Vacuum Stability in the Extended Higgs Model

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Science with Honors in
Physics from the College of William and Mary in Virginia,

by

David J. Leichtman

Accepted for ______________________
(Honors, High Honors or Highest Honors)

____
Advisor: Dr. Marc Sher

____

____

____

Williamsburg, Virginia

April 2000

Contents

1 Introduction **2**

2 Theory **5**
- 2.1 Spontaneous Symmetry Breaking 5
- 2.2 The Higgs Mechanism 5
- 2.3 The Standard Model 8
- 2.4 Renormalization Group Equations 12
- 2.5 Bounds on the Higgs Mass 13

3 The Two Higgs Doublet Model **15**
- 3.1 Two Higgs Doublet Models I and II 17
- 3.2 Computational Results for Models I and II 22

4 Two Higgs Doublet Model III **27**

5 Conclusion **28**

A C++ code for determining mass bounds in the 2HDM **36**

1 Introduction

The fundamental goal of theoretical particle physics is to find a theory that describes all observed particles and interactions. To that end, particle theorists have developed what is known as the standard model. This model describes the constituents and dynamics of the electromagnetic, weak, and strong interactions. The standard model consists of two types of particles, those that participate in an interaction (fermions), and those that mediate the interaction (gauge bosons).

The standard model is a quantum field theory. A quantum field theory describes particles as fields and their interactions as the overlap of these fields. These fields are quantized in that they are allowed only certain oscillatory states. The oscillatory modes of such fields are associated with particle masses, thus tying a given field theory to real particle dynamics.

When defining a quantum field theory, we need only state two things about a system: the fields involved, and the symmetries that they obey. All information about the system falls out of this fairly straightforward initial information. If one knows what kinds of fields are involved (scalar, vector, psuedoscalar, etc.) and whether they are commuting or anticommuting, then it is possible to define the action of the system. The action is the integral over all space-time of the Lagrangian, the kinetic energy minus the potential energy of the system. Requiring certain symmetries imposes conditions of invariance on the action, and therefore allows or disallows certain terms in the Lagrangian. Thus by writing down the Lagrangian and how the various fields transform under certain symmetries, we have a complete theory.

Incorporating experimental constants into the theory makes it possible to predict the outcome of any processes (particle interactions) allowed in the system. One necessary feature of a field theory is the ability to tie certain coupling constants (strengths of interactions) to observed phenomena thus giving predictive power to that theory. To that end, a quantum field theory, when constructed, must be renomalizable, that is, infinite divergences in quantum mechanical amplitudes (the likelihood of a given process) can be removed from the theory by redefinition of couplings. An example of such a theory is the gauge theory of quantum electrodynamics (QED). By requiring the Lorentz invariance of a single field in the Lagrangian in conjunction with a simple gauge symmetry, we produce the rules that govern electromagnetism.

The dynamics are more complicated, however, in trying to unify the electromagnetic and weak forces into a single theory. The electromagnetic interaction is mediated by the massless photon. Because the photon is massless, the uncertainty principle places no restriction on the range of the force. However, the weak interaction is mediated by the vector bosons, the $W^{\pm}$ and the Z. The weak interaction is short ranged, implying that these particles are massive.

Experimentally, the vector bosons have mass. However, massive bosons present a problem in defining a field theory. There is no way to simply write down mass terms in the Lagrangian and still maintain renormalizablility. Without renormalization, the theory loses its predictive power. The solution lies in spontaneous symmetry breaking. It is a means of generating particle masses while maintaining the renormalizability of the theory.

The Glashow-Weinberg-Salam (GWS) unified electroweak theory accounts for the massive vector bosons by incorporating an instance of spontaneous symmetry breaking called the Higgs mechanism. This mechanism, in the form of a broken $SU(2) \times U(1)$ symmetry, assigns a nonzero vacuum expectation value to a self-interacting complex scalar field. The particles resulting from this theory are identified as the free, massive neutral Z boson, the massive, charged $W^{\pm}$ bosons, and the massless photon.

An extra consequence of this theory, however, is a massive neutral scalar boson often referred to as the Higgs boson which has yet to be identified. Upper and lower bounds can be placed on the scalar mass simply by requiring that perturbation theory remain valid up to the unification scale and that the vacuum be stable, i.e. that the ground state of the universe itself does not decay spontaneously. In particular, this paper will explore several extensions of the standard model and the bounds that can be placed on the neutral scalars of those theories.

2 Theory

2.1 Spontaneous Symmetry Breaking

The idea of spontaneous symmetry breaking is quite simple. Spontaneous symmetry breaking (SSB) occurs when we require that the Lagrangian of a theory obey a certain symmetry while allowing the vacuum state to violate that symmetry. A good example is the classical ferromagnet. At high energy, the ferromagnet has no preferred spin alignment. However, at low energy, the ferromagnet's spin points in a particular direction. The ferromagnet breaks rotational symmetry at minimum energy [10]. Another good example is that of a knitting needle aligned lengthwise with a particular axis. Given lateral compression, the needle will, at some point, buckle in an arbitrary direction in the perpendicular plane, sacrificing rotational invariance for a lower energy configuration.

2.2 The Higgs Mechanism

When applied to a set of scalar fields, SSB is known as the Higgs mechanism. A way to illustrate the Higgs mechanism is by considering the simplest case of ϕ^4 theory, a complex scalar field under global $U(1)$ symmetry ($\phi \to e^{-i\theta}\phi$). We first construct the simplest invariant Lagrangian

$$\mathcal{L} = (\partial^\mu \phi)^\dagger (\partial_\mu \phi) - V(\phi), \tag{1}$$

with the potential given as

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2, \tag{2}$$

where μ^2 and λ are arbitrary couplings.

Considering the case where $\mu^2 > 0$, the potential is a simple paraboloid with minimum $\phi^\dagger \phi = 0$. However, considering the case where $\mu^2 < 0$ and $\lambda > 0$, the potential takes the shape that is popularly called the "Mexican Hat" (Fig. 1). There is a local maximum at $\phi^\dagger \phi = 0$, and the minimum is $\phi^\dagger \phi = -\mu^2/2\lambda$. The field, therefore, can be said to have a nonzero vacuum expectation value (VEV) [10, 9].

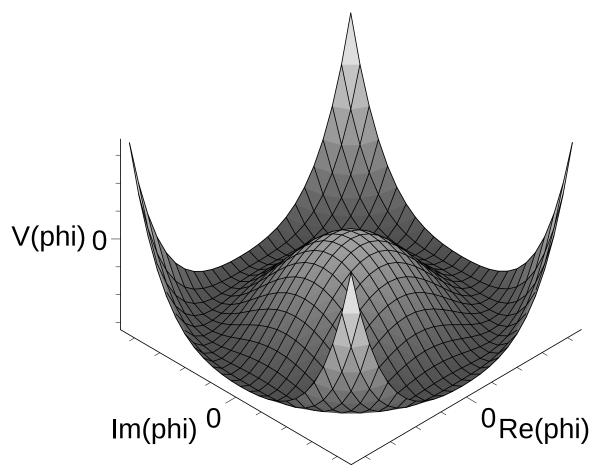

Figure 1: The "Mexican Hat" Potential

The minimum, without loss of generality, can be chosen to lie in the direction of $\Re(\phi)$. A perturbation about the vacuum can be performed, expanding about the minimum, to redefine the theory in terms of the nonzero VEV. The field can be expanded as,

$$\phi(x) = \frac{1}{\sqrt{2}} \left(\sigma + \eta(x) + i\chi(x) \right), \tag{3}$$

where $\sigma^2 = -\frac{\mu^2}{\lambda}$, and where $\eta(x)$ and $\chi(x)$ are both real fields, each with a zero VEV, representing perturbations in the radial and angular directions respectively. Substituting the expanded field back into the Lagrangian and throwing out higher order terms, we have

$$\begin{aligned} \mathcal{L}' &= \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}(2\lambda\sigma^2)\eta^2 - \lambda\sigma\eta\chi^2 \\ &\quad - \frac{1}{4}\lambda\eta^4 - \frac{1}{4}\lambda\chi^4 - \frac{1}{2}\lambda\eta^2\chi^2 \end{aligned} \tag{4}$$

Now, in the redefined theory, we have mass terms of the form $\frac{1}{2}m^2\phi^2$. The mass-squared of the η is then $2\lambda\sigma^2$ and the χ is massless.

The massless boson is the direct result of SSB, known as Goldstone's theorem. A spontaneously broken symmetry will always result in a massless particle (a Goldstone particle). For every degree of freedom lost by a broken symmetry, there will be a resulting Goldstone particle in the theory. While this is not generally obvious, it is readily apparent in the previous example. The massive η corresponds to the positive-frequency oscillatory mode in the radial direction, while the massless χ corresponds to the zero-frequency mode in the angular direction [10].

The idea of the Goldstone boson is an important one in SSB. A theory with a global symmetry, such as the previous example, produces a massless scalar boson. However, we know that there are no such particles in the physical universe. Real massless particles, like the photon, have two degrees of freedom, while massive vector bosons have three, and massive scalars have one. The question then arises as to what this Goldstone boson actually represents.

The answer lies in introducing a local gauge symmetry to the theory ($\phi \rightarrow e^{i\alpha(x)}\phi$). With such a symmetry imposed, we find that the new theory, with a properly defined gauge, now has no massless particles. Instead, there is a massive vector boson with three degrees of freedom, and a single massive scalar. We say, then, that the Goldstone boson has been "eaten" to provide the third degree of freedom to the vector field. The fact of the matter is that the Goldstone boson is simply a placeholder for degrees of freedom, and that, given a properly, physically defined gauge, it disappears with a redefinition of the theory's fields.

2.3 The Standard Model

The GWS electroweak theory is the core component of the Standard Model. This model incorporates the idea of SSB into a locally gauge-invariant $SU(2) \times U(1)$ theory. This model requires the vector bosons to acquire mass and unifies the weak and electromagnetic interactions into a single theory. This model, as previously stated, also produces the yet undetected Higgs boson [5].

Let us now consider the *minimal* standard model, the simplest allowed Higgs field. Let Φ be a complex $SU(2)$ doublet such that

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{5}$$

The kinetic and potential terms of the simplest locally $SU(2)$ invariant Lagrangian are

$$\mathcal{L}_S = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi),\ D_\mu = \partial_\mu - i\frac{g}{2}\tau^i A^i_\mu - i\frac{g'}{2}B^i_\mu. \tag{6}$$

The most general potential is

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2. \tag{7}$$

Higher order terms in the potential lead to renormalization problems, and are therefore excluded from the potential.

The field couples to the fermions to generate fermion masses. The Yukawa coupling terms in the Lagrangian that give mass to the charge -1/3 quarks and leptons are of the form

$$h^d_{ab}\bar{Q}^w_{aL}\Phi d^w_{bR} + h^e_{ab}\bar{L}^w_{aL}\Phi e^w_{bR} + h.c., \tag{8}$$

with

$$Q^w_{aL} = \begin{pmatrix} u^w_a \\ d^w_a \end{pmatrix}_L,\ L^w_{aL} = \begin{pmatrix} \nu^w_a \\ e^w_a \end{pmatrix}_L. \tag{9}$$

Here, Q^w_{aL} are the $SU(2)$ doublets representing the left-handed quarks (indexed over the 3 u and the 3 d generations), and Q^w_{aL} are the left-handed leptons (indexed over the 3 ν and the 3 e generations). The right-handed quarks are given by the $SU(2)$ singlets u^w_{aR} and d^w_{aR}, while the right-handed e is denoted by e^w_{aR}. A second field

$$\Phi_c = \begin{pmatrix} \phi^{0_*} \\ -\phi^- \end{pmatrix} = -i\tau_2\Phi^* \tag{10}$$

can be constructed from the first to couple to charge 2/3 as such

$$h^u_{ab}\bar{Q}^w_{aL}\Phi_c u^w_{bR} + h.c. \tag{11}$$

The h matrices above are 3×3 coupling matrices. The Lagrangian is then the sum of the kinetic and potential terms and the Yukawa coupling terms [10].

