Studies of NOνA neutrino flux with NDOS

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by

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Abstract

The Neutrinos at Main Injector (NuMI) beam is an intense neutrino source formed by firing protons from the Fermilab Main Injector into a graphite target. The NuMI Off-Axis $\nu_e$ Appearance (NO$\nu$A) experiment is a pair of neutrino detectors being constructed downstream of NuMI. The main purpose of NO$\nu$A is to study neutrino oscillations by analyzing the change in the neutrino energy spectrum and/or composition over the distance between the near detector and the far detector. It is difficult to model the spray of particles exiting the graphite target that drives NuMI but it is important to understand it because it determines the neutrino energy spectrum. I analyzed the NuMI beamline model for the NO$\nu$A experiment with two approaches. A hadron-production reweighting algorithm was developed for a previous neutrino experiment and I applied it to the model for the neutrino flux at NO$\nu$A. I found that there are more positive kaons generating neutrinos in the NuMI beamline than previously supposed. In my second analysis, I find that tracking the trajectory of a muon created by a charged current (CC) neutrino interaction in the detector can be used to infer where in the NuMI beamline the neutrino originated.
1 Introduction and Motivation

1.1 Neutrino Interactions and Oscillations

Neutrinos are fundamental particles with no charge, with very little mass (less than 0.6 eV[1]), and which only interact with other particles through the weak force. Neutrino mass is not considered in kinematics and neutrinos effectively move at the speed of light. They interact very infrequently and can only be detected indirectly by measuring the daughter particles produced in weak interactions. There are two types of neutrino interactions, the neutral current (NC) interaction and the charged current (CC) interaction, as shown in Figure 1. In a NC interaction, a neutrino exchanges a Z boson with a target particle and transfers momentum to it. In a CC neutrino interaction a W boson is exchanged and results in a charged lepton (electron, muon, or tau) that can be detected.

![Feynman Diagrams](image)

Figure 1: (a) The Feynman diagram for a charged current converting a neutrino into an electron. (b) the Feynman diagram for a neutral current weak interaction is shown.[2]

The “reverse” CC interaction can occur as well, in which a charged lepton is converted into a neutrino. If a neutrino recently created by a charged lepton is observed in a charged current interaction immediately afterwards then the same type of charged lepton is detected. Because of this, neutrinos can be organized by flavor: A muon-neutrino is defined to be a neutrino which is created by a muon and will create a muon in a CC interaction and the same terminology applies for electrons and taus. If a longer distance is traveled however, then the charged lepton involved in the creation of the neutrino and the charged lepton created in the CC interaction of its detection may not be of the same flavor. In these cases, neutrinos are said to “oscillate” from one neutrino flavor to another. In NC interactions the flavor of the interacting neutrino cannot be determined because no charged lepton is generated but NC event rates still play a role in neutrino oscillation experiments by providing an indication of total neutrino flux.

Neutrino oscillations are a consequence of the fact that neutrinos have mass and the three mass states do not coincide with the flavor states. The change of basis matrix between the mass eigenbasis and the weak
eigenbasis is called the Pontecorvo-Maki-Nakagawa-Sakata Matrix \[3\][4][5] and is given by:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= 
\begin{pmatrix}
c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
-c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\
s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\] (1)

where \(c_{ij} = \cos(\theta_{ij})\), \(s_{ij} = \sin(\theta_{ij})\). By measuring neutrino oscillation rates of each flavor over a variety of energies and distance traveled, experimenters can constrain and measure the neutrino oscillation parameters, including the separation between the mass states. Solar and atmospheric neutrino experiments have already provided good measurements of \(\theta_{23}\), \(\theta_{12}\), \(\Delta m^2_{32}\), \(\Delta m^2_{21}\), and \(|\Delta m^2_{31}|\) (where \(\Delta m^2_{ij} = m^2_i - m^2_j\)). The NO\(\nu\)A experiment has been designed to measure or constrain the remaining unknown neutrino oscillation parameters, \(\theta_{13}\) and \(\delta\). The \(\delta\) given in Eq. 1 is a CP (charge-parity) violating phase that has important theoretical implications. If \(\delta\) is not a multiple of \(\pi\) then it would mean the oscillation rates between neutrinos and anti-neutrinos would differ and this difference might help explain why our universe is dominated by matter.

