Improving Unification in the Lee-Wick Standard Model

by

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Abstract

Lee-Wick field theories are a category of quantum field theories containing higher-derivative terms in their Lagrangians. Adding higher-derivative terms to the Standard Model yields a model that solves the hierarchy problem, but that is not compatible with gauge coupling unification. The goal of this research is to determine if adding additional particles to the Lee-Wick Standard Model will improve unification. Improved unification was determined by calculating the adjustments to the running of the coupling constants for the $SU(3)$, $SU(2)$, and $U(1)$ gauge groups. We find several solutions in which nearly exact unification occurs. For example, adding two spin-0 $SU(2)$ triplets with $U(1)$ hypercharge $+1$ to the minimal Lee-Wick Standard Model generates a theory which unifies.

Contents

1 Introduction

1.1 The Standard Model

The Standard Model of particle physics describes elementary particles and the forces governing their interactions. The three forces relevant to these subnuclear particles are the strong, weak, and electromagnetic forces. The fourth fundamental force, gravity, is negligible for elementary particles, since their masses are small. The interactions are mediated by force-carrier particles, which are usually massless. For the electromagnetic force, such a particle is the photon. Similarly, the interactions of the weak force can be described with the W and Z bosons and the gluons for the strong force.

The action for elementary particles is required to be gauge invariant. Gauge invariance requires that the fields transform as $\psi \mapsto U\psi$ where

$$U = e^{i\alpha_a T^a}$$

is a unitary matrix, and T^a are hermitian matrices that generate the symmetry group. The commutators of the generators satisfy

$$[T^a, T^b] = if^{abc}T^c \ , \tag{1}$$

where f^{abc} are the structure constants [1]. One group satisfying these properties is the group of all unitary $N \times N$ matrices, denoted $U(N)$. The subgroup of unitary $N \times N$ matrices, such that $\det(U) = 1$, and hence $\mathrm{Tr}\,(T^a) = 0$, is known as the $SU(N)$ symmetry group [1]. The three forces of the Standard Model can be described using gauge groups. The Glashow-Salam-Weinberg theory of electroweak interactions uses the symmetry group $SU(2)_L \otimes U(1)$ to describe both the electromagnetic and weak forces [2]. In order for this theory to be gauge invariant, the presence of the photon, and W and Z bosons is required. This theory alone predicts that the W and Z bosons are massless. Since this is not the case experimentally, it motivates the inclusion of the Higgs mechanism in the Standard Model. The other gauge group included in the Standard Model is $SU(3)$, used to describe the strong force. The theory of quantum chromodynamics is described using this group. Here, local gauge invariance requires that eight massless spin-1 particles, known as gluons, exist [2].

The fermions in the Standard Model have spin-1/2 and are either quarks or leptons. Only quarks are affected by the strong force and transform as triplets under the $SU(3)$ interaction. There are three generations of particles organized by masses; the first generation are the lightest and are the main constituents of ordinary matter. For example, the two first generation quarks, up and down, compose protons and neutrons. Heavier quarks form more massive bound states. The particles of the Standard Model fall into representations of the symmetry groups. These representations are listed in the Table 1 [3].

The $U(1)$ hypercharge, Y, is related to the electromagnetic charge as

$$Q = T_3 + Y \ , \tag{2}$$

where T_3 is the third generator of $SU(2)$ [3]. Also included in the Standard Model are the bosons required for gauge invariance and an additional spin-0 scalar, known as the Higgs Boson. The presence of the Higgs scalar and requiring invariance under $U(1)$ yields the nonzero mass of the W and Z bosons, but not the photon.

The Standard Model is widely accepted as the current model of particle physics, however, there is also much agreement that the Standard Model is not sufficient. There are several problems with the model, and among them is the hierarchy problem (discussed later). The problem concerns extending the current model to much higher energies; the Standard Model works well for the energies encountered in current experiments. We turn now to the Lee-Wick Standard Model, which can be shown to solve this problem.

