Holographic QCD

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by

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Abstract

This project describes a model of quantum chromodynamics (QCD) that uses a holographic approach. A five-dimensional holographic model was constructed to include key traits of QCD such as chiral symmetry breaking. In order to explain the motivations behind this model, this paper will cover some basics of quantum chromodynamics, relevant string theory, the holographic principle, and the AdS/CFT correspondence. The model will then be presented, the analogies between the mathematics and the physical world explained, and finally results will be presented and compared to experimental data. The main result is a calculation of a ratio involving the decay constant and mass of the rho meson which was found to agree with measured values to about 4.5 percent. This result is roughly independent of model parameters and thus it provides a stringent test of the AdS/QCD model.

1 Introduction

Quantum Chromodynamics

Quantum chromodynamics (QCD) is a theory of the strong interaction, the fundamental force governing the combination of quarks and gluons into hadrons, such as the proton and the neutron, as well as the combination of hadrons into nuclei. A hadron is a composite particle and will fall into one of two categories: mesons and baryons. A meson is made up of a quark and an antiquark pair, while a baryon is made up of three quarks. A quark is a spin 1/2 particle, a fermion. Thus mesons, being made of two quarks, are bosons, while baryons are fermions.

Particles governed by the strong interaction can have one of three types of charge, called color charge, which is in some sense analogous to positive and negative charge in electrodynamics. The three types of charge are red, green, and blue. As in electrodynamics, a charge attracts opposite charges, so a blue charge will attract red and green charge. In electrodynamics, the photon, which mediates interactions between particles of various charge, is itself chargeless. However, gluons, which mediate interactions between quarks with different color charge, carry color charge themselves. This causes them to interact with each other. These interactions significantly complicate QCD, making it a nonlinear theory that cannot be solved analytically.

Quantum chromodynamics has two properties that other quantum field theories do not. The first is asymptotic freedom, wherein at high energies and short distances the interactions fall off asymptotically to zero. The second is color confinement, which refers to the strengthening of the interaction as distances between quarks increase. The energy required to isolate a quark would be infinite, thus quarks are eternally bound into hadrons. Confinement explains the fact that quarks have never been observed in isolation and thus studied only indirectly. The color force between quarks is constant and extremely large.

There are six different types of quarks: up, down, charm, strange, top, and bottom. These types are also known as flavors and the types vary in mass. The up and down are the lightest quarks, the charm and strange are the next lightest, and the top and bottom quarks are the heaviest. The heavier quarks tend to decay into the lighter quarks, thus up and down quarks are the most stable and common of the flavors.

The quark mass is a strange idea, since due to color confinement a quark cannot be isolated in order to investigate its mass. A proton, for example, is made up of two up quarks and a down quark, but its mass is greater than the sum of the masses of the three quarks. This extra mass is due to the energy of the gluon field between the quarks (according to $e = mc^2$) as well as to the motion of the quarks within a particle. The quark masses are determined by scattering experiments on hadrons containing quarks.

Since QCD cannot be solved analytically, theorists have tried to take a perturbative approach similar to those used in describing other fundamental forces. Due to the strength of the interactions, this approach fails in many contexts, working only at high energies. Thus theorists turn to the non-perturbative approach, lattice QCD. Lattice QCD requires a great deal of time and computing power in order to calculate simple observables such as particle masses. The difficulty of lattice QCD has motivated theorists to develop models that approximate QCD.

Models of QCD are intended to simplify calculations at the expense of accuracy. An example include the large N or 1/N model, in which calculations are simplified by assuming a large number of colors, rather than the three seen in QCD. Another example is the bag model, in which the color confinement is viewed as a bag holding together the particles governed by the strong interactions. Our model is AdS/QCD (anti-de Sitter space/quantum chromodynamics) and it utilizes a holographic approach to build a model that includes some key features of the strong interactions.

One important aspect of the strong interactions is chiral symmetry. Chiral symmetry has to do with handedness of particles. If a particle has spin in the same direction as its direction of motion, it is right-handed, while if a particle has spin in the opposite direction as its direction of motion it is left handed. If a fermion is massless than its chirality is the same as its handedness, but generally, the concept of chirality is more abstract.