1.2 NO\(\nu\)A Experiment

In its final form, the NO\(\nu\)A experiment will consist of two neutrino detectors, the near detector and the far detector. The near detector will be housed at Fermilab, an accelerator lab outside of Chicago, and the far detector will be located 810 km away at Ash River, Minnesota. Both detectors are to be downstream of the Neutrinos at the Main Injector (NuMI) beam, an intense neutrino source. The same population of neutrinos can be compared at two different times (in the flight of the neutrinos). The NO\(\nu\)A detector is designed to be sensitive to electron neutrino CC interactions in addition to muon CC interactions and NC interactions. The neutrino population from NuMI is predominately from muon sources, but the neutrino flux at far detector “oscillates” to have a higher probability of producing electrons or taus in CC weak interactions than at the near detector. The increase in electron CC events at the NO\(\nu\)A far detector will be interpreted as muon-to-electron neutrino oscillations and the decrease in muon CC events at the NO\(\nu\)A far detector will be interpreted as muon-to-tau neutrino oscillations (the tau CC interactions will be undetected).

In order to minimize systematic uncertainty in the oscillation measurement, the near and far detectors are designed to be as similar as possible. The far detector is made of the same materials as the near detector, but is to be over 100 times the volume of the near detector to maximize the count rates. The detectors are composed of rigid PVC shaped into columnar cells filled with liquid scintillator (95.8% mineral oil, 4.1% pseudocumene) as an active target. When the scintillator in a cell is struck by a charged particle the light emitted is collected by the wave-shifting optical fiber for that cell, which transmits the light to an avalanche photodiode (APD).
1.3 NuMI Beamline

The NuMI beamline is composed of a graphite target, a set of focusing horns, and decay pipe as shown in Figure 2. The 120 GeV protons from Fermilab’s Main Injector can be directed to a graphite target and the resulting collisions create a shower of hadrons. The charged stream is focused by two beamline devices, called horns, which generate toroidal magnetic fields that focus the charged particles depending on their momentum and original trajectory. Next, the charged particles enter a 700m decay pipe so that the kaons and pions decay into a mixture of leptons. The neutrinos in this population, created from these decay in the NuMI beamline, pass through the rock and to the neutrino detectors downstream of the beam. Because of the kinematics of relativistic decay, the energy of a neutrino in the beam depends on the angle between the direction of the neutrino and the direction of its parent meson as well as the mass of its parent meson. The energy of a neutrino is approximately given by:

\[ E_\nu = \frac{(1 - (m_\mu/m_{\pi,K})^2)E_{(\pi,K)}}{1 + \gamma^2 \theta^2} \]  

where \( \theta \) is the angle between the neutrino and the meson direction and \( \gamma = E_{(\pi,K)}/m_{(\pi,K)} \).

Manipulating the current running through the horns and changing the relative distance between the target and horns changes the energy spectrum of the resultant neutrino beam. Because the interaction cross-sections of neutrinos are so small, the rock has a negligible effect on the neutrino flux but filters out, by absorption, most of the other particles from NuMI and from atmospheric sources. Any charged particles that fail to decay into neutrinos in the decay pipe are absorbed by a beam dump at the end.

Figure 2: The NuMI beamline generates hadrons from the energy of the Main Injector and focuses them with magnetic horns to result in an intense, directed beam of muon-neutrinos with an energy spectrum dependent on angle. [7]
2 NO$\nu$A NDOS Assembly

An intermediate step in the assembly of the NO$\nu$A near detector in its underground location is the Near Detector on Surface (NDOS). The NDOS was built to the specifications of the NO$\nu$A near detector, but is on the surface rather than underground in the MINOS tunnel, which is where it will need to be to conduct the oscillation measurement. It was originally planned that NDOS would be transported underground to serve as the NO$\nu$A near detector for oscillation analysis but those plans are currently under review. In the meantime, the NDOS allows for calibration and debugging to take place using NuMI and cosmic sources of data.

This past summer I worked at Fermilab on the NO$\nu$A experiment at the time that the NDOS was being assembled. Four 4.2m x 2.9m x 2.1 m PVC detector blocks arrived at Fermilab after being assembled and pressure tested at University of Minnesota and Argonne National Laboratory. One task that had to be performed at Fermilab is that the detector blocks had to be made “light tight” to prevent light from leaking into the cells. The only safe way to achieve this was to paint the blocks by hand because paint sprays could damage the detector and covering the detector would block access to the electronics. I was one member of a 6-8 person team that accomplished painting four blocks with one coat of primer and two coats of black paint over uneven surfaces. These blocks were put together by gluing together planes of alternating orientation, so there were seams where two planes met that had to be injected with black paint or covered with RTV adhesive. In addition, I aided the electronics group by affixing grounding strips to the detector. Now the NDOS has all six blocks, is fully electronically instrumented, filled with liquid scintillator and is currently taking data.