Particle	$SU(3)_C \times SU(2)_L \times U(1)_Y$
$(u, c, t)_L$	(3,2,1/6)
$(u, c, t)_R$	(3,1,2/3)
$(d, s, b)_L$	(3,2,1/6)
$(d, s, b)_R$	(3,1,-1/3)
$(e^-, \mu^-, \tau^-)_L$	(1,2,-1/2)
$(e^-, \mu^-, \tau^-)_R$	(1,1,-1)
$(\nu_e, \nu_\mu, \nu_\tau)_L$	(1,2,-1/2)
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	(1,2,1/2)

Table 1: Listing of the particles in the Standard Model with their quantum numbers. Those particles which interact under the strong force transform as a triplet under the $SU(3)$ group. Note the quarks form doublets by generation, i.e. the up quark and down quark transform as a doublet under $SU(2)$. The second and third generation quarks form the analagous doublet. Similarly, the left-handed leptons doublets consist of the electron, muon, or tauon and the corresponding neutrino.

1.2 Lee-Wick Standard Model

In the Lee-Wick extension of the Standard Model, higher-derivative terms are introduced in the Lagrangian for each particle. In order to accommodate the higher derivatives, a mathematically equivalent approach is to introduce a partner state. Therefore, each left- and right-handed fermion in the Standard Model has a Dirac partner particle with the same charges, although the partner particle has a much higher mass. It can also be shown that each scalar will have one scalar partner particle [4].

The Lee-Wick partner particles are 'ghosts', particles with wrong-sign kinetic and mass terms. The Lagrangian for theories with ghosts is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}m^2\phi^2 + interactions. \tag{3}$$

To address the problem with wrong-sign kinetic terms, the states in Lee-Wick theories can have negative norms [5]. This assures that the expectation value of the Hamiltonian is positive and the theory is stable. If the Lee-Wick particles decay, it can be shown that the S matrix of the theory is unitary. It appears that this approach leads to a quantum theory that violates causality at a microscopic level because the unitary quantum theory requires the presence of future boundary conditions. However, causality is not violated in macroscopic measurements [4].

The Lee-Wick Standard Model can then be shown to solve the hierarchy problem. In the Standard Model, the mass of the Higgs boson is given by

$$m_H^2 = m_{H_0}^2 + \Delta m^2 \ , \tag{4}$$

where $m_{H_0}^2$ is the mass appearing in the Lagrangian and

$$\Delta m^2 \propto \frac{\Lambda^2}{16\pi^2} \tag{5}$$

represents quantum corrections. The energy scale, Λ, is that of the heaviest particles in the theory. It follows that m_H^2 may receive corrections as large as $M_{Pl}^2/16\pi^2$, where M_{Pl} is the Planck scale. The Planck scale is about 10^{19} GeV and is the energy at which quantum gravity begins to affect elementary particles. However, we want m_H to be on the order of 100 GeV, corresponding to the W and Z boson masses. Thus $m_{H_0}^2$ must be fine-tuned to cancel the quantum corrections. The quadratic divergence in Eq. (5) is one the primary reasons the Standard Model is not a satisfactory explanation of particle interactions.

In the Lee-Wick Standard Model, an additional parameter is introduced representing the masses of the partner particles. The presence of these new degrees of freedom changes the quantum corrections:

$$\Delta m^2 \propto \frac{m_{LW}^2}{16\pi^2} \ln \frac{\Lambda^2}{m_{LW}^2} \tag{6}$$

where m_{LW} represents the mass scale of the Lee-Wick particles. Having a logarithmic divergence in Λ, the quantum corrections are not large, even when $\Lambda \approx M_{Pl}$. So, when

$m_{LW} \approx 100$ GeV, the mass of the Higgs is of the order of the mass of the W and Z bosons without great fine-tuning [4]. In this way, the Lee-Wick Standard Model solves the hierarchy problem.