A field exhibits chiral symmetry if it is possible for the right-handed and left-handed components of the field to transform independently of one another. Massless particles have chiral symmetry, and the mass of quarks and hadrons breaks this symmetry. Chiral symmetry is an approximate symmetry of the strong interactions because the two lightest and most common quarks, up and down, are very light compared to the bound states of the particles that they constitute.

If quarks were truly massless they would undergo the transformations:

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow (SU(2)_L) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow (SU(2)_R) \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

However, the mass of the quarks spontaneously breaks the symmetry, thus

$$\langle \overline{q}_L q_R \rangle \neq 0$$

because the lagrangian is not invariant under separate transformations for its left-handed and right-handed components. The chiral condensate spontaneously breaks the chiral symmetry.

Chiral symmetry breaking is included in our AdS/QCD model since it is related to color confinement, physical properties of the proton, and the scale of particle masses.

The AdS/CFT Correspondence

Our model is motivated by the AdS/CFT correspondence [1]. The AdS/CFT correspondence was introduced by Juan Maldecena in the late 1990s. Maldecena conjectured that there is a duality between string theory in a certain dimensional space, and a conformal field theory that does not include gravity in a space with one fewer spatial dimension.

The AdS/CFT correspondence was motivated by an observation in string theory. String theory was first proposed as a theory for the strong interactions, but it was abandoned for this purpose when QCD was developed. In the last few decades, theorists have returned to string theory as a potential theory of everything. However, string theory has also proved useful as a theoretical tool for understanding field theories more generally. The model used in this project applies string theory back to the strong interactions.

Currently, three of the four fundamental forces have quantum descriptions that fit together without contradiction. The united theory of these three, the strong, weak, and electromagnetic forces, makes up the standard model. In quantum theory the momentum and exact location of a particle are not definite, they are probabilities and waves. The force of gravity has yet to be described in terms of a quantum field theory. Currently the best theory of gravity is Einstein's general relativity. General relativity describes how concentrations of mass or energy in the universe curve spacetime, resulting in the effects of gravity. Attempts to describe gravity from a quantum perspective results in a inconsistency due to the fact that gravity is nonrenormalizable.

There are several different types of string theory, and the field is still developing, but principles of all of them include that they involve not point particles, but rather oscillating strings, the modes of which correspond to physical observables. These strings can either be closed loops, or they can end on surfaces called D-branes. The D stands for Dirichlet boundary conditions, since the string is constrained by its ending points on the brane. A string may run from one brane to another or may have both of its ends on the same brane.

String theory accounts for both quantum and gravitational effects. Therefore, in the AdS/QCD correspondence, a string theory including gravity is considered on the higher dimensional ($(d+1)$dimensional) space, anti de-Sitter space, while the theory considered on the lower dimensional (d dimensional) space does not include gravity [5]. The fact that there can be a duality between two different theories in two different spacetimes is based on the holographic principle.

The story behind the holographic principle begins with the question of entropy in a black hole. Black holes were originally thought not to have entropy, since they are exact solutions of Einstein's equations of relativity. An energetic gas has entropy, and if such a gas was to fall into a black hole, the entropy cannot just disappear, since that would violate the second law of thermodynamics. It was thus postulated that a black hole is in fact an object with entropy. It seemed intuitive that the maximum entropy contained in a black hole would be proportional to its volume thus the radius cubed. However, it was found that the maximum entropy is actually proportional to the surface area of its event horizon, and thus the radius squared [2]. This loss of dimension is the inspiration for the holographic principle.

The holographic principle is the idea that the universe can be thought of as information from an $(d+1)$ dimensional space encoded onto an d dimensional space. The d dimensional space is the boundary of the $(d+1)$ dimensional space. For example, if the the $(d+1)$ dimensional space is spherical, the corresponding d dimensional space is the surface of the sphere. This principle enables certain physical theories in one space to be translated into another. This is useful because calculations may differ between the spaces of the two theories, and be considerably easier in one of them.

The AdS/CFT correspondence is one of the most well known realizations of the holographic principle. This correspondence introduces a duality between strongly coupled string theory in anti-de Sitter space and weakly coupled conformal field theories [3][4].