3 SKZP Weighting

Previous efforts to predict the properties of a neutrino beam flux from first principles have been highly inaccurate, because it depends on the hadronic shower exiting the graphite target. Instead, the Monte Carlo (MC) simulation of the neutrino beam models the regimes that it can rely on and fits experimentally where it cannot model so easily. It is important to understand how uncertainties in the production of the charged particles within the target affect the neutrino energy spectrum.

MINOS is a currently running neutrino experiment also downstream of the NuMI beam. The MCReweight is a software package for MINOS written by Sacha Kopp, Žarko Pavlović, and Patricia Vahle. MCReweight implements an algorithm known as “SKZP reweighting” which attempts to make the beamline model more accurate with respect to uncertainties in the parameters for hadronic decay and horn current. By fitting the beam model to MINOS data taken in a variety of run configurations for MINOS, these parameters are tuned and the neutrino energy spectrum is reweighted to reflect this tuning.
I wrote an adaption of the the SKZP reweighting algorithm for use in the NOνA experiment. The code reweights the neutrino flux incident at the NDOS (rather than the MINOS detectors) based on data from MINOS. In addition, the program can be used in the future to take into account an independent analysis of the NuMI beamline parameters from NOνA data. In the process of adapting the SKZP reweighting algorithm I also made some improvements. The algorithm now accepts a broader range of input data and the structure of the algorithm has been reorganized to be both more efficient and readable. I have commented this code thoroughly, I have presented it at three NOνA software meetings, and I maintain a wiki page on its usage. This package, called NuBeamWeights, has been uploaded into the CVS software repository and I modified another package, NovaBeamMats, so the SKZP reweighting algorithm is easy to incorporate into the existing software routine.

To ensure that the code is correctly implemented I compared the plots generated by the NuBeamWeights package (for NOνA) with plots generated by the MCReweight package (for MINOS) from a paper by Mark Dorman[9], as shown in Figure 3 and Figure 4. I then applied the NuBeamWeights package to the NDOS (for which there are no previous plots), as shown in Figure 5 and Figure 6, as well as for other NOνA detector locations (see Appendix). Figure 7 and Figure 8 break down Figure 5 by parent particles. The ∼ 30% increase in the number of neutrino interactions detected at 2 GeV should be an effect that the NDOS can test experimentally.