1.3 Unification

It has been suggested that the three symmetry groups of the Standard Model are subgroups of a larger symmetry group governing these interactions. Such a theory, known as a Grand Unified Theory (GUT), proposes that at some energy higher than observed, the strong, weak, and electromagnetic forces can be represented by one force and single symmetry group, G. Each symmetry group in the Standard Model has a coupling strength, denoted as

$$\alpha_i = \frac{g_i^2}{4\pi} \tag{7}$$

where g_i is the gauge coupling. These couplings are analogous to the fine structure constant in electromagnetism. Their strengths depend logarithmically on energy. As it stands there is no point on the energy scale where the three values for the gauge coupling are the same. Thus, there could be no single symmetry group describing all three forces. Figure 1 shows the coupling constants of the Standard Model varying logarithmically with energy. The existence of a such a unified group would have some energy, noted here as M_{gut}, where these coupling constants converge to a single value, α_{gut}. Below this value the symmetry of the group is broken spontaneously.

Suggestions for unified groups include SU(5) and SO(10) [3]. Both of these symmetry groups require additional fermions and gauge bosons to preserve the invariance of the Lagrangian. They also rely on the Higgs mechanism for breaking the electroweak symmetry and giving mass to the fermions. The theory of supersymmetry has also been proposed in the context of grand unification. Supersymmetry is an appealing theory because it solves the hierarchy problem while also predicting accurate unification of the gauge couplings [3].

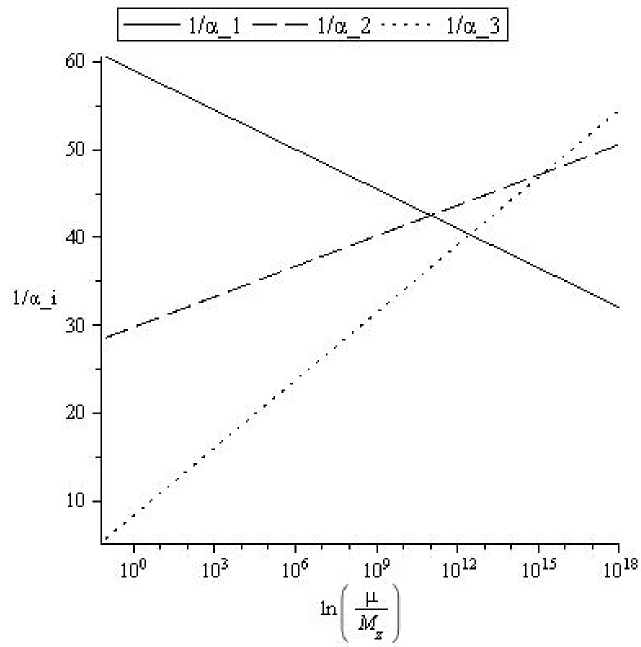

Figure 1: The logarithmic running of the inverse of the coupling coefficients in the Standard Model. Here μ is the energy scale and $M_z = 91.1876$ GeV is the mass of the Z boson [3].

The running of the gauge coupling coefficients depend on the particle content of the theory. Therefore, adding particles would change the running of the couplings and affect the unification of the theory. For example, it has been shown that adding six Higgs doublets to the Standard Model creates a theory that unifies [5]. The gauge couplings in the Lee-Wick Standard Model do not appear to unify naturally [5]. The approach of this paper is to use the Lee-Wick Standard Model as a starting point and add more particles. Since the couplings depend on particle content, additional particles will affect the running of the $SU(3)$, $SU(2)$, and $U(1)$ gauge couplings. Adjusting these values can improve gauge coupling unification so that the theory is an appealing alternative to supersymmetry.