A conformal field theory is one that remains invariant under certain coordinate transformations called conformal transformations. In particular a conformal field theory is invariant under both scale transformations, or dilatations, and under the special conformal transformations. The scaling transformation looks like $x^\mu \rightarrow \lambda x^\mu$. Other conformal transformations involve an inversion: $x^\mu \rightarrow \frac{x^\mu}{x^2}$ and/or a translation: $m^\mu \rightarrow x^\mu - b^\mu$.

Many quantum field theories exhibit conformal symmetry, while the theory governing gravity, general relativity, is not conformally symmetric because is it not scale invariant. $\mathcal{N} = 4$ Yang Mills theory, for example, is a conformal field theory.

The conformal field theory used in Maldecena's duality is also a gauge theory, giving his idea the alternate name the gauge/gravity duality.

A gauge theory is a field theory characterized by a Lagrangian that is invariant under gauge transformations. A gauge transformation is a local symmetry tranformation. The classic example of gauge invariance is in electromagnetism in which the vector field, A^μ, undergoes the gauge transformation $A^\mu \rightarrow A^\mu + \partial^\mu f$, leaving the Lagrangian invariant. The function f can be chosen so as to eliminate one component of the gauage field. This works with any gauge theory, and can be utilized to reduce the degrees of freedom of a problem involving fields.

The gravitational theory involved in the AdS/QCD duality lives in anti-de Sitter space. Anti-de Sitter space is a spacetime with constant negative curvature. Positive curvature is like the boundary of a sphere, while negative curvature resembles a hyperbolic shape. Constant curvature just means that the space has the same degree of curvature everywhere. For example, a sphere has constant positive curvature, while an ellipsoid or egg shape also has positive curvature but it is not constant [5]. Anti-de Sitter space is a solution to Einstein's equations of gravity with the cosmological constant negative. Einstein's theory of general relativity says that the presence of mass or energy curves spacetime, resulting in gravitational effects. A non zero cosmological constant means that spacetime is curved even without the presence of matter or energy. A spacetime with negative curvature, such as anti-de Sitter space, is considered attractive because spacetime is curved in like a bowl.

De Sitter space, on the other hand, has a positive curvature like a sphere and thus is repulsive. The gravitational effects created by mass and energy take place on an antigravitational backdrop. De Sitter space then, is a similar spacetime to that of our universe, which is expanding at an increasing rate despite the gravitational effects acting on its massive constituents. If our universe had a negative curvature like anti-de Sitter space the mass and energy it contains would cause it to collapse in on itself at an increasing rate instead of expanding. Both de-Sitter and anti-de Sitter space are static (not expanding or contracting) despite their curvature because they are empty space, without massive contents.

The hyperboloid nature of anti de-Sitter space is demonstrated by the relation between its dimensional components. A two dimensional hyperbola is described by $y^2 = x^2 + R^2$. AdS_5 is a hyperboloid in 6 dimensions, and is described by

$$-x_o^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 - x_{-1}^2 = -R^2 \tag{1}$$

where x_1, x_2, x_3, and x_4 are spacelike dimensions and x_0 and x_{-1} are timelike dimensions.

Anti-de Sitter space is the simplest of all negatively curved spacetimes. Our model used an anti-de Sitter space in five dimensions, four spatial plus time (AdS_5).

$$\text{metric} : ds^2 = \frac{1}{z^2}(dt^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2) \tag{2}$$

or in short hand:

$$\text{metric} : ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu)[6] \tag{3}$$

where $dx^\mu d_{x\mu} = dt^2 - dx_1^2 - dx_2^2 - dx_3^2$ or $dx^\mu d_{x\mu} = \eta_\mu \nu dx^\mu dx^\nu$. $\eta_{\mu\nu}$ is the Minkowski tensor

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

which functions in the shorthand to

$$x^{\mu} = \eta^{\mu\nu} x_{\nu} \text{ and } x_{\mu} = \eta^{\mu\nu} x^{\nu}. \tag{4}$$

Anti de-Sitter space is infinite, but still has a boundary at $z = 0$. The idea behind the AdS/CFT duality is that a conformal field theory on this d dimensional boundary is completely equivalent to a theory that includes gravity within the $d + 1$ dimensional anti-de Sitter space. An object in the anti-de Sitter space has equivalent particles on the boundary.