To incorporate spontaneous symmetry breaking, we must minimize the potential and expand about the nonzero minimum, as in the previous example. The complex doublet must first be expressed in terms of four real scalar fields

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \tag{12}$$

The direction of the minimum is arbitrary. For simplicity, we will choose the minimum to lie along the real part of the neutral component of Φ, ϕ_3. We perform an $SU(2)$ rotation of the field such that $\phi_1 = \phi_2 = \phi_4 = 0$. The minimum is then given in terms of the real parameter σ as

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \sigma \end{pmatrix}. \tag{13}$$

Again, choosing $\mu^2 < 0$, provides a minimum at $\sigma^2 = -\frac{\mu^2}{\lambda}$. Now the Higgs field is shifted to the vacuum state, and a gauge is chosen to eliminate the remnants of the broken generators. The new field to be put back into the Lagrangian is then

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma + \eta(x) \end{pmatrix}. \tag{14}$$

Substituting this field into the Lagrangian, and making the following definitions

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(A_\mu^1 \mp A_\mu^2 \right), \tag{15}$$

$$Z_\mu = \sin\theta_W B_\mu - \cos\theta_W A_\mu^3, \tag{16}$$

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W A_\mu^3, \tag{17}$$

$$\tan\theta_W = \frac{g'}{g}, \tag{18}$$

we can read the masses of the bosons from the quadratic terms. They are

$$M_W^2 = \frac{1}{4} g^2 \sigma^2, \tag{19}$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2)\sigma^2, \tag{20}$$

$$M_A^2 = 0. \tag{21}$$

The masses of the quarks also arise from Yukawa terms in the redefined Lagrangian. They take the form $m_\mu = \frac{g_Y^\mu \sigma}{\sqrt{2}}$, where the coupling constants g_Y^μ

are determined from the simultaneous diagonalization of the h matrices with the mass matrices. The Higgs's mass, identified from the η field, can also be read off from terms quadratic in η as $m_\eta^2 = 2\lambda\sigma^2$. It should be noted that σ is a scale determined to high precision to be $\sigma = 247\,GeV$ [10].

The fate of the massless Goldstone bosons is the same as in our previous example. While the Higgs field started with four degrees of freedom, the single Higgs boson is a massive scalar with one degree of freedom. The three remaining degrees of freedom have been absorbed by the three vector fields $(W^{\pm}, Z)$, one by each.

2.4 Renormalization Group Equations

Renormalization Group Equations (RGEs) define, for a given theory, how the potential changes with the energy scale. With increasing energy, the couplings in the Lagrangian vary, leading to an "effective" potential. The variations of the couplings are due to first-order corrections to the single-vertex self-interaction of the scalar field. At higher energies these corrections become a significant factor when determining resulting amplitudes. These corrections then provide new fixing conditions for renormalization. This requires the notion of the effective potential in the redefined Lagrangian [9].

The first-order contributions to the effective self-interaction potential are of five types: a scalar coupling of order λ^2, gauge couplings of orders λg^2 and g^4, and Yukawa couplings of order λh^2 and h^4. The RGE for this simple single scalar model, therefore, has the form

$$\frac{d\lambda}{dt} = a\lambda^2 - b\lambda g^2 + cg^4 + d\lambda h^2 - eh^4,\ t = \ln\left(\frac{E}{\sigma}\right) \tag{22}$$

where the minus sign in the last term is due to the Feyman rules for a fermion loop. Ignoring, for the time being, how the gauge and Yukawa terms scale with increased energy, it is easy to see how bounds on the scalar coupling arise. If λ_0 is too large, λ, and thus $V(\Phi)$, will blow up too quickly. Imposing the condition that perturbation theory must be valid all the way up to the Unification Scale (taken here to be $E = 10^{16}\,GeV$), rules out this possibility. Also, vacuum stability is an observed characteristic of our universe. We therefore impose the condition that λ, and thus $V(\Phi)$, cannot go negative which restricts how large the Yukawa coupling term can be.

2.5 Bounds on the Higgs Mass

To place upper and lower bounds on the standard model Higgs mass, we need only write down the RGEs. Simply taking the RGEs as given, and keeping in mind that in the previous representation of the RGE form $g^2 \to 3g^2 + g'^2$, $h \to g_Y$, and $\beta_x = \frac{dx}{dt}$, we have

$$\begin{aligned} \beta_\lambda &= \frac{1}{16\pi^2}(24\lambda^2 - 3\lambda(3g^2 + g'^2) + \frac{3}{4}g^4 + \frac{3}{8}(g^2 + g'^2)^2 \\ &\quad + 12\lambda g_Y^2 - 6g_Y^4), \end{aligned} \tag{23}$$

$$\beta_{g_Y} = \frac{g_Y}{16\pi^2}\left(\frac{9}{2}g_Y^2 - 8g_c^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\right), \tag{24}$$

$$\beta_{g_c} = -\frac{1}{16\pi^2} g_c^3 \left(11 - \frac{4}{3}N\right), \tag{25}$$

$$\beta_g = -\frac{19}{96\pi^2} g^3 \left(\frac{22}{3} - \frac{4}{3}N - \frac{1}{6}N_H\right), \tag{26}$$

$$\beta_{g'} = \frac{41}{96\pi^2} g'^3 \left(\frac{20}{9}N + \frac{1}{6}N_H\right), \tag{27}$$

where g_c is the QCD coupling constant, N is the number of fermion generations, and N_H is the number of Higgs doublets. Boundary conditions for this series of differential equations, at $t = 0$, are given by

$$\left(\frac{dV}{d\phi_i}\right) = 0, \tag{28}$$

$$\left(\frac{d^2V}{d\phi_i d\phi_j}\right) = M_{Hij}^2, \tag{29}$$

where M_H^2 is the mass-squared matrix with nonzero eigenvalue m_H^2 representing the Higgs mass-squared, and where each equation is evaluated at $\phi_{i,j} = 0$ for $i, j \neq 3$ and $\phi_3 = \sigma$ [3, 7].

A simple numerical integration can be performed using Eqs. (23) - (29) to determine the allowed range of λ. Considering that the t quark has $m_t = 175\, GeV$, its Yukawa coupling strength dwarfs those of the other fermions (though, the b quark and τ should be included if the calculation is done rigorously). Therefore, the calculation can be performed, without much loss of precision, taking $g_Y = g_t$. Enumerating two other practical considerations, the Unification Scale, if taken to be $10^{16}\, GeV$ gives a limit of integration of $t \approx 32$, and a perturbative parameter value is taken to be $\lambda < 10$ (since generally

$\lambda < 1$). Taking into account all of these factors, λ can be constrained to $0.30 \leq \lambda \leq 0.54$ which corresponds to a Higgs mass of $135\,GeV \leq m_H \leq 182$ [8].

3 The Two Higgs Doublet Model

A straightforward extension of the standard model Higgs mechanism is to consider the case of two scalar doublets. This is known as the Two Higgs Doublet Model (2HDM). The reason for considering this case is that it is the simplest possible extension of the MSM without considering supersymmetry. The new self-interaction potential now takes the form

$$\begin{aligned} V(\Phi_1,\Phi_2) &= \mu_1^2\Phi_1^\dagger\Phi_1 + \mu_2^2\Phi_2^\dagger\Phi_2 + (\mu_3^2\Phi_1^\dagger\Phi_2 + h.c.) + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ &+ \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+ \{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)]\Phi_1^\dagger\Phi_2 + h.c.\}, \end{aligned} \tag{30}$$

where the two doublets take the form

$$\Phi_1 = \begin{pmatrix} \chi_1^+ \\ \phi_1 + i\chi_1 \end{pmatrix},\ \Phi_2 = \begin{pmatrix} \chi_2^+ \\ \phi_2 + i\chi_2 \end{pmatrix}. \tag{31}$$

The phenomena known as Flavor Changing Neutral Currents (FCNC) arise when we consider what is called Model III, the entire potential with no discrete symmetries imposed. The Lagrangian now has two separate Yukawa coupling terms given as

$$\mathcal{L}_Y = h_{ij}^1\bar{\psi}_i\psi_j\Phi_1 + h_{ij}^2\bar{\psi}_i\psi_j\Phi_2 \tag{32}$$

where i and j represent quark generations. The mass matrix for the Yukawa terms is

$$M_{ij} = h^1_{ij} v_1 + h^2_{ij} v_2, \tag{33}$$

where v_1 and v_2 are the VEVs of ϕ_1 and ϕ_2 respectively. Because there are now two Yukawa terms, diagonalizing M does not necessarily simultaneously diagonalize both h_1 and h_2. This has the effect of introducing interactions of the field with a quark and antiquark of different flavor (e.g. $\bar{d}s\phi$).

FCNC have, in the past, been considered dangerous, as they tend to lead to certain phenomenological problems. Also, since very strict upper bounds have been placed on these FCNC by experiment, they are not a desirable element of our theory. However, models have been suggested to suppress FCNC sufficiently such that Model III is worth consideration [4].

Generally, in what are called Models I and II, one of two symmetries is imposed to eliminate these FCNC. The two symmetries are

$$(I)\ \phi_2 \to -\phi_2,\ d^i_R \to -d^i_R,\ (II)\ \phi_2 \to -\phi_2. \tag{34}$$

The symmetry of Model I has the effect of requiring all of the charge 2/3 quarks to couple to the first doublet and all of the -1/3 quarks to couple to the second. Model II requires that all of the quarks couple to the first doublet. Both have the effect of removing from the potential terms that are linear or cubic in one field, that is $\lambda_6 = \lambda_7 = \mu_3 = 0$ [10, 6]. As Model III is a fairly straightforward extension of the others, we will first examine Models I and II.

3.1 Two Higgs Doublet Models I and II

To analyze the potential, it is convenient to express the two doublets in terms of their real components as

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \ \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}. \tag{35}$$

Performing an SU(2) rotation on the first doublet, we choose the minimum to lie along the real part of the neutral component, ϕ_3. This has the effect of setting $\phi_1 = \phi_2 = \phi_4 = 0$ at the minimum. Taking $\frac{dV}{d\phi_i} = 0$, and assigning $\phi_3 = v_1$ at the minimum (making v_1 the VEV of Φ_1), there are three possible solutions:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \ \langle\Phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{36}$$

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \ \langle\Phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ iv_2 \end{pmatrix}, \tag{37}$$

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \ \langle\Phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} a + ib \\ 0 \end{pmatrix}, \tag{38}$$

where a and b are arbitrary real constants.

We can disregard the third solution (Eq. 38) on the grounds that it breaks the $U(1)$ symmetry. The second solution (Eq. 37) can also be discounted with the argument that it is equivalent to the first solution (Eq. 36) within a redefinition of the fields being relatively imaginary and a change of sign of λ_5. Eq. 36 is then the minimum solution with v_2 as the VEV of Φ_2 and where $v_1^2 + v_2^2 = \sigma^2$ [10].

To determine the scalar masses, we take the second derivative of the potential

$$M^2 = \frac{\partial^2 V}{\partial\phi_i \partial\phi_j} \tag{39}$$

and evaluate it at the minimum. This yields (after a simple redefinition of indices) an 8×8 matrix of 4, 2×2 submatrices. The eigenvalues of the 4 submatrices are then the eigenvalues of the matrix, M, and they represent the scalar masses.