2 Calculations

To determine the degree to which a model unifies, the dependence of the gauge coupling on energy scale is needed. The usual gauge coupling constants are g_3, g_2, g_1 for the groups $SU(3)$, $SU(2)$, $U(1)$ respectively. Usually the inverse of α_i as defined in Eq. (7) is considered, which scales logarithmically with energy. The values for these coefficients at measurable energies are taken to be $\alpha_1^{-1} = 59.00 \pm 0.01$, $\alpha_2^{-1} = 29.57 \pm 0.02$, and $\alpha_3^{-1} = 8.2169 \pm 0.1148$ where α_1^{-1} is GUT normalized.

The inverse of the coefficients scale according to the energy, μ, as

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_z) - \frac{b_i}{4\pi} \ln \frac{\mu^2}{M_z^2} \ , \tag{8}$$

where b_i are the beta functions and $M_z = 91.1876$ GeV is the Z boson mass. Beta functions depend on the particles in the theory as well as the gauge group and the representations of the particles. For the gauge groups of the Standard Model, the beta functions are each given by

$$b = -\frac{11}{3} C_2(G) + \frac{2}{3} n_\chi C(r) \ . \tag{9}$$

Here n_χ is the number of chiral fermions in the model, $C_2(G)$ is a Casimir operator that depends on the Lie group G, and $C(r)$ is another Casimir operator that depends on the representation of the particles in the theory [1].

For the groups $SU(N)$, the smallest representations of interest to us are the fundamental representation and the adjoint representation. The fundamental representation of $SU(N)$ can be thought of as an N-dimensional vector, on which the group element U acts. In the fundamental representation, the T^a are $N \times N$ matrices that satisfy

$$\mathrm{Tr}\, T^a T^b = \frac{\delta_{ab}}{2} \ . \tag{10}$$

The Casimir, $C(r)$, is defined by

$$\mathrm{Tr}\, T^a T^b = C(r) \delta_{ab} \ , \tag{11}$$

so that

$$C(r) = 1/2 \tag{12}$$

in the fundamental representation. The Casmir, $C_2(G)$, is defined from the structure constants of G, by

$$f^{acd} f^{bcd} = C_2(G)\delta_{ab} \ . \tag{13}$$

For $SU(N)$, $C_2(G) = N$. For the U(1) symmetry group, the values are

$$C_2(G) = 0 \tag{14}$$

and

$$C(r) = \mathrm{Tr}\, Q^2 \ , \tag{15}$$

where Q is the diagonal charge matrix for all the particles in the model. Therefore, $C(r)$ is the sum of squares of the $U(1)$ hypercharges.

Particles can also be in the adjoint representation. The structure constants of a group belong to this representation. For the $SU(N)$ groups, the dimension of this representation is $N^2 - 1$. The adjoint representation of $SU(2)$ is the familiar spin triplet of quantum mechanics. For a general $SU(N)$,

$$C(r_{adj}) = C_2(G) = N \ . \tag{16}$$

We do not consider an adjoint representation of $U(1)$ because the structure constants vanish for an Abelian group [1].

When scalars contribute to the beta functions, Eq. (9) is modified such that $\frac{2}{3} n_\chi C(r) \mapsto \frac{1}{3} n_s C(r)$ where n_s is the number of complex scalars. The GUT normalization of α_1^{-1} requires b_1 be such that $b_1^{GUT} = \frac{3}{5} b_1$. For example, the beta functions for the Standard Model are

$$b_3 = -\frac{11}{3}(3) + \frac{2}{3}(12)\left(\frac{1}{2}\right) = -7$$
$$b_2 = -\frac{11}{3}(2) + \frac{2}{3}(12)\left(\frac{1}{2}\right) + \frac{1}{3}(1)\left(\frac{1}{2}\right) = -\frac{19}{6}$$

$$b_1 = \frac{3}{5} \cdot \frac{2}{3} \cdot [18 \left(\frac{1}{6}\right)^2 + 9 \left(\frac{2}{3}\right)^2 + 9 \left(\frac{1}{3}\right)^2 + 6 \left(\frac{1}{2}\right)^2 + 3(1)^2] + \frac{3}{5} \cdot \frac{1}{3} \cdot [2 \left(\frac{1}{2}\right)] = \frac{41}{10} \, .$$

Note that there are 12 chiral fermions in b_3 because 6 Dirac fermions, each with a left-handed and right-handed component, are affected by the strong force. Also, $SU(2)$ only acts on the left-handed fermions, and n_χ corresponds to the number of doublets that transform under $SU(2)$.