In this way, calculations in the boundary theory give insight into the equivalent interior, higher dimensional, gravitational theory and calculations in the high dimensional space correspond to physical truths on in the lower dimensional boundary space. Though there is hope that AdS/CFT correspondence can yield a quantum theory of gravity in the interior AdS, our model uses the reverse approach. A five dimensional spacetime that includes gravity is considered in the interior in order to calculate experimentally determined physical truths in the boundary theory, quantum chromodynamics.

Our Model: AdS/QCD

In our model, the conformal field theory considered is QCD, and the AdS space is AdS_5. The model was built to include some features of QCD, and thus the relationship between the two theories in the two spacetimes is not truly duality, but rather correspondence. The extra dimension in AdS_5 is represented by the extra coordinate z. Thus operators $\mathcal{O}(x)$ in the four dimensional (4D) theory correspond to a field $\phi(x, z)$ in the 5D theory. The fields are $A^a_{L\mu}$, $A^a_{R\mu}$, and X.

The metric for our model is simply a slice of the AdS metric, equation 3, above.

The extra coordinate, z, corresponds to the energy scale, or momentum transfer Q^2, in QCD, where $Q \sim 1/z$. The boundary of Ads_5 considered is not at $z = \infty$, instead the extra dimension z is constrained to the region $0 < z < z_m$ where $z = z_m$ is the infrared (IR) brane, and indicates the location of boundary conditions on the fields. The constraint on z is an analogy to the color confinement of QCD. The ultraviolet (UV) brane is represented by the boundary $z = \epsilon$, with the limit $\epsilon \to 0$. The action of the string theory in the five dimensional (5D) space is

$$S = \int d^5x \sqrt{g} \text{Tr} \left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\} \tag{5}$$

where $D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu}$, $A_{L,R} = A^a_{L,R}t^a$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ [6]. The function X is a nonzero background to the five dimensional space that functions to model the chiral symmetry breaking of QCD while preserving the isospin (a subgroup of the chiral symmetry). The two sets of gauge fields A_L and A_R correspond to the $\text{SU}(2)_L$ and $\text{SU}(2)_R$ symmetry transformations. These transformations correspond to the global symmetries of QCD in four dimensions and the gauge invariance in AdS_5. In QCD the chiral symmetry is broken explicitly by the quark masses and then also broken spontaneously by the chiral condensate. The field X is the bifundamental charged under left and right symmetries and it covers bothtypes of chiral symmetry breaking. X has the form $X(z) = \frac{m_q}{2}z + \frac{\sigma}{2}z^3$ where m_q accounts for the chiral symmetry breaking due to quark mass and σ accounts for the chiral symmetry breaking due to the chiral condensate.

$D^\mu X$ is the gauge covariant derivative, meaning that it transforms like X under gauge transformations. The $|DX|^2$ term in the action makes X dynamical. The $3|X|^2$ term is a mass term. Tr is the trace, where $\text{Tr}|X|^2 = X^i_j X^{\dagger j}_i$, g_5 is the gauge coupling and is fixed by QCD. $g^{\mu\nu} = \frac{z^2}{R^2}\eta^{\mu\nu}$, and $\sqrt{g}$ is $\frac{R^5}{z^5}$.

At the IR brane ($z = z_m$) the boundary conditions are $(F_L)_{z\mu} = (F_R)_{z\mu} = 0$. The gauge of the model is $A_z = 0$.

The holographic correspondence is specifically

$$W_{4D}[\phi_0(x)] = S_{5D,eff}[\phi(x,\epsilon)] \text{ at } \phi(x,\epsilon) = \phi_0(x) \tag{6}$$

at the UV boundary conditions (the boundary $z = \epsilon \to 0$). $S_{5D,eff}[\phi(x,\epsilon)]$ is the effective action of the 5D theory, and $W_{4D}[\phi_0(x)]$ is the generating functional of the connected correlators in the 4D theory [6].

With a vector field $V^\mu = (A_L + A_R)/2$, and the gauge $V_z(x,z) = 0$ we can derive the equation of motion for the transverse part of the gauge field to be:

$$[\partial_z(\frac{1}{z}\partial_z V^a_\mu(q,z)) + \frac{q^2}{z}V^a_\mu(q,z)]_\perp = 0 \tag{7}$$

where $V^a_\mu(q,z)$ is the 4D Fourier transform of $V^a_\mu(x,z)$ [6][7], and the $\perp$ refers to the fact that the transverse part is what is being considered.