The charged boson mass matrix is

$$-\frac{1}{2}(\lambda_4 + \lambda_5) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}. \tag{40}$$

This submatrix is repeated twice in the mass matrix, once for each charge. This submatrix also has a zero eigenvalue which corresponds to the Goldstone boson absorbed as a degree of freedom of the $W^{\pm}$. The charged scalar mass-squared is

$$m^2_{\chi^{\pm}} = -\frac{1}{2}(\lambda_4 + \lambda_5)(v_1^2 + v_2^2). \tag{41}$$

The psuedoscalar mass matrix is given by

$$-\lambda_5 \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}. \tag{42}$$

This submatrix also has a zero eigenvalue, corresponding to the Goldstone boson absorbed by the Z. The mass-squared of the neutral psuedoscalar is, similarly,

$$m^2_{\chi^0} = -\lambda_5(v_1^2 + v_2^2). \tag{43}$$

The neutral scalar mass matrix is slightly more complicated. It is

$$\begin{pmatrix} \lambda_1 v_1^2 & (\lambda_3+\lambda_4+\lambda_5)v_1v_2 \\ (\lambda_3+\lambda_4+\lambda_5)v_1v_2 & \lambda_2 v_2^2 \end{pmatrix}. \tag{44}$$

The two scalar masses are then

$$m^2_{\phi^0,\eta^0} = \frac{1}{2}\left(\lambda_1 v_1^2 + \lambda_2 v_2^2 \pm \sqrt{(\lambda_1 v_1^2 - \lambda_2 v_2^2)^2 + 4((\lambda_3+\lambda_4+\lambda_5)v_1v_2)^2}\right). \tag{45}$$

These masses lead to some obvious conditions at the minimum [10, 8]. Requiring that the mass-squared of a particle remain positive, we have that

$$\lambda_5 < 0, \tag{46}$$

$$\lambda_4 + \lambda_5 < 0. \tag{47}$$

Also, it is obvious that both λ_1 and λ_2 be positive so that the potential remains bounded in the ϕ_3 and ϕ_7 directions respectively. The final condition arises if we pick an arbitrary direction $\phi_3 = \alpha\phi_7$ and require that the potential be bounded in that direction as well. Minimizing with respect to the arbitrary parameter α and requiring that the resulting potential be positive, we arrive at the condition

$$\lambda_1\lambda_2 > (\lambda_3+\lambda_4+\lambda_5)^2. \tag{48}$$

It remains, then, to define the RGEs for this theory in order to determine how the various λ_i scale. The RGEs for all models of the 2HDM are as follows [6]:

$$
\begin{aligned}
\beta_{\lambda_1} &= \frac{1}{16\pi^2}\{12\lambda_1^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + 24\lambda_6^2 + \frac{3}{2}g^4 \\
&\quad + \frac{3}{4}(g^2 - g'^2)^2 - 12h_{1t}^4 - 12h_{1b}^4 - 4\lambda_1\gamma_1\}, \qquad (49) \\
\beta_{\lambda_2} &= \frac{1}{16\pi^2}\{12\lambda_2^2 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + 24\lambda_7^2 + \frac{3}{2}g^4 \\
&\quad + \frac{3}{4}(g^2 - g'^2)^2 - 12h_{2t}^4 - 12h_{2b}^4 - 4\lambda_2\gamma_2\}, \qquad (50) \\
\beta_{\lambda_3} &= \frac{1}{16\pi^2}\{(2\lambda_1 + 2\lambda_2)(3\lambda_3 + \lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 \\
&\quad + 4\lambda_6^2 + 4\lambda_7^2 + 16\lambda_6\lambda_7 + \frac{3}{2}g^4 + \frac{3}{4}(g^2 - g'^2)^2 \\
&\quad - 12h_{1t}^2h_{1b}^2 - 12h_{2t}^2h_{2b}^2 - 2\lambda_3(\gamma_1 + \gamma_2)\}, \qquad (51) \\
\beta_{\lambda_4} &= \frac{1}{16\pi^2}\{\lambda_4(2\lambda_1 + 2\lambda_2 + 8\lambda_3 + 4\lambda_4) + 8\lambda_5^2 + 10\lambda_6^2 + 10\lambda_7^2 \\
&\quad + 4\lambda_6\lambda_7 + 3g^2g'^2 + 12h_{1t}^2h_{1b}^2 + 12h_{2t}^2h_{2b}^2 - 2\lambda_4(\gamma_1 + \gamma_2)\}, \qquad (52) \\
\beta_{\lambda_5} &= \frac{1}{16\pi^2}\{\lambda_5(2\lambda_1 + 2\lambda_2 + 8\lambda_3 + 12\lambda_4) + 8\lambda_5^2 + 10\lambda_6^2 + 10\lambda_7^2 \\
&\quad + 4\lambda_6\lambda_7 - 2\lambda_5(\gamma_1 + \gamma_2)\}, \qquad (53) \\
\beta_{\lambda_6} &= \frac{1}{16\pi^2}\{\lambda_6(12\lambda_1 + 6\lambda_3 + 8\lambda_4 + 10\lambda_5) \\
&\quad + \lambda_7(6\lambda_3 + 4\lambda_4 + 2\lambda_5) - \lambda_6(3\gamma_1 + \gamma_2)\}, \qquad (54) \\
\beta_{\lambda_7} &= \frac{1}{16\pi^2}\{\lambda_7(12\lambda_2 + 6\lambda_3 + 8\lambda_4 + 10\lambda_5) \\
&\quad + \lambda_6(6\lambda_3 + 4\lambda_4 + 2\lambda_5) - \lambda_6(\gamma_1 + 3\gamma_2)\} \qquad (55)
\end{aligned}
$$

where

$$
\begin{aligned}
\gamma_1 &= 9g^2 + 3g'^2 - 12h_{1t}^2 - 12h_{1b}^2, \qquad (56) \\
\gamma_2 &= 9g^2 + 3g'^2 - 12h_{2t}^2 - 12h_{2b}^2. \qquad (57)
\end{aligned}
$$

The RGEs for the Yukawa couplings and gauge couplings are

$$\beta_{h_{it}} = \frac{h_{it}}{16\pi^2}\{\frac{9}{2}h_{it}^2 + \frac{1}{2}h_{ib}^2 - 8g_c^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2\}, \tag{58}$$

$$\beta_{h_{ib}} = \frac{h_{ib}}{16\pi^2}\{\frac{9}{2}h_{ib}^2 + \frac{1}{2}h_{it}^2 - 8g_c^2 - \frac{9}{4}g^2 - \frac{15}{12}g'^2\}, \tag{59}$$

$$\beta_g = -\frac{3g^3}{16\pi^2}, \tag{60}$$

$$\beta_{g'} = \frac{7g^3}{16\pi^2}, \tag{61}$$

$$\beta_{g_c} = -\frac{7g^3}{16\pi^2}. \tag{62}$$

$$\tag{63}$$

In the two field notation, the Yukawa couplings are given by

$$\frac{\sqrt{2}m_t}{\sigma} = h_{1t}v_1 + h_{2t}v_2, \; \frac{\sqrt{2}m_b}{\sigma} = h_{1b}v_1 + h_{2b}v_2 \tag{64}$$

In both Models I and II, $\mu_3^2 = \lambda_6 = \lambda_7 = 0$. In Model I, where the charge 2/3 quarks couple to the first field, and the charge -1/3 quarks couple to the second, we have that $h_{1b} = h_{2t} = 0$. Likewise, in Model II, where all quarks couple to the first field, we have that $h_{2t} = h_{2b} = 0$.

Considering that the t quark dominates the quark mass spectrum, it is not even necessary to include the b quark in the calculation of scalar mass bounds. While it is less precise, this aproximation allows us to consider both Model I and Model II simultaneously, as the bounds that arise are equivalent given $m_t >> m_b$.

3.2 Computational Results for Models I and II

Placing bounds on the Higgs scalars is now only a matter of numerical computation. A convenient starting point is a set value for m_{χ^0} and $m_{\chi^\pm}$ and a value for $\tan\beta = \frac{v_1}{v_2}$. The charged and psuedoscalar masses fix λ_4 and λ_5, and $\tan\beta$ fixes the Yukawa couplings. Then, for each triplet $(\lambda_1, \lambda_2, \lambda_3)$, the given values of λ_i are integrated up to the Unification Scale, checking at each step to make sure that the conditions of Eqs. 46 - 48 and the positivity of λ_1 and λ_2 are not violated. A successful parameter set implies a point in the allowed region of the four-dimensional scalar mass space (four, because the charged scalars are alway equal in mass). The most convenient way to examine this mass space is to look at slices of the allowed region in the m_η vs. m_ϕ plane for varying values of m_{χ^0}, $m_{\chi^\pm}$, and $\tan\beta$.

The results of this computation are shown in Figures 2 - 4. In each graph, the allowed regions of m_η are generally bounded by Eq. 48. The allowed regions of m_ϕ have an upper bound due to the fact that above that region some λ_i's become too large (nonperturbative). Lower bounds are imposed by requiring the positivity of λ_1 and λ_2. m_ϕ, for all parameter configurations, is constrained between 140 and 200 GeV, the Standard Model range for a single Higgs scalar. There are also no parameter configurations that allow $m_\eta > 80\,GeV$.

Allowed values of m_η and m_ϕ for different values of $\tan\beta$, given that m_{χ^0} and $m_{\chi^\pm}$ are fixed at 100 GeV, are displayed in Figure 2. For values of

$\tan\beta < 2.0$, the region disappears. For large values of $\tan\beta$ it is obvious that m_η becomes very small, and m_ϕ attains the full Standard Model range (not exactly, though, as m_{χ^0}, $m_{\chi^\pm} \neq 0$). This is not surprising, as $\tan\beta \to \infty$ represents the single doublet model.

Allowed values of m_η and m_ϕ for different values of $m_{\chi^\pm}$, with $m_{\chi^0} = 100\,GeV$ and $\tan\beta = 3$, are displayed in Figure 3. For values of $m_{\chi^\pm} > 150\,GeV$, there is no allowed mass region. Varying the charged scalar mass has much the same effect as varying $\tan\beta$, in that larger values of $m_{\chi^\pm}$ allow smaller ranges of m_η but larger ranges of m_ϕ.

Allowed values of m_η and m_ϕ for different values of m_{χ^0}, with $m_{\chi^\pm} = 100\,GeV$ and $\tan\beta = 3$, are displayed in Figure 4. For values of $m_{\chi^0} > 140\,GeV$, there is no allowed mass region. The allowed region is very insensitive to the psuedoscalar mass, varying only slightly for larger values of m_{χ^0}.

It should be noted here that, with this information Model II can be excluded experimentally, if it is assumed to be valid up to the Unification Scale. If all quarks couple to one Higgs field, then the branching ratio of $B \to X_s\gamma$ implies a charged scalar mass which is greater than 165 GeV [2]. This is impossible with vacuum stability constraints imposed on the model. Unfortunately, no such further constraints can be placed on Model I [1].

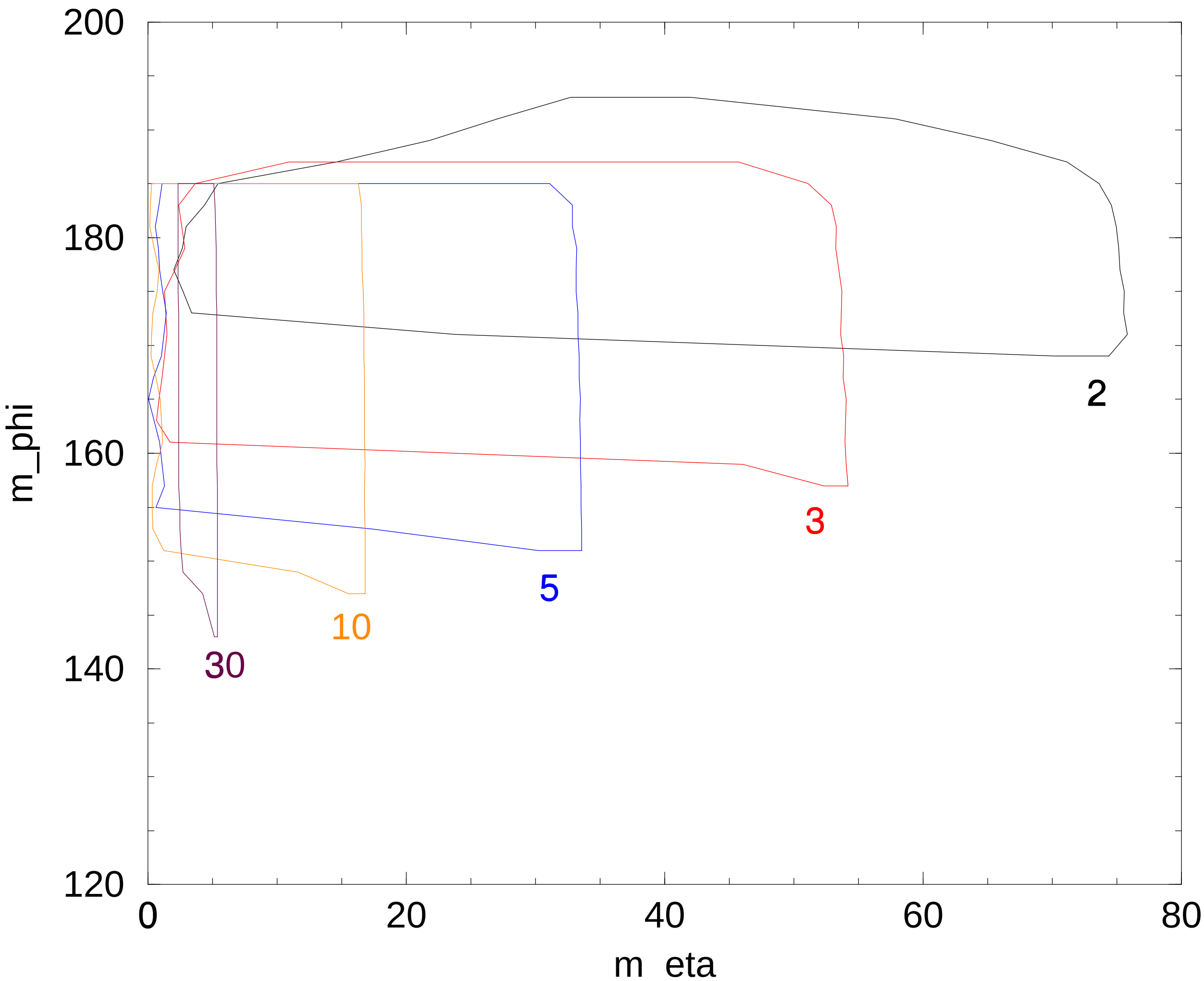

Figure 2: Allowed regions of m_η and m_ϕ (Gev) for varying values of $\tan\beta$ with $m_{\chi^0} = m_{\chi^\pm} = 100\,GeV$. Values of $\tan\beta$ shown here are 2 (black), 3 (red), 5 (blue), 10 (yellow), and 30 (purple).