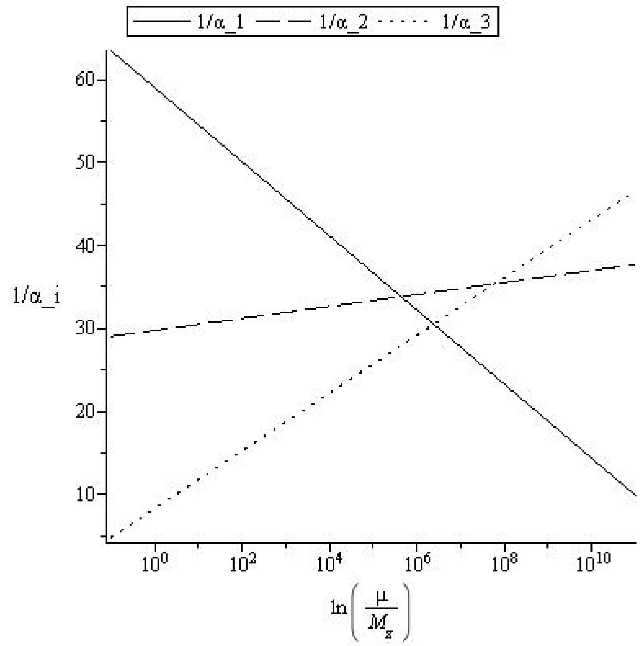

Figure 2: The coupling constants in the Lee-Wick extension of the Standard Model. Each coefficient scales logarithmically with energy. There is no point at which all gauge couplings are the same. In our approach, we take the energy at the intersection of α_1^{-1} and α_2^{-1} as M_{gut} and predict the value for $\alpha_3^{-1}(M_z)$; the minimal Lee-Wick Standard Model predicts a value that is 50.6 standard deviations from the experimental value.

The form of the higher-derivative terms in the Lee-Wick theory requires the addition of a Dirac partner for each chiral fermion and a scalar partner particle for each scalar [4]. This affects the n_χ and n_s terms in the beta functions such that $n_\chi \mapsto 3n_\chi$ and $n_s \mapsto 2n_s$when we implement the Lee-Wick extension [5]. A modification to the gauge boson interactions in the Lee-Wick theory requires, in addition,

Added Particle	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
None	$(-19/2, -2, 61/5)$	4×10^7	14.03	$+50.6\sigma$
$(1, 1, -1/2)_L$, $(1, 1, -1/2)_R$	$(-19/2, -2, 64/5)$	2×10^7	14.66	$+56.1\sigma$
$(1, 2, -1/2)_L$, $(1, 2, -1/2)_R$	$(-19/2, 0, 67/5)$	9×10^7	8.71	$+4.26\sigma$
$(1, 3, -1/2)_L$, $(1, 3, -1/2)_R$	$(-19/2, 6, 14)$	10×10^{11}	-27.45	-311σ
$(3, 1, -1/2)_L$, $(3, 1, -1/2)_R$	$(-15/2, -2, 67/5)$	1×10^7	19.06	$+94.5\sigma$
$(3, 2, -1/2)_L$, $(3, 2, -1/2)_R$	$(-11/2, 4, 79/5)$	6×10^8	5.88	-20.4σ
$(8, 1, -1/2)_L$, $(8, 1, -1/2)_R$	$(5/2, -2, 17)$	2×10^6	36.54	$+246\sigma$
$(8, 2, -1/2)_L$, $(8, 2, -1/2)_R$	$(29/2, 14, 109/5)$	2×10^{12}	31.46	$+202\sigma$