The derivation begins with:

$S = -\frac{1}{4}\int d^5x\sqrt{g}F_{MN}F^{MN}$

$= -\frac{1}{4}\int d^5x\sqrt{g}(\partial_M V_N - \partial_N V_M)(\partial^M V^N - \partial^N V^M)$

$= -\frac{1}{4}\int d^5x\sqrt{g}(\partial_M V_N - \partial_N V_M)(\partial_P V_Q - \partial_Q V_P)g^{MP}g^{NQ}$

The Euler-Lagrange equation is

$$\partial_M\left(\frac{\partial\mathcal{L}}{\partial(\partial_M V_N)}\right) - \frac{\partial\mathcal{L}}{\partial V_N} = 0 \tag{8}$$

where the $\frac{\partial\mathcal{L}}{\partial V_N}$ term goes to zero.

We will use the identity $\frac{\partial(\partial_M V_N)}{\partial(\partial_A V_B)} = \delta^A_M\delta^B_N$ where δ^A_M is a delta function and thus equal to one if $M = A$ and equal to zero otherwise.

Inside the parenthesis from the Euler-Lagrange equation:

$\frac{\partial\mathcal{L}}{\partial(\partial_A V_B)} = -\frac{1}{4}\sqrt{g}\left(\delta^A_M\delta^B_N - \delta^A_N\delta^B_M\right)(\partial_P V_Q - \partial_Q V_P)g^{MP}g^{NQ} - \frac{1}{4}\sqrt{g}\left(\partial_M V_N - \partial_N V_M\right)\left(\delta^A_P\delta^B_Q - \delta^A_Q\delta^B_P\right)$

$= -\frac{1}{4}\sqrt{g}\left(g^{AP}g^{BQ} - g^{BP}g^{AQ}\right)(\partial_P V_Q - \partial_Q V_P) - \frac{1}{4}\sqrt{g}\left(\partial^P V^Q - \partial^Q V^P\right)\left(\delta^A_P\delta^B_Q - \delta^A_Q\delta^B_P\right)$

$= -\frac{4}{4}\sqrt{g}\left(\partial^A V^B - \partial^B V^A\right) = -\sqrt{g}\left(\partial_M V_N - \partial_N V_M\right)g^{MA}g^{NB}$

This result then gets plugged into the nonzero term left in the Euler-Lagrange equation:

$0 = \partial_A \left[\frac{\partial \mathcal{L}}{\partial(\partial_A \partial_B)}\right] = -\partial_A \left[\sqrt{g}\left(\partial_M V_N - \partial_N V_M\right) g^{MA} g^{NB}\right]$

in the fifth dimensional AdS: $g_{MN} = \frac{R^2}{z^2}\eta_{MN}$, $\sqrt{g} = \frac{R^5}{z^5}$, $g^{MN} = \frac{z^2}{R^2}\eta^{MN}$

$0 = \partial_A \left[\frac{R^2}{z^2}\left(\partial_M V_N - \partial_N V_M\right)\left(\frac{z^2}{R^2}\eta^{MA}\right)\left(\frac{z^2}{R^2}\eta^{NB}\right)\right]$

$0 = \partial_A \left[\frac{R}{z}\left(\partial_M V_N - \partial_N V_M\right)\eta^{MA}\eta^{NB}\right]$

then, when we make a gauge choice of $V_z = 0$ we get

$0 = \partial_z \left[\frac{R}{z}\left(-\partial_z V_N + \partial_N V_z\right)\eta^{NB}\right] + \frac{R}{z}\partial_\mu \left[\left(\partial^\mu V_N - \partial_N V^\mu\right)\eta^{NB}\right]$

where for the transverse part the $\partial_\mu V^\mu$ term goes to zero.

