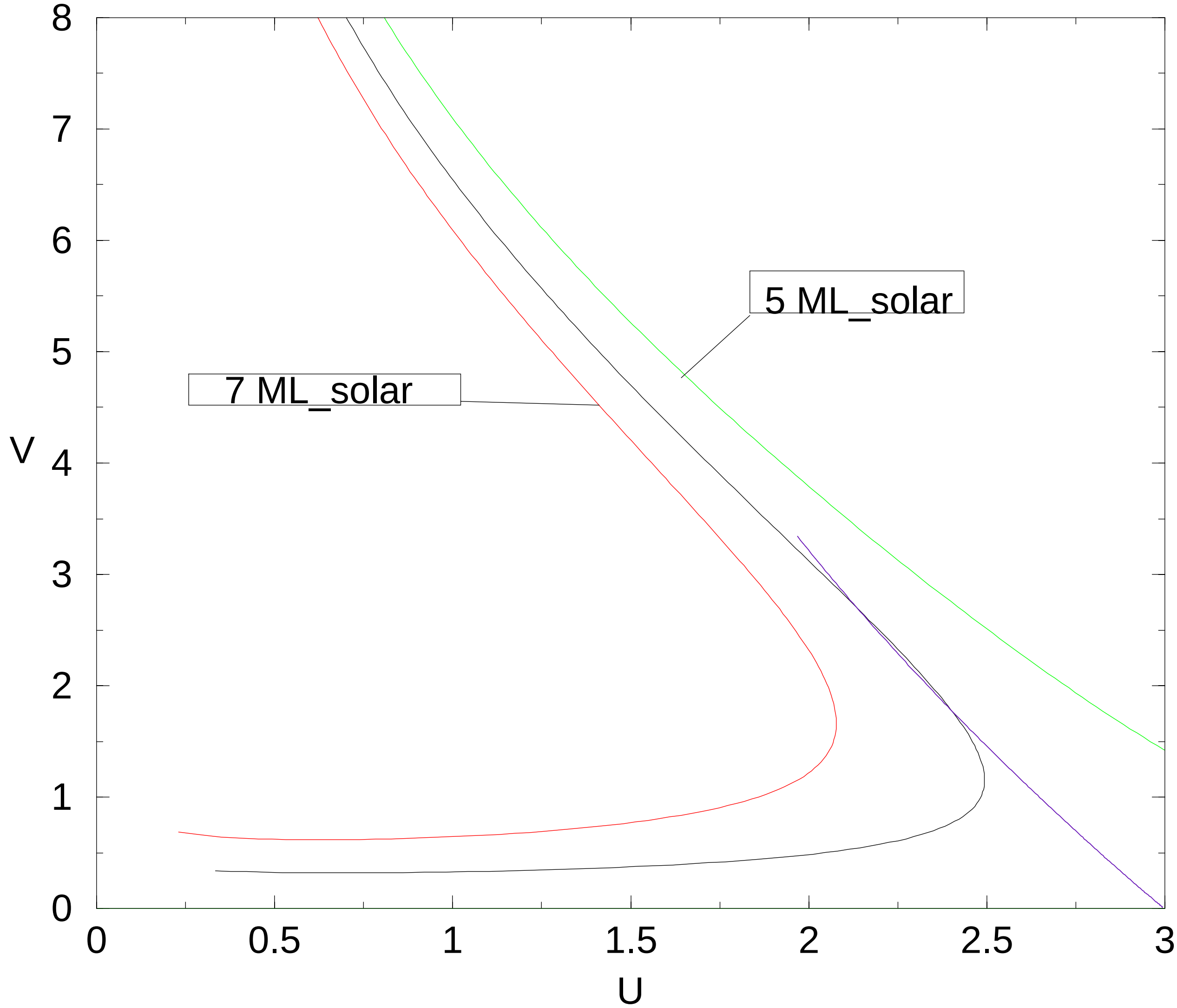

Figure 1: Results for Various Luminosity Values for the 100 Solar Mass Star.
Purple Curve-Inside-Out Integrations for All Three Luminosity Values
Red Curve-Outside-In Integration for $7ML_{\odot}$ Star
Green Curve-Outside-In Integration for $5ML_{\odot}$ Star
Black Curve-Outside-In Integration for $6ML_{\odot}$ Star

8 Methodology for the Development and Study of Computer Models

In calculating the stellar results for this paper, a simple numerical finite differentiation technique was utilized. To accomplish this, the derivative forms of the equations were approximated with the finite differential form. The expressions devised from applying this technique were the primary component of the analysis code in our computer models. All of the modeling work was done in C++ utilizing the Gnu compilers available on the department Linux and Unix machines. In addition, the code generated and stored the 2-D coordinates needed to generate Fig. 1.

In order to determine the validity of the models, a series of test runs using known values from [1] was conducted. After some debugging of the model to remove all of the mathematical errors inherent in the initial model, the system began to give perfect matches to the known data. Given the age of the Schwartzchild reference used, and its continued relevance, it is doubtful that there are any errors in the reference data given in source [1]. Based on these test simulations, we feel that the models we utilize are valid in the low-mass range and can find no reason to doubt the validity of the model in the high-mass range, for the zero angular momentum condition. Therefore, we feel that the model we are utilizing is a valid, if simple model, that should produce accurate results for the given input case.