4 Distribution of Parent Particle Decay Positions

For the ideal measurement of neutrino oscillation, the only difference between the neutrino flux incident at the NOνA near detector and the NOνA far detector would be the distance between where the neutrinos are generated at the NuMI beamline and where they are detected at each location. We’ve mentioned already how the detector designs are similar and the detectors are on the same axis with respect to the orientation of the NuMI beam. However it is also important to consider the fact that the NuMI beamline acts as a line source to the NOνA near detector but acts as a point source to the NOνA far detector. The solid angle subtended by the NOνA near detector is orders of magnitude larger than that of the NOνA far detector and is more sensitive to changes in the decay position. Relativistic effects cause the higher energy mesons to have longer characteristic decay lengths and therefore cause the energy spectrum to depend on the decay position. In addition, the angle at which a given neutrino leaves the decay processes from its parent particle will dramatically affect the energy of the neutrino spectra according to Eq. 2 - with lower angles having higher momentum boosts. The NOνA experiment will look for neutrino oscillations by comparing the energy spectra between the NOνA near detector and NOνA far detector so it is important to know that the distribution of decay positions is correctly modeled [10]. If NOνA data indicates a distribution of decay positions that differs from the MC model, than the hadron production model can be tuned.
Figure 3: The top plot is generated by NuBeamWeights, the software package I wrote for NO\(\nu\)A, and below it is a plot\cite{9} generated using the MCReweight package for MINOS. The MINOS near detector muon neutrino energy spectrum weighted by the SKZP reweighting algorithm is in red and the unmodified spectrum is in blue. The fact that the ratio between the blue line and the red line for a given energy is the same in each plot verifies my implementation of the SKZP reweighting algorithm for use by NO\(\nu\)A.
Figure 4: The top plot shows the weighted MINOS near detector muon neutrino energy spectrum divided by that of the unweighted spectrum and it is generated by NuBeamWeights. Below it is a plot generated using the MCReweight package showing the ratio of the weighted and unweighted spectrum to data. The two plots match if the ratio between the blue line and the red line on the bottom plot is equal to the value plotted on the top plot. They do appear to match and this verifies my implementation of the SKZP reweighting algorithm for use by NOνA.
Figure 5: The above plot is the NOνA energy spectrum in the NDOS of muon-neutrinos. It is on a logarithmic scale with the plot weighted by the NuBeamWeights package in red and unweighted in blue. The plot below it is the ratio of the weighted to the unweighted, with the blue line marking the ratio of one.
Figure 6: The above plot is the NO$\nu$A energy spectrum in the NDOS of anti-muon-neutrinos. It is on a logarithmic scale with the plot weighted by the NuBeamWeights package in red and unweighted in blue. The plot below it is the ratio of the weighted to the unweighted, with the blue line marking the ratio of one and the green line marking the average ratio.
Figure 7: The above plot is the NOνA energy spectrum in the NDOS of muon-neutrinos from kaon decays. It is on a logarithmic scale with the plot weighted by the NuBeamWeights package in red and unweighted in blue. The plot below it is the ratio of the weighted to the unweighted with the blue line marking the ratio of one and the green line marking the average ratio, with the blue line marking the ratio of one and the green line marking the average ratio.
Figure 8: The above plot is the NOνA energy spectrum in the NDOS of muon-neutrinos from pion decays. It is on a logarithmic scale with the plot weighted by the NuBeamWeights package in red and unweighted in blue. The plot below it is the ratio of the weighted to the unweighted, with the blue line marking the ratio of one.
I looked through MC simulations of events in the NDOS detector to identify data that could be used to test the distribution of parent particle decay positions currently used in the MC model. It would only be possible to use the real NDOS detector data to make inferences about the actual distribution of parent particle decay positions, if there was a detector observable strongly correlated with the model of parent particle decay position. In our analysis, we use the muon direction to infer the neutrino direction and by extension, the position in the decay pipe that produced the interacting neutrino. The muon trajectory can typically be reconstructed fairly accurately, but some of the neutrino energy from the neutrino will go into the nucleon which cannot be tracked so easily. The nucleon produced by the CC event interacts strongly with the detector creating a short, wide hadronic shower that makes it nearly impossible to assign an accurate momentum direction to the nucleon. Instead we can calculate the hadronic parameter, $y$, which is the fraction that the energy into the hadronic recoil system. CC events with the lower values of $y$ have muon direction that more highly correlates with the incoming neutrino direction as shown in Figure 9, Figure 10, and Figure 11 (see Appendix for anti-neutrino data). The correlation becomes sharper with more extreme cuts on $y$, but that must be balanced with the loss of statistical power.

**Figure 9:** This plot compares the momentum in the direction of the beamline for a neutrino interacting in the detector and that of the resulting muon. If there were a perfect correlation between the two, the plot would make a straight line of slope 1. Instead some of the interaction energy goes into the hadronic shower and as a result the neutrino energy can be significantly greater than the muon energy. The black line is a linear fit (without the $y$-offset) and has a slope of $1.35974 \pm (5.61292 \times 10^{-4})$.

The MC gives us access to the parent particle decay position in the decay pipe at which each simulated neutrino originates. Figure 12 shows the distribution of parent particle decay positions from neutrino events and Figure 24 shows the same plot on a logarithmic scale. A critical part of our analysis was comparing this MC parent particle decay position with the angle (relative to the beamline) of a simulated detected neutrino
Figure 10: This plot compares the momentum in the direction of the beamline for a neutrino interacting in the detector and that of the resulting muon, for interaction events with $y < 0.1$. If there were a perfect correlation between the muon momentum and neutrino momentum, the plot would make a straight line of slope 1. Instead some of the interaction energy goes into the hadronic shower and as a result the neutrino energy can be significantly greater than the muon energy. The black line is a linear fit (without the y-offset) and has a slope of $1.07348 \pm (8.62632 \times 10^{-4})$. The slope is closer to 1, because the $y > 0.1$ events were cut out.
Figure 11: This plot compares the momentum in the direction of the beamline for a neutrino interacting in the detector and that of the resulting muon, for interaction events with $y < 0.05$. If there were a perfect correlation between the muon momentum and neutrino momentum, the plot would make a straight line of slope 1. Instead some of the interaction energy goes into the hadronic shower and as a result the neutrino energy can be significantly greater than the muon energy. The black line is a linear fit (without the y-offset) and has a slope of $1.04186 \pm (1.09219 \times 10^{-3})$. The slope is closer to 1, because the $y > 0.05$ events were cut out.
to see if the NDOS detector would be sensitive to the parent particle decay positions from real data. We used the cosine of the neutrino-angle, rather than the angle itself. Figure 13 and Figure 14 show these 2D histogram plots for neutrinos and anti-neutrinos respectively.