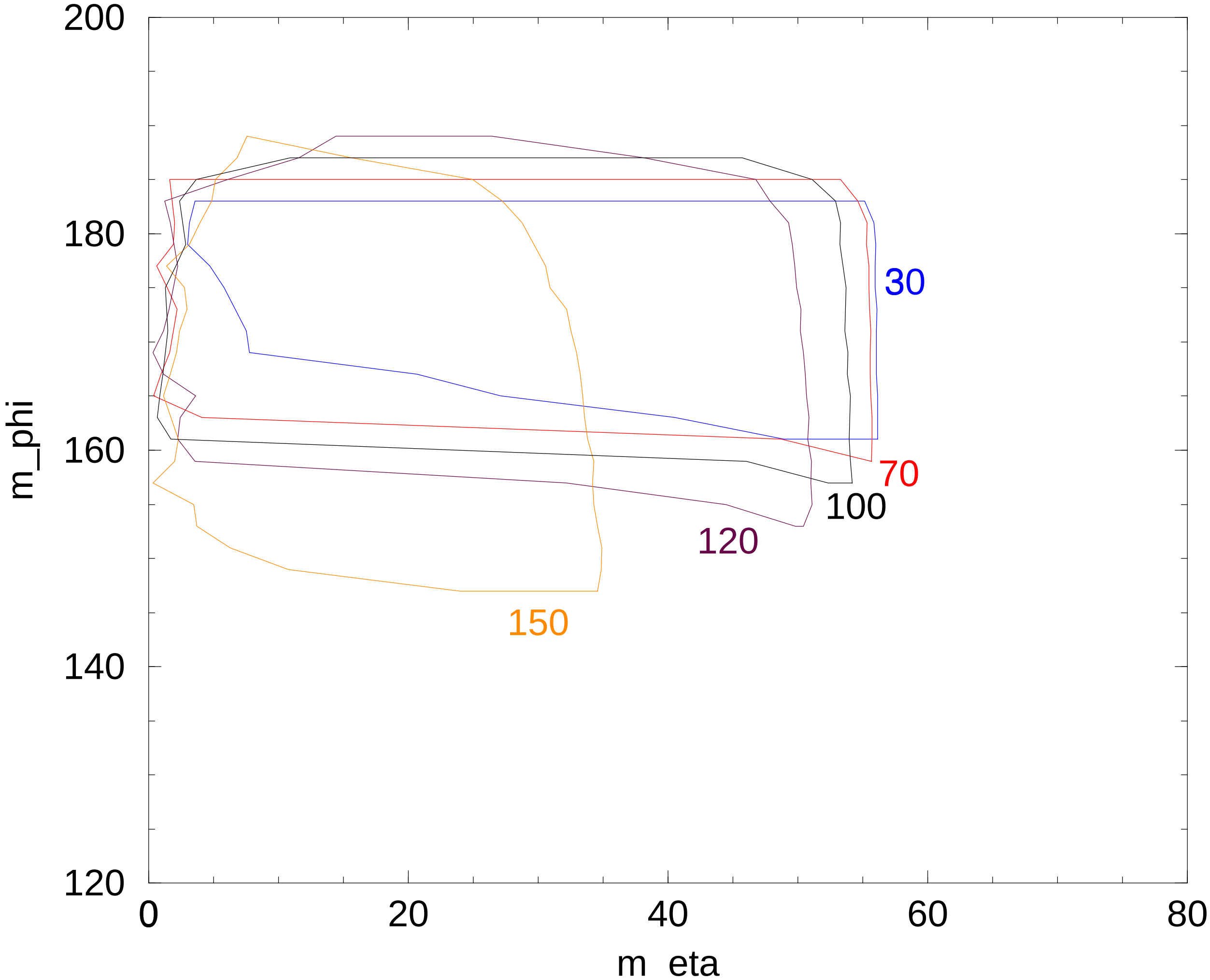

Figure 3: Allowed regions of m_η and m_ϕ (Gev) for varying values of $m_{\chi^\pm}$ with $m_{\chi^0} = 100\,GeV$ and $\tan\beta = 3$. Values of $m_{\chi^\pm}$ shown here are 30 (blue), 70 (red), 100 (black), 120 (purple), and 150 (yellow).

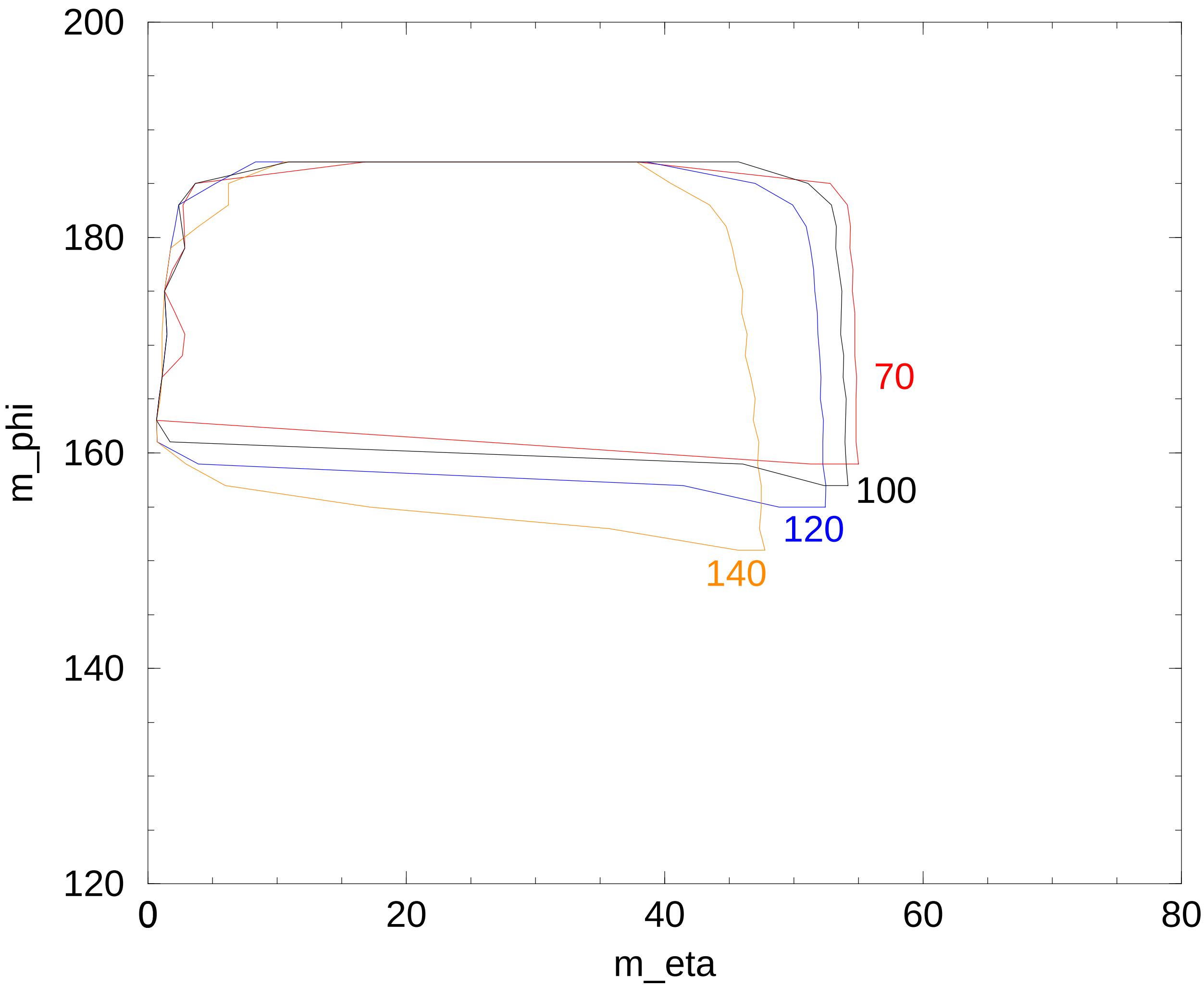

Figure 4: Allowed regions of m_η and m_ϕ (Gev) for varying values of m_{χ^0} with $m_{\chi^\pm} = 100\, GeV$ and $\tan\beta = 3$. Values of m_{χ^0} shown here are 70 (red), 100 (black), 120 (purple), and 140 (yellow).

4 Two Higgs Doublet Model III

Model II of the 2HDM allows the all of the quarks to couple to both fields. This allows the potential to have $\mu_3^2, \lambda_6, \lambda_7 \neq 0$. This also allows all of the Yukawa couplings to be nonzero. Therefore, there are significantly more parameters to vary. This tends to have the effect of expanding the allowed regions of mass space.

In Figures 5 - 7, λ_6 and λ_7 are varied. The behavior of the allowed regions is, not surprisingly, quite similar to Models I and II. The main difference is that the general range of m_ϕ is expanded to be between 140 and 200 GeV, and the range of m_η is extended to less than 100 GeV.

In Figure 5, the allowed region for the neutral scalars is shown as $\tan\beta$ varies. The main feature of the variation is that the allowed region expands and shifts to lower m_ϕ. There are no allowed regions for $\tan\beta < 1$.

In Figure 6, the neutral scalar slice of allowed mass space is given for varying values of the charged scalar mass. The region exhibits the same evolution with charged scalar mass as the other models. As before, the charged mass is constrained to lie below 150 GeV.

In Figure 7, the neutral scalar slice of allowed mass space is given for varying values of the psuedoscalar mass. The psuedoscalar is, as in the other models, constrained to be less than 150 GeV. Likewise, the region behaves as it does in the other models, not a surprising result since we have simply allowed the parameters more freedom, without having changed the coupling strengths.

The data presented in Figure 8 is preliminary. Though there is not much change expected, the regions could expand slightly as the program continues to generate more data. Allowing the t quark to couple to the "down" Higgs field (conventional 2HDM nomenclature), and allowing the b quark to couple to the "up" Higgs field, it is obvious that the regions in mass space are greatly expanded. The one notable feature of this data is that there is an allowed region for $\tan\beta = 1$, implying that the two fields can have equal VEVs. Because of the cross-coupling of the quarks, the "up" field need not dominate to make the t quark significantly heavier. In fact, this specific region is interesting because it gives an allowed region where m_η, the mass of the lightest scalar, is greater than 100 GeV, a mass range which has yet to be experimentally constrained. Thus Model III remains a viable theory even given current experimental constraints on the lightest scalar.

5 Conclusion

As of now, this is the extent of my results. The process of determining the allowed regions becomes computationally much more complex and time-consuming with additional parameters. Due to a lack of powerful computing resources, the process of generating data has been slow. Still to be determined are the mass space bounds imposed on m_{χ^0} and $m_{\chi^\pm}$ when the Yukawa cross-coupling stregths are allowed to vary, and finally the considerations of the full model, including $\mu_3^2 \neq 0$.

It should be noted that a large amount of computational time was devoted to reproducing the results of Nie and Sher (1999) [8]. My results and the results of that paper differ significantly, most notably in the lower bounds on m_η. Further computations (varying other parameters) have been performed on the assumption that my data for Models I and II are correct. It should also be noted, however, that though the regions in m_η and m_ϕ space are expanded, the data do not suggest a modification to the upper bounds previously places on m_{χ^0} and $m_{\chi^\pm}$ and the lower bound previously placed on $\tan\beta$.

As for the data currently being computed, it is obvious that the regions in m_η and m_ϕ space will be significantly expanded in Model III as compared to Models I and II. The upper bounds on m_{χ^0} and $m_{\chi^\pm}$ will most likely be higher, and it is already evident that the lower bound on $\tan\beta$ is smaller. With the addition of $\mu_3^2 \neq 0$, the regions of mass space will be expanded upward, possibly with no upper bound. Unfortunately, there are no experimental constraints on the charged scalar mass that would help to rule out this model (under the assumption that it is valid up to the Unification Scale).