9 Longevity of Stars based on Current Model Data

Initially, we felt that the models would not give a viable solution in the $100_{\odot}$ range and were surprised at the ease in which a solution that appears viable came into existence. This then led to a study of the possible lifetime of a star this large and a comparison to the collapse time for the stellar material to come together. This study was conducted through the use of a Maple worksheet to get an estimate of the time needed for the material to collapse and then how long the star would survive before it would complete hydrogen burning. While it is true that there can be shell burning after the hydrogen burning is completed, the time for shell burning is much lower than the time for hydrogen burning and can be neglected from these calculations. The results we found indicate that it would take approximately 180000 years for the cloud of material to come together and form a star yet the lifetime of the star from the initial protostar stage would only be 600000 years. These calculations are, for the collapse time in years,

$$0.1 \times 0.007 \times 2 \times 10^{33} \frac{9 \times 10^{20}}{3.9 \times 10^{33} * 3.7 \times 10^{100} * 6000000} = 180000 \tag{35}$$

and for the time of stellar burning, again in years:

$$6.26 \times 10^{4^{3/2}} \times 0.1 \times 0.5 = 780000 \tag{36}$$

Therefore, stars in this high mass range are around for less than one million years before they run out of fuel, if they are able to come together. Another consideration that we were unable to cover but is covered in full detail in [3] is the effect of angular momentum. If there is a large amount of angular momentum, as is common in the clouds from which large stars form, then there is less likelihood for the star to form and, if the star does form, it will be around for less than 400000 years before it exhausts its nuclear fuel. The angular momentum state can also combine with the radiation pressure from any core burning in the protostar state to expel some of the outer layers of gas. This then results in a smaller cloud than would initially exist and lower the mass of the star. However, from all of the collected data and modeling, it appears that high-mass stars are a very rare event in nature and will not be fully understood observationally due to the lack of observational possibilities.

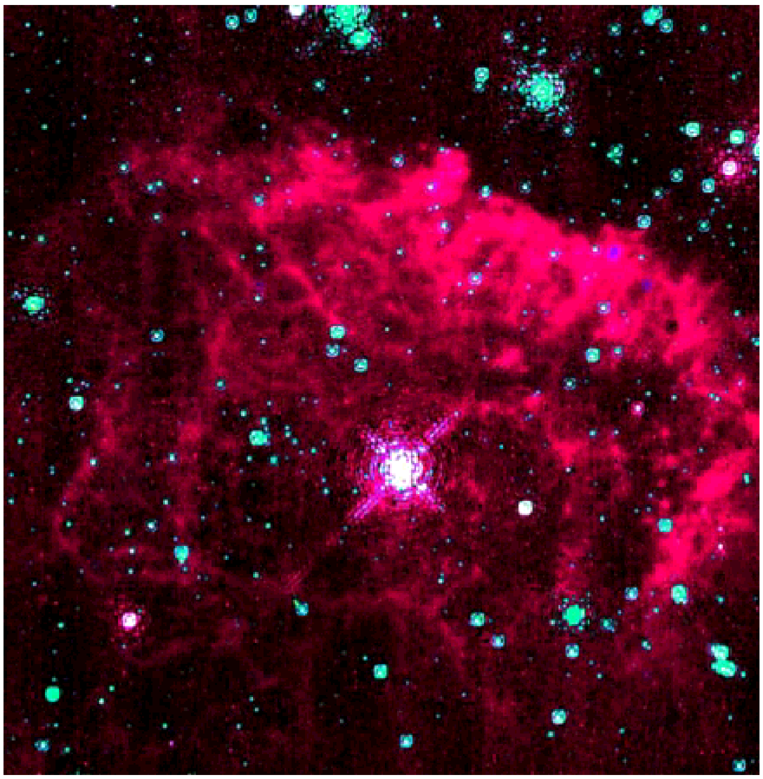

Figure 2: A Hubble Space Telescope Image of the Pistol Star

9.1 The Pistol Star

The Pistol Star is the only known star with a mass of around $100M_\odot$ that has been seen. It is currently in the middle of a large nebula that is either from the formation of the star, or it is the result of stellar expulsion. However, current observational data places the star at having a luminosity of approximately $10ML_\odot$ but this value is somewhat uncertain due to surrounding dust. This value is well outside the range of possibility for stellar luminosity for our model conditions. The discrepancy indicates that there may be a problem with the observational data on the Pistol Star, such as an overcompensation for dust in the calculation of the luminosity of the star. The Pistol Star does however show all of the signs of a star that is throwing off material and the surrounding nebula appears to be hydrogen gas, given its red emission, that was thrown out from the protostar's cloud before infall was complete. However, given our lifetime estimates, the probability of the star burning a large amount of material before collapse seems somewhat likely than an expulsion of material due to the radiation pressure or angular momentum. For instance, the high luminosity value compared to our model data suggests that the most likely cause of the Pistol Star's matter loss is due to radiative expulsion from the star and the gravitational force not being able to compensate. Yet, another plausible suggestion is that the luminosity is greater from there being less absorbing layers, that would have resulted from infall, lowering the stellar luminosity.

10 Conclusion

The purpose of this paper was the cover the instabilities inherent in high-solar mass stars and the reasons for their relative scarcity. The primary hypotheses which we chose to investigate were whether the radiation pressure would be too great for gravity to overcome and this would result in an expulsion of matter until the star cooled down to a level where gravity could compensate for radiation; that the star begins to burn before collapse is complete, therefore, the star will never fully collapse before burning begins and the burning will be almost complete when the collapse ends. Of these , it appears that in the only case for which we have evidence, the Pistol Star, that the overabundance of radiation pressure to gravitational infall is the main reason for the lack of high-mass stars. However, given the models presented in [3] it appears as though the angular momentum state of the cloud is important as well, because if the cloud has too high of an angular momentum, then the time for formation of the stars will be longer or nearly as long as their burning period, which will lead to the stars disappearing before they have time to form. However, a longer term period of studying larger computer models, including those that fully account for the angular momentum of the cloud and the star, will be needed before a definitive answer for the paucity of extremely large stars will be found. It is my hope that the results found here will be able to assist in the development of further work in this field that will one day lead to a definitive answer.

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