For four dimensional plane wave solutions of the form $V_\mu(x,z) = e^{(iqx)}\epsilon_\mu(q)\psi_\rho(z)$,

$\partial_\mu \partial^\mu V_\nu = q^2 V_\nu$

yielding the equations of motion:

$-\partial_z \left[\frac{R}{z}\partial_z V_\mu\right] - q^2 \frac{R}{z} V_\mu = 0.$

We then factor out the R from each term, as well as the $e^{(iqx)}\epsilon_\mu(q)$ part of V_μ, to obtain

$-\partial_z \left[\frac{1}{z}\partial_z \psi_\rho\right] - \frac{q^2}{z}\psi_\rho = 0.$

The boundary conditions for the Kaluza Klein modes are

$\psi_\rho(\epsilon) = 0$ and $\psi'_\rho(z_m) = 0$, since $\psi'(z_m) \to F_{Mz}|_{z_m} = 0 = \partial_M A_z - \partial_z A_M$

When the action is calculated on the solution to equation 7, it reduces to [6]

$$S = -\frac{1}{2g_5^2}\int d^4x \left(\frac{1}{z}V_\mu^a \partial_z V^{\mu a}\right)_{z=\epsilon} \tag{9}$$

This V_μ has different boundary conditions from ψ_ρ since V_μ is the solution to the equations of motion satisfying that $V_\mu^a(x,\epsilon)$ is fixed. This corresponds to the ϕ and ϕ_0 functions of equation (6), where $V_\mu^a(x,\epsilon)$ is essentially ϕ and ϕ_0 is its fixed value.

With this action as the generating functional of the connected correlators for the AdS/CFT correspondence, a two-point function for the vector current, J_μ^a, is found to be

$$\int_z e^{iqz}\left\langle J_\mu^a(x) J_\nu^b(0)\right\rangle = \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(Q^2). \tag{10}$$

where

$$\Pi_V(-q^2) = -\frac{1}{g_5^2 Q^2}\frac{\partial_z V(q,z)}{z}|_{z=\epsilon} \tag{11}$$

In equation (10) $\left\langle J_\mu^a(x) J_\nu^b(0)\right\rangle$ is a correlation function describing how the isospin current at point x effects the isospin current at 0, and in both (10) and (11), $Q^2 = -q^2$. From equation (11), a comparison with a perturbative QCD calculation at large Q^2 gives a value for the gauge coupling constant of

$$g_5^2 = \frac{12\pi^2}{N_c} = 4\pi^2 \tag{12}$$

where $N_c = 3$ is the number of quark colors in QCD [6][7].

The normalizable modes of the 5D field correspond to the hadrons of QCD. The boundary condition at the IR brane yields a set of solutions to equation (7) that is a discrete tower of normalizable modes, similar to the solutions to Schrodinger's equation for a particle in a box. These eigenvalues of the solutions are the Kaluza-Klein modes of the meson being investigated, and correspond to the masses seen in experiment.

For the rho meson, the various solutions correspond to different excited states of the meson that have different characteristic masses. A wavefunction the solution to equation (7) for an arbitrary component of V_μ is ψ_ρ with the eigenvalue corresponding to mass: $q^2 = m_\rho^2$. The boundary conditions on the wavefunction are $\psi(\epsilon) = 0$ and $\partial_z\psi_\rho$ and ψ_ρ is normalized by $\int(dz/z)\psi_\rho(z)^2 = 1$. These parameters yield a Green's function of

$$G(q;z,z') = \sum_\rho \frac{\psi_\rho(z)\psi_\rho(z')}{q^2 - m_\rho^2} \tag{13}$$

Reconsidering equation (11), it is the case that $V(q,z') = -\frac{1}{z}\partial_z G(q;z,z')$ at $z = \epsilon$, and the equation is rewritten as

$$\Pi_V(-q^2) = -\frac{1}{g_5^2}\sum_\rho \frac{\left[\psi'_\rho(\epsilon)/\epsilon\right]^2}{(q^2 - m_\rho^2)m_\rho^2} \tag{14}$$

From which we obtain the equation for the decay constants F_ρ:

$$F_\rho^2 = \frac{1}{g_5^2}\left[\psi'_\rho(\epsilon)/\epsilon\right]^2 = \frac{1}{g_5^2}\left[\psi''_\rho(0)\right]^2 \tag{15}$$

where for a rho meson with polarization ε_μ, $\left\langle 0|J_\mu^a|\rho^b\right\rangle = F_\rho \delta^{ab}\varepsilon_\mu$ [6][7].