Figure 12: This plot is a 1D histogram of the decay positions from which neutrinos interacting in the detector originate. In particular, muon-neutrino CC interactions events are shown.

We can also make similar such plots comparing the MC parent particle decay position with the angle (relative to the beamline) of a simulated muon resulting from a CC interaction. Figure 15 and Figure 16 show these 2D histogram plots for neutrinos and anti-neutrinos events with $y < 0.05$. Figure 25 through Figure 26 in the Appendix shows these same 2D histogram plots for $y < 0.1$. The correlation is much weaker in muon-angle plots, but the relationship can still be detected.

5 Conclusion

Figure 3 and Figure 4 verify that the generalization of the SKZP reweighting algorithm that I wrote, NuBeamWeights, was implemented correctly. This means that we can compare this tuned model of the neutrino spectrum to the neutrino spectrum detected by NO$
u$A and reanalyze the hadron production model. By comparing Figure 7 with Figure 5, one can see that a major effect of the SKZP tuning was to indicate that there was $\sim 30\%$ more neutrinos from positive kaons than previously expected in the NDOS energy spectrum.

Figure 15 and Figure 16 show a weak but visible correlation between the parent particle decay positions and the cosine of the muon-beamline angle. Because the muon angle can be constructed in the detector, this means NO$
u$A data will be able to provide some information about the distribution of particle decays in the NuMI beamline. This regime of the beamline is both difficult to model and highly relevant to neutrino
Figure 13: This plot is a 2D histogram of CC events relates the cosine of the angle that the neutrino makes with the beamline and the decay position from which the interacting neutrino originates. The plot has a logarithmic ‘z’ axis and shows muon-neutrino CC events with any $y$ value. The black lines are the x-profile of this 2D histogram - each horizontal line is the mean of a bin on the x-axis. The plot shows that the neutrinos that interact with the detector at angles transverse to the beamline originate from further down the beamline - this clearly matches the relationship that we expect from the geometry of the NuMI beamline.
Figure 14: This plot is a 2D histogram of CC events relates the cosine of the angle that the neutrino makes with the beamline and the decay position from which the interacting neutrino originates. The plot has a logarithmic ‘z’ axis and shows muon-neutrino CC events with any $y$ value. The black lines are the x-profile of this 2D histogram - each horizontal line is the means of a bin on the x-axis. The plot shows that the neutrinos that interact with the detector at angles transverse to the beamline originate from further down the beamline - this clearly matches the relationship that we expect from the geometry of the NuMI beamline.
Figure 15: This plot is a 2D histogram of CC events relates the cosine of the angle that the interaction muon makes with the beamline and the parent particle decay position (from which the interaction neutrino originates). The plot has a logarithmic ‘z’ axis and shows muon-neutrino CC events with $y < 0.05$. The black lines are the x-profile of this 2D histogram - each horizontal line is the mean of a bin on the x-axis and each vertical line is the standard error about the mean. As in Figure 13, the decay position rises with more transverse angles, however the effect is less dramatic because there is a distribution of muon angles corresponding to a given neutrino angle as shown in Figure 11.
Figure 16: This plot is a 2D histogram of CC events relates the cosine of the angle that the interaction muon makes with the beamline and the parent particle decay position (from which the interaction anti-neutrino originates). The plot has a logarithmic ‘z’ axis and shows anti-muon-neutrino CC events with $y < 0.05$. The black lines are the x-profile of this 2D histogram - each horizontal line is the mean of a bin on the x-axis and each vertical line is the standard error about the mean. As in Figure 14, the decay position rises with more transverse angles, however the effect is less dramatic because there is a distribution of muon angles corresponding to a given neutrino angle as shown in Figure 23.
oscillation analysis.

The effort to improve the MC model of the NuMI beamline is a continuously on-going effort and this analysis furthers what we know about the neutrino flux from NuMI at the NDOS. When the NO$\nu$A near and far detectors become operational the NO$\nu$A experiment will be able to further tune the beamline model using the NuBeamWeights package and analysis of distribution of particle decays in the beamline. Because an accurate analysis of neutrino oscillation in NO$\nu$A depends on understanding the distribution of neutrino energies from the NuMI beamline, I expect that verifying and tuning the NuMI beamline model will be an active part of the project for years to come.