For future computations, it will be necessary to find ways to speed up the runtime of the program. Optimization, apart from normal computational rearrangements and the like, might be achieved by looking at ways to find the allowed parameter space by random starting places and probing (Monte Carlo) or other such methods. Also, more advantage could be taken of the fact that it is much faster to find points that are not in the allowed region than those that are. The C++ code used is given as an appendix.

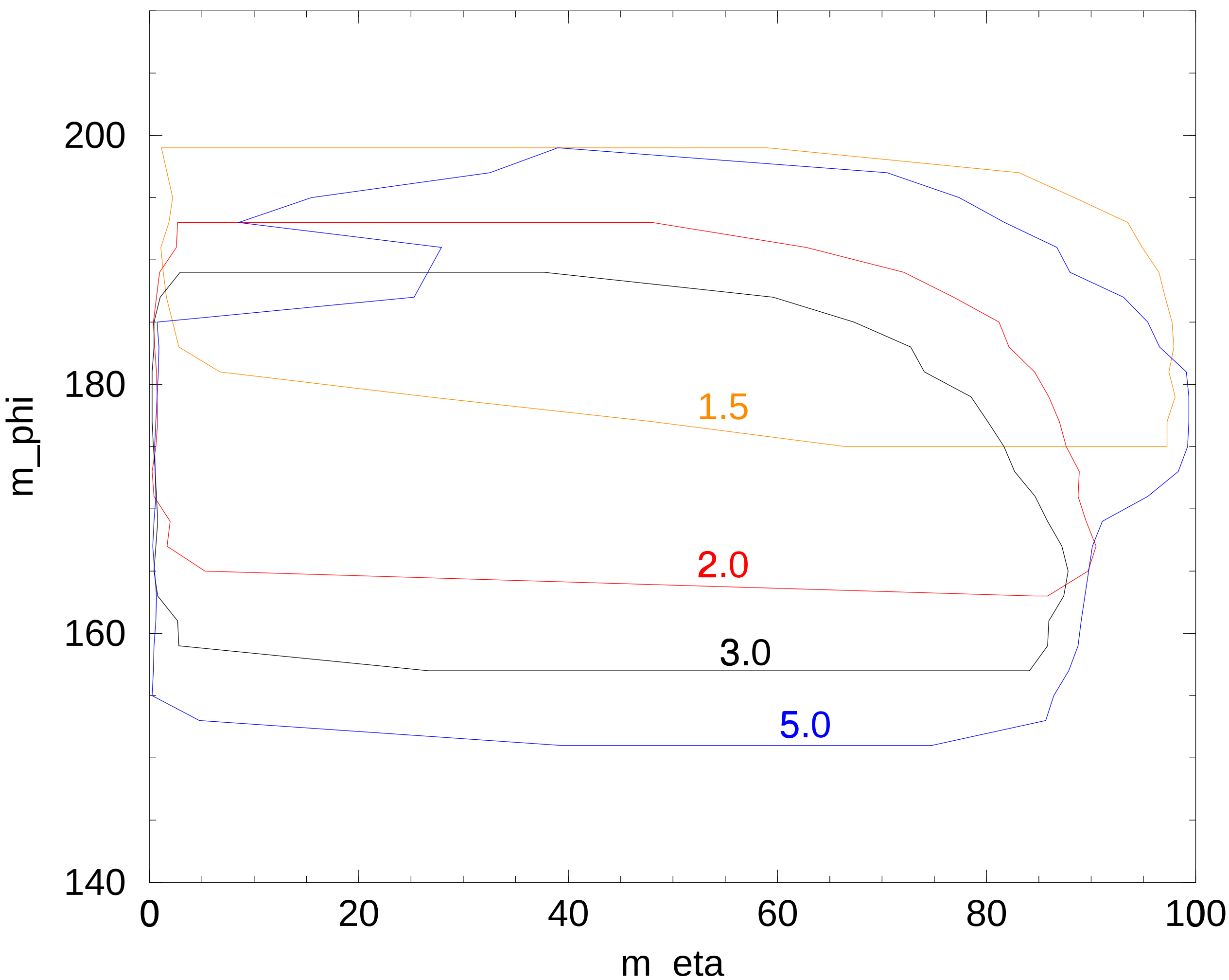

Figure 5: Allowed regions of m_η and m_ϕ (Gev) for varying values of $\tan\beta$ with $m_{\chi^0} = m_{\chi^\pm} = 100\,GeV$, allowing that $\lambda 6$, $\lambda 7 \neq 0$. Values of $\tan\beta$ shown here are 1.5 (yellow), 2.0 (red), 3.0 (black), and 5.0 (blue).

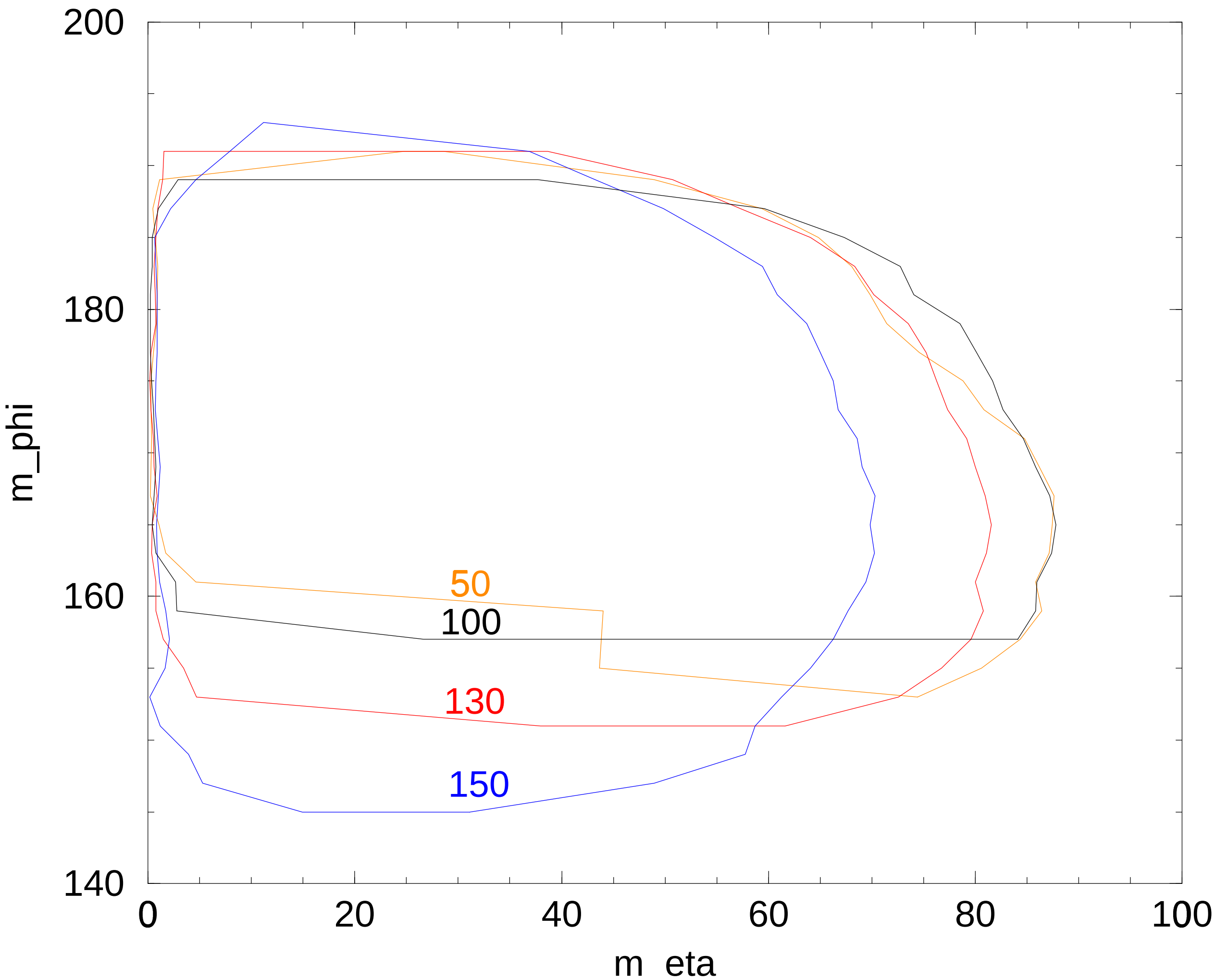

Figure 6: Allowed regions of m_η and m_ϕ (Gev) for varying values of $m_{\chi^\pm}$ with $m_{\chi^0} = 100\,GeV$ and $\tan\beta = 3$, allowing that $\lambda 6$, $\lambda 7 \neq 0$. Values of $m_{\chi^\pm}$ shown here are 50 (yellow), 100 (black), 130 (red), and 150 (blue).

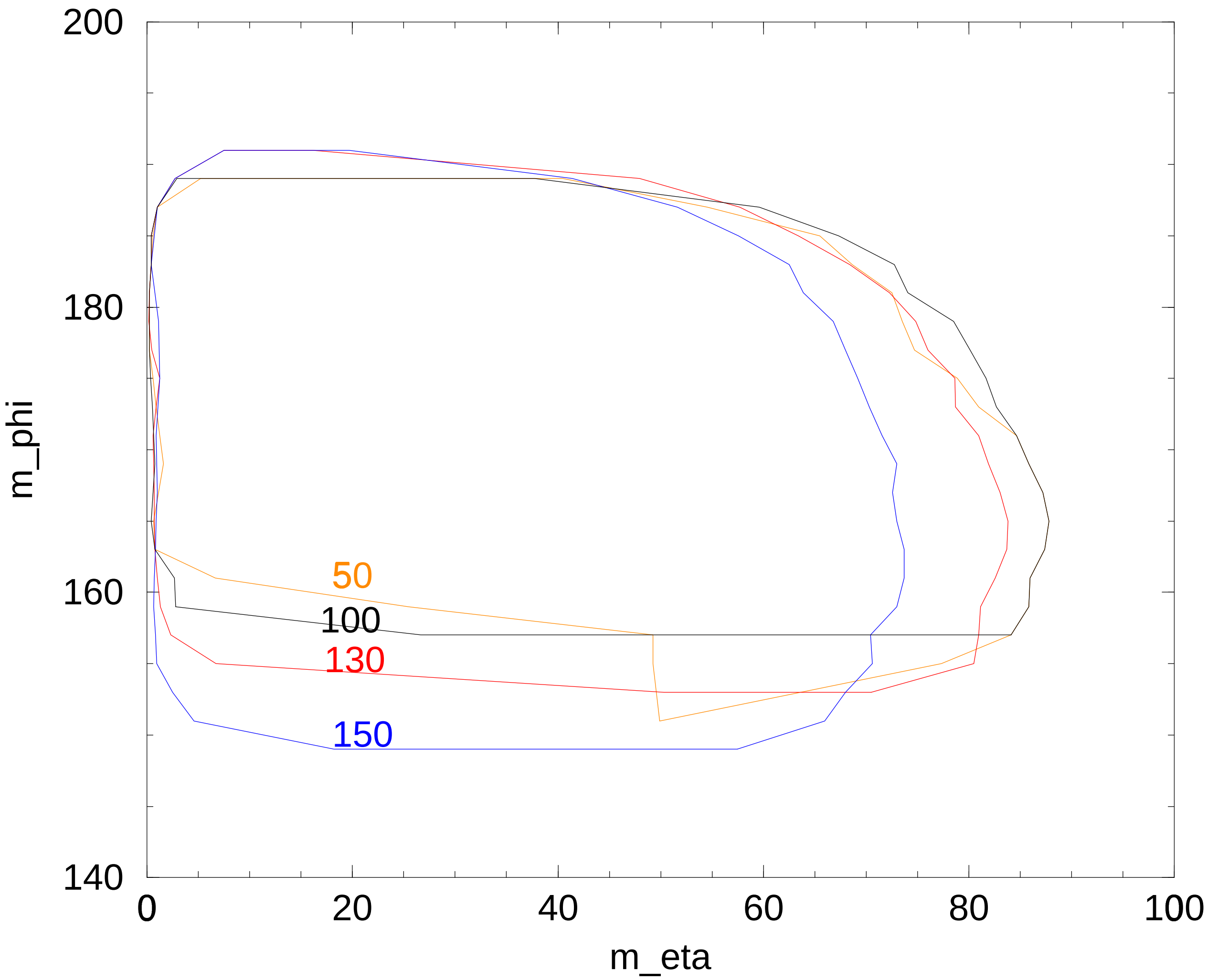

Figure 7: Allowed regions of m_η and m_ϕ (Gev) for varying values of m_{χ^0} with $m_{\chi^\pm} = 100\,GeV$ and $\tan\beta = 3$, allowing that $\lambda 6$, $\lambda 7 \neq 0$. Values of m_{χ^0} shown here are 50 (yellow), 100 (black), 130 (red), and 150 (blue).