The decay constant of the rho meson is related to the decay of the particle into charged particles. Most of the time a rho meson decays into two pions, a π^+ and π^-. On occasion, however, the rho meson decays into two electrons or muons through an intermediate photon. This type of decay is what the decay constant describes.

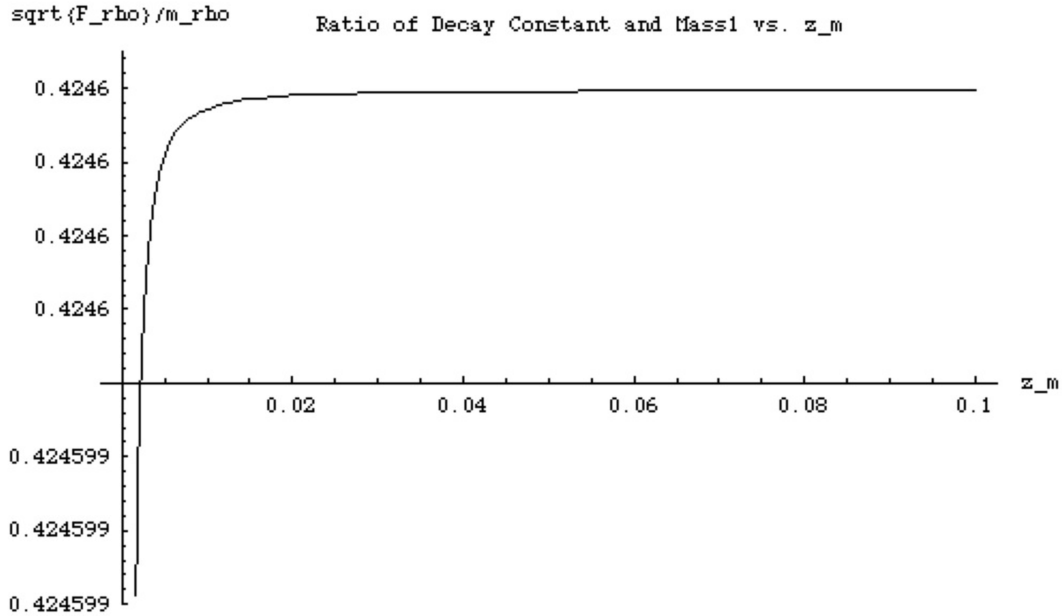
Using this model, the masses and decay constants were calculated for the rho meson. The lowest two masses were calculated, as well as the corresponding decay constants. Considering equation (7) and the Dirichlet boundary conditions, it is found that the solutions $\psi_\rho(z)$ are Bessel functions. The physical masses of the wavefunctions are found by the zeros of $J_0(qz_m)$

Since the tower of wavefunction solutions to equation (7) are produced by the boundary condition of the IR brane, at $z = z_m$, both the masses and decay constants were functions of z_m. For the calculation of observables, z_m is fixed by the value of the lowest rho mass: $m_\rho = 2.405/z_m = 776$ MeV gives $z_m = 1/(323$ MeV). So for the ground state of the rho meson:

Ground State of Rho Meson

Observable	Measured (MeV)	Calculated (MeV)
m_ρ	775.8 ± 0.5[6]	776.8
$F_\rho^{1/2}$	345 ± 8[6]	329.8