References


Figure 17: The top plot is the energy spectrum in NOνA near detector for muon-neutrinos. It is on a linear scale with the plot weighted by the NuBeamWeights package in red and unweighted in blue. The plot below it is the ratio of the weighted to the unweighted, with the blue line marking the ratio of one. The effect of the reweighting is not dramatic.
Figure 18: The top plot is the energy spectrum in NOνA near detector for anti-muon-neutrinos. It is on a linear scale with the plot weighted by the NuBeamWeights package in red and unweighted in blue. The plot below it is the ratio of the weighted to the unweighted, with the blue line marking the ratio of one and the green line marking the average ratio. The effect of the reweighting is not dramatic.
Figure 19: The top plot is the energy spectrum in NOνA far detector for muon-neutrinos. It is on a linear scale with the plot weighted by the NuBeamWeights package in red and unweighted in blue. The plot below it is the ratio of the weighted to the unweighted, with the blue line marking the ratio of one and the green line marking the average ratio. The effect of the reweighting is not dramatic.
Figure 20: The top plot is the energy spectrum in NOvA far detector for anti-muon-neutrinos. It is on a linear scale with the plot weighted by the NuBeamWeights package in red and unweighted in blue. The plot below it is the ratio of the weighted to the unweighted, with the blue line marking the ratio of one and the green line marking the average ratio. The effect of the reweighting is not dramatic.
Figure 21: This plot compares the momentum in the direction of the beamline for an anti-neutrino interacting in the detector and that of the resulting muon. If there were a perfect correlation between the two, the plot would make a straight line of slope 1. Instead some of the interaction energy goes into the hadronic shower, and as a resulting the neutrino energy can be significantly greater than the muon energy. The black line is a linear fit (without the y-offset) and has a slope of $1.45749 \pm (5.41835 \times 10^{-4})$.

Figure 22: This plot compares the momentum in the direction of the beamline for an anti-neutrino interacting in the detector and that of the resulting muon, for interaction events with $y < 0.1$. If there were a perfect correlation between the muon momentum and neutrino momentum, the plot would make a straight line of slope 1. Instead some of the interaction energy goes into the hadronic shower, and as a resulting the neutrino energy can be significantly greater the muon energy. The black line is a linear fit (without the y-offset) and has a slope of $1.07353 \pm (8.80608 \times 10^{-4})$. The slope is closer to 1, because the $y > 0.1$ events were cut out.
Figure 23: This plot compares the momentum in the direction of the beamline for an anti-neutrino interacting in the detector and that of the resulting muon, for interaction events with $y < 0.05$. If there were a perfect correlation between the muon momentum and neutrino momentum, the plot would make a straight line of slope 1. Instead some of the interaction energy goes into the hadronic shower, and as a resulting the neutrino energy can be significantly greater than the muon energy. The black line is a linear fit (without the y-offset) and has a slope of $1.04132 \pm (1.12532 \times 10^{-3})$. The slope is closer to 1, because the $y > 0.05$ events were cut out.

Figure 24: This plot is a 1D histogram of the decay positions from which neutrinos interacting in the detector originate. In particular, muon-neutrino CC interactions events are shown. The y-axis is logarithmic, so the negative linear trend in this plot indicates that the distribution falls off exponentially.
Figure 25: This plot is a 2D histogram of CC events relating the cosine of the angle that the interaction muon makes with the beamline and the parent particle decay position (from which the interaction neutrino originates). The plot has a logarithmic ‘z’ axis and shows muon-neutrino CC events with $y < 0.1$. The black lines are the x-profile of this 2D histogram - each horizontal line is the mean of a bin on the x-axis and each vertical line is the standard error about the mean. As in Figure 15, the decay position rises with more transverse angles, however the effect is not as dramatic because the distribution of muon angles corresponding to a given neutrino angle is wider in Figure 10 than in Figure 11.
Figure 26: This plot is a 2D histogram of CC events relates the cosine of the angle that the interaction muon makes with the beamline and the parent particle decay position (from which the interaction anti-neutrino originates). The plot has a logarithmic ‘z’ axis and shows anti-muon-neutrino CC events with $y < 0.1$. The black lines are the x-profile of this 2D histogram - each horizontal line is the mean of a bin on the x-axis and each vertical line is the standard error about the mean. As in Figure 16, the decay position rises with more transverse angles, however the effect is not as dramatic because the distribution of muon angles corresponding to a given neutrino angle is wider in Figure 22 than in Figure 23.