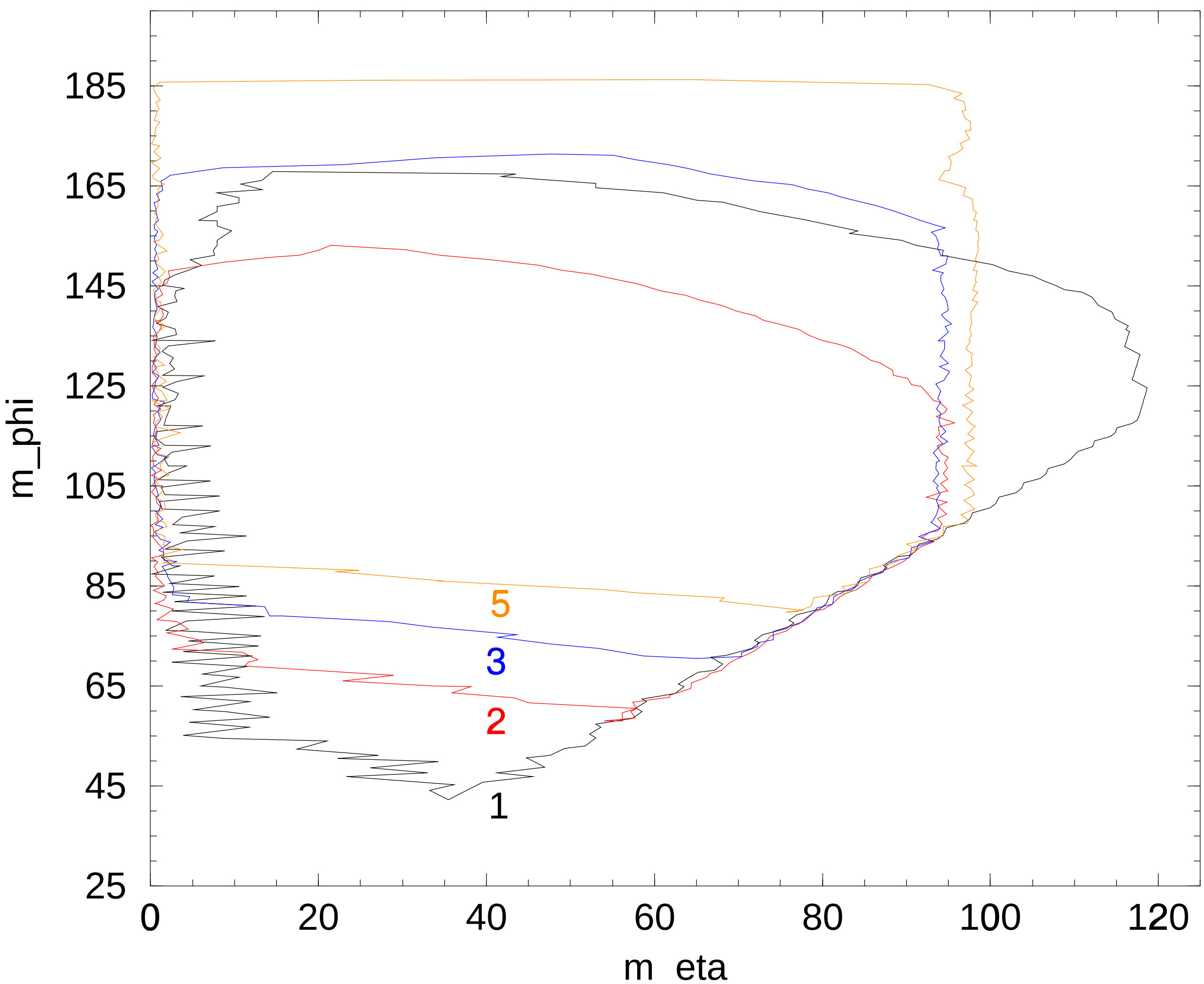

Figure 8: Allowed regions of m_η and m_ϕ (Gev) for varying values of $\tan\beta$ with $m_{\chi^0} = m_{\chi^\pm} = 100\,GeV$, allowing that $\lambda6$, $\lambda7 \neq 0$, and h_{2t}, $h_{1b} \neq 0$. Values of $\tan\beta$ shown here are 1 (black), 2 (red), 3 (blue), and 5 (yellow).

References

[1] T. M. Aliev and E. O. Iltan. b $\rightarrow$ s γ decay in the two higgs doublet model with flavor changing neutral currents. *hep-ph/9803272*.

[2] F. M. Borzumati and C. Greub. Lessons from $\overline{B} \rightarrow x_s\gamma$ in two higgs doublet models. *hep-ph/9810240*.

[3] E. Eichten Cheng, T.P. and Ling-Fong Li. Higgs phenomena in asymptotically free gauge theories. *Phys. Rev. D*, 9(8):2259–73, April 1974.

[4] T.P. Cheng and M. Sher. Mass-matrix ansatz and flavor nonconservation in models with multiple higgs doublets. *Phys. Rev. D*, 35(11), June 1987.

[5] W.N. Cottingham and D.A. Greenwood. *An Introduction to the Standard Model of Particle Physics*, pages 98–111. Cambridge University Press, 1998.

[6] H. Haber. *Perspectives on Higgs Physics*, pages 44–128. World Scientific, 1993.

[7] Akira Kakuto Inoue, Kenzo and Yashimasa Nakano. Perturbation constraint on particle masses in the weinberg-salam model with two massless higgs doublets. *Progress of Theoretical Physics*, 63(1):234–45, January 1980.

[8] Shuquan Nie and Marc Sher. Vacuum stability bounds in the two-higgs doublet model. *Phys. Lett. B*, (449):89–92, March 1999.

[9] M. Peskin and D. Schroeder. *An Introduction to Quantum Field Theory*, pages 347–88,689–91. 1995.

[10] M. Sher. Electroweak higgs potentials and vacuum stability. *Physics Reports*, 179(5 and 6):273–418, August 1989.

Acknowledgments

I would like to thank my advisor Dr. Marc Sher for all of his help. Thanks also to Dr. Chris Carone for letting me audit Field Theory. I would also like to thank the other members of my committee Dr. Bill Bynum and Dr. Gina Hoatson. In addition, thanks go to Ian Steplowski, Lisa Kaufman, Ruth Van de Water, and Tony Padula for LaTeX help.