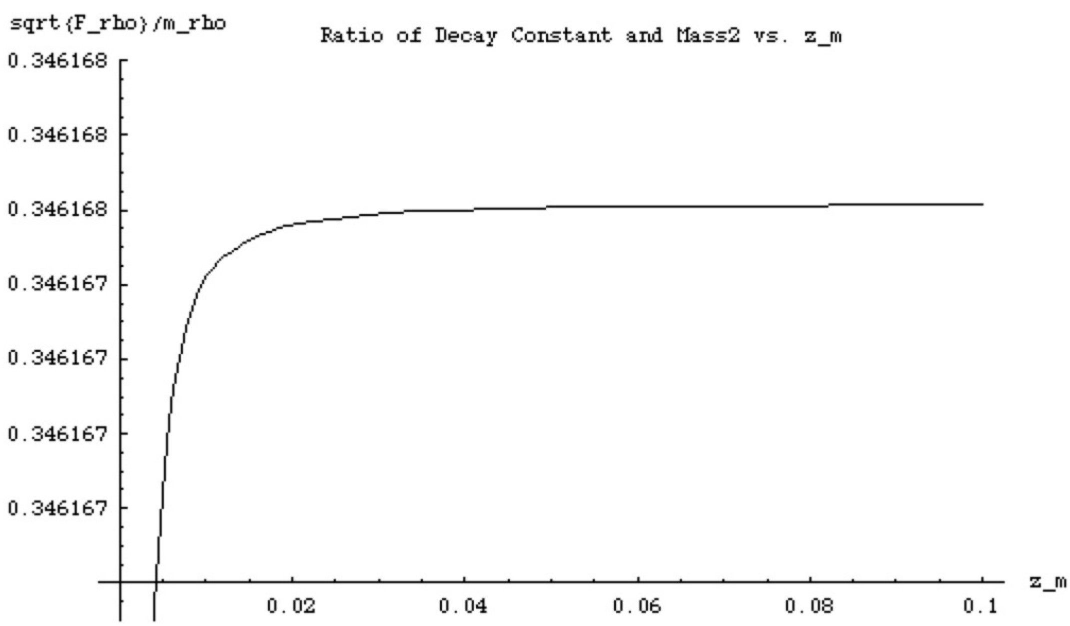
As a way of investigating the performance of the model, the ratio $F_\rho^{1/2}/m_\rho$ was plotted against z_m. The more accurate the model, the closer this plot should have been to a flat line. This plot is for the ground state of the rho meson.



The plot levels off at about 0.4246. The ratio based on measured values is 345/776 = 0.445. The percent difference between the measured value of the ratio of the observables and that predicted by the model is $\frac{0.444588-0.4246}{.444588} = .045$, or 4.5 percent. The mass calculated by the model is .1 percent different from the measured value and the decay constant calculated is 4.4 percent different from the measured value. The plot shows that the ratio is constant to 1 part in 10^6 in the range plotted. The non constant portion on the left of the plot is exaggerated because the scale is so small, but it is the case that as $z_m \to \epsilon$ there should be a deviation from the result for $z_m/\epsilon >> 1$.

The plot shows the ratio not holding for very small values of z_m, since the modes are not clear when $z_m \to \epsilon$.

The same plot was also made for the first excited state of the rho meson.



The model is only supposed to hold for low energies, so it is expected that the values calculated for the second ρ mass and decay constant should not agree as well with physical measurements. Though there is an excepted value of the second ρ mass, there is not one for its decay constant, because unlike the ground state of the rho meson, the first excited state can decay in several ways and is less predictable. This is demonstrated by the fact that the value calculated in the model for the mass of the first excited state of the rho meson is 21.7 percent different from the measured value. Despite this discrepancy, it is still possible to use the ratio found in this plot to predict a value for the decay constant.

The plot is again seen to level off, this time at about 0.346. This yields a value for the decay constant of $F_\rho^{1/2} = 0.346 * 1780 = 617$ MeV.

First Excited State of Rho Meson

Observable	Measured (MeV)	Calculated (MeV)
m_ρ	1465 ± 25	1782.99
$F_\rho^{1/2}$	(none available)	617.209

Conclusions

This paper describes a holographic model of quantum chromodynamics. A five dimensional model has been developed to include characteristics of QCD such as chiral symmetry breaking. This AdS/QCD (anti-de Sitter space/quantum chromodynamics) model is motivated by the AdS/QCD (anti-de Sitter space/conformal field theory) correspondence, which is the conjecture that a field theory in a certain dimensional space can have an equivalent theory that includes gravity in a space with one extra dimension. AdS/QCD utilizes this concept to carry out calculations that are very difficult in four dimensional QCD, but much easier in the corresponding five dimensional theory.

Our model was used to calculate the mass and decay constant of the rho meson, both in its ground state and in its first excited state. We then calculated a dimensionless ratio involving the rho meson decay constant and rho meson mass. This was then plotted against the input parameter that corresponded to the size of the extra dimension. The ratio was found to be within about 4.5 percent of the experimental value. This result is roughly independent of the input parameters and other details, thus it provides a stringent test of our AdS/QCD model.

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