A C++ code for determining mass bounds in the 2HDM

The C++ code presented here is for determining bounds on the allowed regions in mass space for m_η and m_ϕ given initial values for m_{χ^0}, $m_{\chi^\pm}$ and $\tan\beta$. The code can be easily generalized for all Models of the 2HDM, as models I and II can be considered special cases of the full Model III.

```
#include <fstream.h>
#include <math.h>
#include <stdio.h>
#include <string.h>

// Global Initial Values for Program

// Scaling Variables
double t_step_size = 00.05;
double t     = 00.00;
double t_final     = 32.00;

double m_cc = 100.0;
double m_nc = 100.0;
double tanbeta = 010.0;
char *filename = "htb10.0.dat";
ofstream outfile;
ofstream outfile2;
ofstream outfile3;
ifstream infile;

// RGE Variables
double lambda_1_0 = 0.00;
double lambda_1_f = 1.01;
double lambda_1_step = 0.01;
double lambda_2_0 = 0.00;
double lambda_2_f = 0.81;
double lambda_2_step = 0.02;
```

```
double lambda_3_0 = -0.50;
double lambda_3_f = 0.71;
double lambda_3_step = 0.02;
double lambda_6_0 = -0.40;
double lambda_6_f = 0.41;
double lambda_6_step = 0.03;
double lambda_7_0 = -0.40;
double lambda_7_f = 0.41;
double lambda_7_step = 0.03;
double mu_3_squared_0 = 0.00;
double mu_3_squared_f = 0.01;
double mu_3_squared_step = 0.01;
double h_2_t_0 = 0.00;
double h_2_t_f = 0.51;
double h_2_t_step = 0.10;
double h_1_b_0 = 0.00;
double h_1_b_f = 0.51;
double h_1_b_step = 0.10;
double g_c_0       = sqrt(M_PI*4*0.113);
double g_prime_0  = sqrt(M_PI*4/(129*0.77));
double g_0      = sqrt(M_PI*4/(129*0.23));

double M_SQRT3 = 1.732050808;

// Stability Condition Variables
double sigma   = 247.0;
double m_t     = 175.0;
double m_b     = 004.7;

// Globals for all functions to use

double lambda_1;
double lambda_2;
double lambda_3;
double lambda_4;
double lambda_5;
double lambda_6;
```

```
double lambda_7;
double mu_3_squared;
double h_1_t;
double h_2_t;
double h_1_b;
double h_2_b;
double g;
double g_prime;
double g_c;
double v_1;
double v_2;

// Function Prototypes
double d_lambda_1();
double d_lambda_2();
double d_lambda_3();
double d_lambda_4();
double d_lambda_5();
double d_lambda_6();
double d_lambda_7();
double d_g();
double d_g_prime();
double d_g_c();
double d_h_1_t();
double d_h_2_t();
double d_h_1_b();
double d_h_2_b();
double get_part_1();
double get_part_2();
double get_part_a();
double get_part_b();
double get_part_c();
double getV(double x);
int potential_check();
int check_conditions();

// Main processing function
```

```
int main (void) {

int success = 1;
int writeout, firsti;

double delta_lambda_1;
double delta_lambda_2;
double delta_lambda_3;
double delta_lambda_4;
double delta_lambda_5;
double delta_lambda_6;
double delta_lambda_7;
double delta_g;
double delta_g_prime;
double delta_g_c;
double delta_h_1_t;
double delta_h_2_t;
double delta_h_1_b;
double delta_h_2_b;
double part_1;
double part_2;
double mass_3;
double mass_4;
double eta, phi;
double a,b,c,d,e,temp;
int i;

double lambda_1_loop;
double lambda_2_loop;
double lambda_3_loop;
double lambda_4_loop;
double lambda_5_loop;
double lambda_6_loop;
double lambda_7_loop;
double mu_3_squared_loop;
double h_2_t_loop;
```

```
double h_1_b_loop;

double lambda_7_success;
double lambda_6_success;
double lambda_3_success;
double lambda_2_success;
double lambda_1_success;
double h_2_t_success;
double h_1_b_success;

double masses[1600];

char filenameinfo[30];
char filenameerror[30];
strcpy(filenameinfo,filename);
strcat(filenameinfo,".nfo");
strcpy(filenameerror,filename);
strcat(filenameerror,".err");

outfile3.open(filenameerror);

infile.open(filename);
for (i=0; i<1600;i+=4) {
masses[i] = 200.0;
masses[i+1] = 400.0;
masses[i+2] = 0.0;
masses[i+3] = 0.0;
}
/* if (infile) {
while (!infile.eof()) {
infile >> eta;
infile >> phi;
infile.ignore(256,'\n');
i = int(floor(phi))*4;
if (eta < masses[i]) {
masses[i] = eta;
masses[i+1] = phi;
```

```
}
else if (eta > masses[i+2]) {
masses[i+2] = eta;
masses[i+3] = phi;
}
}
infile.close();
}
*/
v_2 = 1/sqrt((tanbeta*tanbeta)+1);
v_1 = tanbeta*v_2;

m_cc = m_cc/sigma;
m_cc = m_cc*m_cc;
m_nc = m_nc/sigma;
m_nc = m_nc*m_nc;

// begin big for
lambda_1_success = 0.0;
for (lambda_1_loop = lambda_1_0;
lambda_1_loop < lambda_1_f;
lambda_1_loop += lambda_1_step) {
lambda_2_success = 0.0;
for (lambda_2_loop = lambda_2_0;
lambda_2_loop < lambda_2_f;
lambda_2_loop += lambda_2_step) {
cout << "\nlambda_1 = " << lambda_1_loop;
cout << " lambda_2 = " << lambda_2_loop << endl;
lambda_3_success = 0.0;
for (lambda_3_loop = lambda_3_0;
lambda_3_loop < lambda_3_f;
lambda_3_loop += lambda_3_step) {
fprintf(stderr,"-");
lambda_6_success = 0.0;
for (lambda_6_loop = lambda_6_0;
lambda_6_loop < lambda_6_f;
lambda_6_loop += lambda_6_step) {
```

```
h_2_t_success = 0.0;
for (h_2_t_loop = h_2_t_0;
h_2_t_loop < h_2_t_f;
h_2_t_loop += h_2_t_step) {
h_1_b_success = 0.0;
for (h_1_b_loop = h_1_b_0;
h_1_b_loop < h_1_b_f;
h_1_b_loop += h_1_b_step) {
lambda_7_success = 0.0;
for (lambda_7_loop = lambda_7_0;
lambda_7_loop < lambda_7_f;
lambda_7_loop += lambda_7_step) {
for (mu_3_squared_loop = mu_3_squared_0;
mu_3_squared_loop < mu_3_squared_f;
mu_3_squared_loop += mu_3_squared_step) {

lambda_1 = lambda_1_loop;
lambda_2 = lambda_2_loop;
lambda_3 = lambda_3_loop;
lambda_6 = lambda_6_loop;
lambda_7 = lambda_7_loop;
mu_3_squared = mu_3_squared_loop;

a =  - ((v_2/v_1)*mu_3_squared)
- (lambda_6*v_1*v_2/4.0)
- (lambda_7*v_2*v_2*v_2/v_1/4.0);
b =  - ((v_1/v_2)*mu_3_squared)
- (lambda_6*v_1*v_1*v_1/v_2/4.0)
- (lambda_7*v_2*v_1/4.0);
c =  mu_3_squared
+ (lambda_6*v_1*v_1/4.0)
+ (lambda_7*v_2*v_2/4.0);

lambda_5 = -((c*c)-((a-m_nc)*(b-m_nc)));
lambda_5 /= ((v_1*v_1*(b-m_nc))+(2*c*v_1*v_2)+(v_2*v_2*(a-m_nc)));

temp = -((c*c)-((a-m_cc)*(b-m_cc)));
```

```
temp /= ((v_1*v_1*(b-m_cc))+(2*c*v_1*v_2)+(v_2*v_2*(a-m_cc)));

lambda_4 = (2.0*temp) - lambda_5;

lambda_4_loop = lambda_4;
lambda_5_loop = lambda_5;

h_2_t = h_2_t_loop;
h_1_b = h_1_b_loop;
h_1_t = ((m_t*M_SQRT2/sigma) - (v_2*h_2_t))/v_1;
h_2_b = ((m_b*M_SQRT2/sigma) - (v_1*h_1_b))/v_2;
g       = g_0;
g_prime  = g_prime_0;
g_c      = g_c_0;
success = 1;
t = 0.00;

part_1 = get_part_1();
part_2 = get_part_2();
mass_3 = part_1 + part_2;
mass_4 = part_1 - part_2;

// d = -(lambda_3+lambda_4+lambda_5)*((v_2*v_2)/(v_1*v_1));
// e = -(lambda_3+lambda_4+lambda_5)*((v_1*v_1)/(v_2*v_2));

if ( (mass_3 < 0) || (mass_4 < 0) ) {
success = 0;
}
// else if ( (lambda_1 < d) || (lambda_2 < e) ) {
// success = 0;
// }
// else if (check_conditions()) {
// success = 0;
// }

while (t <= t_final && success) {
```

```
// cout << "Current t = " << t << endl;

delta_lambda_1 = t_step_size*d_lambda_1();
delta_lambda_2 = t_step_size*d_lambda_2();
delta_lambda_3 = t_step_size*d_lambda_3();
delta_lambda_4 = t_step_size*d_lambda_4();
delta_lambda_5 = t_step_size*d_lambda_5();
delta_lambda_6 = t_step_size*d_lambda_6();
delta_lambda_7 = t_step_size*d_lambda_7();
delta_g = t_step_size*d_g();
delta_g_prime = t_step_size*d_g_prime();
delta_g_c = t_step_size*d_g_c();
delta_h_1_t = t_step_size*d_h_1_t();
delta_h_2_t = t_step_size*d_h_2_t();
delta_h_1_b = t_step_size*d_h_1_b();
delta_h_2_b = t_step_size*d_h_2_b();

lambda_1 += delta_lambda_1;
lambda_2 += delta_lambda_2;
lambda_3 += delta_lambda_3;
lambda_4 += delta_lambda_4;
lambda_5 += delta_lambda_5;
lambda_6 += delta_lambda_6;
lambda_7 += delta_lambda_7;
g += delta_g;
g_prime += delta_g_prime;
g_c += delta_g_c;
h_1_t += delta_h_1_t;
h_2_t += delta_h_2_t;
h_1_b += delta_h_1_b;
h_2_b += delta_h_2_b;

/* cout << "lambda_1 = " << lambda_1 << " "
     << "lambda_2 = " << lambda_2 << " "
     << "lambda_3 = " << lambda_3 << " "
     << "lambda_4 = " << lambda_4 << " "
     << "lambda_5 = " << lambda_5 << " "
```

```
     << "lambda_6 = " << lambda_6 << " "
     << "lambda_7 = " << lambda_7 << " "
     << "v_1 = " << v_1 << " "
     << "v_2 = " << v_2 << " "
     << "h_1_t = " << h_1_t << " "
     << "h_2_t = " << h_2_t << " "
     << "h_1_b = " << h_1_b << " "
     << "h_2_b = " << h_2_b << " " << endl;
*/

if (check_conditions()) {
success = 0;
}

t += t_step_size;

}

if (success) {

eta = sqrt(mass_4)*sigma;
phi = sqrt(mass_3)*sigma;

i = 4*int(floor(phi));

writeout = 0;
if (eta < masses[i]) {
masses[i] = eta;
masses[i+1] = phi;
writeout = 1;
}
else if (eta > masses[i+2]) {
masses[i+2] = eta;
masses[i+3] = phi;
writeout = 1;
}
```

```
firsti = 0;
if (writeout) {
outfile.open(filename);
for (i=0;i<1600;i+=4) {
if (masses[i] != 200.0) {
if (!firsti) {
firsti = i;
}
outfile << masses[i] << " "
<< masses[i+1] << endl;
}
}
for (i=1596;i>-1;i-=4) {
if (masses[i+2] != 0.0) {
outfile << masses[i+2] << " "
<< masses[i+3] << endl;
}
}
outfile << masses[firsti] << " "
<< masses[firsti+1] << endl;
outfile.close();
outfile2.open(filenameinfo);
outfile2 << "last lambda_1 = " << lambda_1_loop
 << endl
 << "last lambda_2 = " << lambda_2_loop
 << endl
 << "last lambda_3 = " << lambda_3_loop
 << endl
 << "last lambda_6 = " << lambda_6_loop
 << endl
 << "last lambda_7 = " << lambda_7_loop
 << endl
 << "last h_1_b = " << h_1_b_loop
 << endl
 << "last h_2_t = " << h_2_t_loop
 << endl
 << "last mu_3_squared = " << mu_3_squared_loop
```

```
 << endl;
outfile2.close();
if (lambda_1_loop == lambda_1_0 ||
    lambda_1_loop == lambda_1_f ||
    lambda_3_loop == lambda_3_0 ||
    lambda_3_loop == lambda_3_f ||
    lambda_6_loop == lambda_6_0 ||
    lambda_6_loop == lambda_6_f ||
    lambda_7_loop == lambda_7_0 ||
    lambda_7_loop == lambda_7_f ||
    h_2_t_loop == h_2_t_f ||
    h_1_b_loop == h_1_b_f ||
//     mu_3_squared_loop == mu_3_squared_0 ||
//     mu_3_squared_loop == mu_3_squared_f ||
//     lambda_2_loop == lambda_2_0 ||
    lambda_2_loop == lambda_2_f) {
outfile3 << "Range bump: "
<< lambda_1_loop << " "
<< lambda_2_loop << " "
<< lambda_3_loop << " "
<< lambda_6_loop << " "
<< lambda_7_loop << " "
<< h_1_b_loop << " "
<< h_2_t_loop << " "
<< mu_3_squared_loop << endl;
}
}

// cout << "successful value recorded" << endl;
lambda_7_success = lambda_7_loop;
lambda_6_success = lambda_6_loop;
lambda_3_success = lambda_3_loop;
lambda_2_success = lambda_2_loop;
lambda_1_success = lambda_1_loop;
h_2_t_success = h_2_t_loop;
h_1_b_success = h_1_b_loop;
}
```

```
else {
if (lambda_7_success != 0.0) {
lambda_7_loop += lambda_7_f;
lambda_7_success = 0.0;
}
}

} // end mu_3_squared
} // end lambda_7
if ( (h_1_b_success != 0.0)
&& (h_1_b_success != h_1_b_loop) ) {
h_1_b_loop += h_1_b_f;
h_1_b_success = 0.0;
}
} // end h_1_b
if ( (h_2_t_success != 0.0)
&& (h_2_t_success != h_2_t_loop) ) {
h_2_t_loop += h_2_t_f;
h_2_t_success = 0.0;
}
} // end h_2_t
if ( (lambda_6_success != 0.0)
&& (lambda_6_success != lambda_6_loop) ) {
lambda_6_loop += lambda_6_f;
lambda_6_success = 0.0;
}
} // end lambda_6
if ( (lambda_3_success != 0.0)
&& (lambda_3_success != lambda_3_loop) ) {
lambda_3_loop += lambda_3_f;
lambda_3_success = 0.0;
}
} // end lambda_3
if ( (lambda_2_success != 0.0)
&& (lambda_2_success != lambda_2_loop) ) {
lambda_2_loop += lambda_2_f;
lambda_2_success = 0.0;
```

```
}
} // end lambda_2
if ( (lambda_1_success != 0.0)
&& (lambda_1_success != lambda_1_loop) ) {
lambda_1_loop += lambda_1_f;
lambda_1_success = 0.0;
}
} // end lambda_1
// end big for

outfile2.open(filenameinfo);
outfile2 << "Done" << endl;

outfile.close();
outfile2.close();
outfile3.close();

return 0;

} // end main


int check_conditions () {

if ( lambda_5 > 0 ) {
// cout << "Failed at mass 1 = " << mass_1 << endl;
return 1;
}
else if ( (lambda_4+lambda_5) > 0 ) {
// cout << "Failed at mass 1 = " << mass_1 << endl;
return 1;
}
else if ( lambda_1 > 10) {
// cout << "Failed at lambda_1 = " << lambda_1 << endl;
return 1;
}
else if ( lambda_1 < 0 ) {
```

```
// cout << "Failed at lambda_1 = " << lambda_1 << endl;
return 1;
}
else if ( lambda_2 > 10 ) {
// cout << "Failed at lambda_2 = " << lambda_2 << endl;
return 1;
}
else if ( lambda_2 < 0 ) {
// cout << "Failed at lambda_2 = " << lambda_2 << endl;
return 1;
}
else if ( (lambda_3 > 10) ) {
// cout << "Failed at lambda_3 = " << lambda_3 << endl;
return 1;
}
else if ( (lambda_4 > 10) ) {
// cout << "Failed at lambda_4 = " << lambda_4 << endl;
return 1;
}
else if ( (lambda_5 > 10) ) {
// cout << "Failed at lambda_5 = " << lambda_5 << endl;
return 1;
}
else if ( (lambda_6 > 10) ) {
// cout << "Failed at lambda_6 = " << lambda_6 << endl;
return 1;
}
else if ( (lambda_7 > 10) ) {
// cout << "Failed at lambda_7 = " << lambda_7 << endl;
return 1;
}
else if (!potential_check()) {
// cout << "Failed potential check" << endl;
return 1;
}
else {
return 0;
```

```
}

} // end check_conditions


double d_lambda_1 () {

return ( (1.0/(16.0*(M_PI*M_PI)))*(
// 0
12.0*(lambda_1*lambda_1)
+ 4.0*(lambda_3*lambda_3)
+ 2.0*(lambda_4*lambda_4)
+ 2.0*(lambda_5*lambda_5)
+ 24.0*(lambda_6*lambda_6)
+ 4.0*(lambda_3*lambda_4)
- 9.0*(lambda_1*(g*g))
- 3.0*(lambda_1*(g_prime*g_prime))
+ (9.0/4.0)*(g*g*g*g)
+ (3.0/2.0)*((g*g)*(g_prime*g_prime))
+ (3.0/4.0)*(g_prime*g_prime*g_prime*g_prime)
+ 12.0*(lambda_1*(h_1_t*h_1_t))
- 12.0*(h_1_t*h_1_t*h_1_t*h_1_t)
+ 12.0*(lambda_1*(h_1_b*h_1_b))
- 12.0*(h_1_b*h_1_b*h_1_b*h_1_b)
) );

} // end d_lambda_1


double d_lambda_2 () {

        return ( (1.0/(16.0*(M_PI*M_PI)))*(
// 0
12.0*(lambda_2*lambda_2)
+ 4.0*(lambda_3*lambda_3)
+ 2.0*(lambda_4*lambda_4)
+ 2.0*(lambda_5*lambda_5)
```

```
+ 24.0*(lambda_7*lambda_7)
+ 4.0*(lambda_3*lambda_4)
                - 9.0*(lambda_2*(g*g))
                - 3.0*(lambda_2*(g_prime*g_prime))
                + (9.0/4.0)*(g*g*g*g)
                + (3.0/2.0)*((g*g)*(g_prime*g_prime))
                + (3.0/4.0)*(g_prime*g_prime*g_prime*g_prime)
+ 12.0*(lambda_2*(h_2_t*h_2_t))
- 12.0*(h_2_t*h_2_t*h_2_t*h_2_t)
+ 12.0*(lambda_2*(h_2_b*h_2_b))
- 12.0*(h_2_b*h_2_b*h_2_b*h_2_b)
) );

} // end d_lambda_2


double d_lambda_3 () {

        return ( (1.0/(16.0*(M_PI*M_PI)))*(
// 0.0
4.0*(lambda_3*lambda_3)
+ 6.0*(lambda_1*lambda_3)
+ 6.0*(lambda_2*lambda_3)
+ 2.0*(lambda_4*lambda_4)
+ 2.0*(lambda_5*lambda_5)
+ 4.0*(lambda_6*lambda_6)
+ 4.0*(lambda_7*lambda_7)
+ 16.0*(lambda_6*lambda_7)
+ 2.0*(lambda_1*lambda_4)
+ 2.0*(lambda_2*lambda_4)
- 9.0*(lambda_3*(g*g))
- 3.0*(lambda_3*(g_prime*g_prime))
+ (9.0/4.0)*(g*g*g*g)
- (3.0/2.0)*((g*g)*(g_prime*g_prime))
+ (3.0/4.0)*(g_prime*g_prime*g_prime*g_prime)
+ 6.0*(lambda_3*(h_1_t*h_1_t))
+ 6.0*(lambda_3*(h_2_t*h_2_t))
```

```
- 12.0*(h_1_t*h_1_t*h_2_t*h_2_t)
+ 6.0*(lambda_3*(h_1_b*h_1_b))
+ 6.0*(lambda_3*(h_2_b*h_2_b))
- 12.0*(h_1_b*h_1_b*h_2_b*h_2_b)
) );


} // end d_lambda_3


double d_lambda_4 () {

        return ( (1.0/(16.0*(M_PI*M_PI)))*(
// 0.0
4.0*(lambda_4*lambda_4)
+ 2.0*(lambda_1*lambda_4)
+ 2.0*(lambda_2*lambda_4)
+ 8.0*(lambda_3*lambda_4)
+ 8.0*(lambda_5*lambda_5)
+ 10.0*(lambda_6*lambda_6)
+ 10.0*(lambda_7*lambda_7)
+ 4.0*(lambda_6*lambda_7)
- 9.0*(lambda_4*(g*g))
- 3.0*(lambda_4*(g_prime*g_prime))
+ 3.0*((g*g)*(g_prime*g_prime))
+ 6.0*(lambda_4*(h_1_t*h_1_t))
+ 6.0*(lambda_4*(h_2_t*h_2_t))
+ 12.0*(h_1_t*h_1_t*h_2_t*h_2_t)
+ 6.0*(lambda_4*(h_1_b*h_1_b))
+ 6.0*(lambda_4*(h_2_b*h_2_b))
+ 12.0*(h_1_b*h_1_b*h_2_b*h_2_b)
) );


} // end d_lambda_4
```

```
double d_lambda_5 () {

        return ( (1.0/(16.0*(M_PI*M_PI)))*(
// 0.0
2.0*(lambda_1*lambda_5)
+ 2.0*(lambda_2*lambda_5)
+ 8.0*(lambda_3*lambda_5)
+ 12.0*(lambda_4*lambda_5)
+ 10.0*(lambda_6*lambda_6)
+ 10.0*(lambda_7*lambda_7)
+ 4.0*(lambda_6*lambda_7)
- 9.0*(lambda_5*(g*g))
- 3.0*(lambda_5*(g_prime*g_prime))
+ 6.0*(lambda_5*(h_1_t*h_1_t))
+ 6.0*(lambda_5*(h_2_t*h_2_t))
+ 6.0*(lambda_5*(h_1_b*h_1_b))
+ 6.0*(lambda_5*(h_2_b*h_2_b))
) );


} // end d_lambda_5


double d_lambda_6 () {

        return ( (1.0/(16.0*(M_PI*M_PI)))*(
12.0*(lambda_1*lambda_6)
+ 6.0*(lambda_3*lambda_6)
+ 8.0*(lambda_4*lambda_6)
+ 10.0*(lambda_5*lambda_6)
+ 6.0*(lambda_3*lambda_7)
+ 4.0*(lambda_4*lambda_7)
+ 2.0*(lambda_5*lambda_7)
- 9.0*(lambda_6*(g*g))
- 3.0*(lambda_6*(g_prime*g_prime))
+ (9.0/2.0)*(lambda_6*(h_1_t*h_1_t))
+ (9.0/2.0)*(lambda_6*(h_2_t*h_2_t))
```

```
+ (3.0/2.0)*(lambda_6*(h_1_b*h_1_b))
+ (3.0/2.0)*(lambda_6*(h_2_b*h_2_b))
) );


} // end d_lambda_6


double d_lambda_7 () {

        return ( (1.0/(16.0*(M_PI*M_PI)))*(
12.0*(lambda_2*lambda_7)
+ 6.0*(lambda_3*lambda_7)
+ 8.0*(lambda_4*lambda_7)
+ 10.0*(lambda_5*lambda_7)
+ 6.0*(lambda_3*lambda_6)
+ 4.0*(lambda_4*lambda_6)
+ 2.0*(lambda_5*lambda_6)
- 9.0*(lambda_6*(g*g))
- 3.0*(lambda_6*(g_prime*g_prime))
+ (3.0/2.0)*(lambda_7*(h_1_t*h_1_t))
+ (3.0/2.0)*(lambda_7*(h_2_t*h_2_t))
+ (9.0/2.0)*(lambda_7*(h_1_b*h_1_b))
+ (9.0/2.0)*(lambda_7*(h_2_b*h_2_b))
) );


} // end d_lambda_7


double d_g () {

return ( (1.0/(16.0*(M_PI*M_PI)))* (
(-3.0)*(g*g*g) ) );

} // end d_g
```

```
double d_g_prime () {

return ( (1.0/(16.0*(M_PI*M_PI)))* (
(7.0)*(g_prime*g_prime*g_prime) ) );

} // end d_g_prime


double d_g_c () {

return ( (1.0/(16.0*(M_PI*M_PI)))* (
(-7.0)*(g_c*g_c*g_c) ) );

}

double d_h_1_t () {

return ( (h_1_t/(16.0*(M_PI*M_PI)))* (
(9.0/2.0)*(h_1_t*h_1_t)
+ (1.0/2.0)*(h_1_b*h_1_b)
- 8.0*(g_c*g_c) - (9.0/4.0)*(g*g)
- (17.0/12.0)*(g_prime*g_prime) ) );

} // end d_h_1_t


double d_h_2_t () {

return ( (h_2_t/(16.0*(M_PI*M_PI)))* (
(9.0/2.0)*(h_2_t*h_2_t)
+ (1.0/2.0)*(h_2_t*h_2_t)
- 8.0*(g_c*g_c) - (9.0/4.0)*(g*g)
- (17.0/12.0)*(g_prime*g_prime) ) );

} // end d_h_2_t
```

```
double d_h_1_b () {

return ( (h_1_b/(16.0*(M_PI*M_PI)))* (
(9.0/2.0)*(h_1_b*h_1_b)
+ (1.0/2.0)*(h_1_b*h_1_b)
- 8.0*(g_c*g_c) - (9.0/4.0)*(g*g)
- (5.0/12.0)*(g_prime*g_prime) ) );

} // end d_h_1_b


double d_h_2_b () {

return ( (h_2_b/(16.0*(M_PI*M_PI)))* (
(9.0/2.0)*(h_2_b*h_2_b)
+ (1.0/2.0)*(h_2_b*h_2_b)
- 8.0*(g_c*g_c) - (9.0/4.0)*(g*g)
- (5.0/12.0)*(g_prime*g_prime) ) );

} // end d_h_2_b


double get_part_a () {

return (
lambda_1*(v_1*v_1)
+ (3.0/4.0)*lambda_6*(v_1*v_2)
- (1.0/4.0)*lambda_7*(v_2*v_2*v_2/v_1)
);

}


double get_part_c () {

return (
(lambda_3+lambda_4+lambda_5)*(v_1*v_2)
```

```
+ (3.0/4.0)*lambda_6*(v_1*v_1)
+ (3.0/4.0)*lambda_7*(v_2*v_2)
);

}


double get_part_b () {

return (
lambda_2*(v_2*v_2)
- (1.0/4.0)*lambda_6*(v_1*v_1*v_1/v_2)
+ (3.0/4.0)*lambda_7*(v_1*v_2)
);

}


double get_part_1 () {

return ( (get_part_a() + get_part_b()) / 2.0 );

}


double get_part_2 () {

double a = get_part_a();
double b = get_part_b();
double c = get_part_c();

double temp = ((a-b)*(a-b)) + (4.0*c*c);

if (temp < 0) {
return -1;
}
else if (temp == 0.0) {
```

```
return 0;
}
else {
return ( sqrt(temp) / 2.0 );
}

}


int potential_check () {

double A,B,r,t,x1,x2,x3,temp1,temp2,temp3,V1,V2,V3;

double a = 2*lambda_2;
double b = 3*lambda_7;
double c = 2*(lambda_3+lambda_4+lambda_5);
double d = lambda_6;

if (a == 0) {
if  (b == 0) {
x1 = -d/c;
V1 = getV(x1);
if (V1 >= 0 ) {
return 1;
}
else {
return 0;
}
}
else {
x1 = (-c + sqrt((c*c) - 4.0*(b*d)))/(2.0*b);
x2 = (-c - sqrt((c*c) - 4.0*(b*d)))/(2.0*b);
V1 = getV(x1);
V2 = getV(x2);
if (V1 <= V2) {
if (V1 >= 0) {
return 1;
```

```
}
else {
return 0;
}
}
else {
if (V2 >= 0) {
return 1;
}
else {
return 0;
}
}
}
}

double f = ((3.0*c/a) - ((b*b)/(a*a)))/3.0;
double g = (2.0*((b*b*b)/(a*a*a)) - 9.0*((b*c)/(a*a))
+ 27.0*(d/a))/27.0;

double h = ((g*g)/4.0) + ((f*f*f)/27.0);

if (h > 0) {
A = cbrt((-g/2.0) + sqrt(h));
B = cbrt((-g/2.0) - sqrt(h));
x1 = A+B-(b/(3.0*a));
V1 = getV(x1);
if (V1 >= 0) {
return 1;
}
else {
return 0;
}
}
else if (h == 0) {
temp1 = cbrt(-g/2.0);
temp2 = b/(3.0*a);
```

```
x1 = (2.0*temp1) - temp2;
x2 = -temp1 - temp2;
V1 = getV(x1);
V2 = getV(x2);
if (V1 <= V2) {
if (V1 >= 0) {
return 1;
}
else {
return 0;
}
}
else {
if (V2 >= 0) {
return 1;
}
else {
return 0;
}
}
}
else {
r = sqrt((g*g/4.0) - h);
t = acos(-g/(2.0*r));
temp1 = cbrt(r);
temp2 = t/3.0;
temp3 = b/(3.0*a);
x1 = (2.0*temp1*cos(temp2)) - temp3;
x2 = ((-temp1)*(cos(temp2) + M_SQRT3*sin(temp2))) - temp3;
x3 = ((-temp1)*(cos(temp2) - M_SQRT3*sin(temp2))) - temp3;
V1 = getV(x1);
V2 = getV(x2);
V3 = getV(x3);
if ( (V1 <= V2) && (V1 <= V3) ) {
if (V1 >= 0) {
return 1;
}
```

```
else {
return 0;
}
}
else if ( (V2 <= V1) && (V2 <= V3) ) {
if (V2 >= 0) {
return 1;
}
else {
return 0;
}
}
else {
if (V3 >= 0) {
return 1;
}
else {
return 0;
}
}
}

}


double getV (double x) {

return ( (lambda_1/2.0) + (lambda_2*x*x*x*x/2.0)
+ ((lambda_3+lambda_4+lambda_5)*x*x)
+ (lambda_6*x)
+ (lambda_7*x*x*x) );

}
```