Table 2: The columns show the beta functions, the energy where unification occurs, the predicted value for α_3^{-1} and the deviation of the prediction from the experimental value of 8.2169 ± 0.1148. The first line gives the minimal LWSM result. Assuming that each added fermion has a $U(1)$ hypercharge of $-1/2$, there are finitely many combinations of $SU(3)$ and $SU(2)$ charges for fermions in the fundamental and adjoint representations. Particles that transform as triplets or octets under the $SU(3)$ group tend to have larger errors than those that don't, so we concentrate later on particles that are $SU(3)$ singlets. For two cases, adding fermions transforming as $SU(2)$ triplets produced a theory that did not even unify; thus, we consider fermions that are $SU(2)$ singlets and doublets.

that $-\frac{11}{3}C_2(G) \mapsto -\frac{43}{6}C_2(G)$ [6]. The beta functions for the LWSM are

$$b_3 = -\frac{43}{6}(3) + \frac{2}{3}(12 \cdot 3)\left(\frac{1}{2}\right) = -\frac{19}{2}$$

$$b_2 = -\frac{43}{6}(2) + \frac{2}{3}(12 \cdot 3)\left(\frac{1}{2}\right) + \frac{1}{3})(1 \cdot 2)\left(\frac{1}{2}\right) = -2$$

$$b_1 = \frac{3}{5} \cdot \frac{2}{3} \cdot 3 \cdot [18\left(\frac{1}{6}\right)^2 + 9\left(\frac{2}{3}\right)^2 + 9\left(\frac{1}{3}\right)^2 + 6\left(\frac{1}{2}\right)^2 + 3(1)^2] + \frac{3}{5} \cdot \frac{1}{3} \cdot 2 \cdot [2\left(\frac{1}{2}\right)] = \frac{61}{5}\ .$$

Upon determining the beta functions for a particular theory, the degree to which the theory unifies can be quantified. First, define the energy where unification occurs,

Added Particle	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
None	$(-19/2, -2, 61/5)$	4×10^7	14.03	$+50.6\sigma$
$(1, 2, -1/2)_L$, $(1, 2, -1/2)_R$	$(-19/2, 0, 67/5)$	9×10^7	8.71	$+4.26\sigma$
$(1, 2, -1/3)_L$, $(1, 2, -1/3)_R$	$(-19/2, 0, 191/15)$	2×10^8	7.61	-5.26σ
$(1, 2, -1/4)_L$, $(1, 2, -1/4)_R$	$(-19/2, 0, 25/2)$	2×10^8	7.20	-8.83σ
$(1, 2, -5/12)_L$, $(1, 2, -5/12)_R$	$(-19/2, 0, 391/30)$	1×10^8	8.19	-0.854σ
$(1, 2, -4/9)_L$, $(1, 2, -4/9)_R$	$(-19/2, 0, 355/27)$	1×10^8	8.31	$+0.777\sigma$

Table 3: Adding color-singlet fermion doublets with rational $U(1)$ hypercharge. The best results was obtained with hypercharge $-4/9$. The result for hypercharge $-1/2$ is included for reference.

M_{gut}, such that

$$\alpha_1^{-1}(M_{gut}) = \alpha_2^{-1}(M_{gut}) \; . \tag{17}$$

Assuming that $\alpha_3^{-1}(M_{gut})$ is equal to the other coupling coefficients at this energy, Eq. (8) can be used to predict the value for $\alpha_3^{-1}(M_z)$, because that is the scale at which the α_i^{-1} are accurately measured. Comparing our predicted $\alpha_3^{-1}(M_z)$ with the experimental value indicates the degree of unification. We compute the deviation,

$$\Delta\alpha_3^{-1} \equiv \frac{\alpha_3^{-1}(M_z)_{th} - \alpha_3^{-1}(M_z)_{ex}}{\sigma} \tag{18}$$

where σ is one-standard deviation experimental uncertainty in $\alpha_3^{-1}(M_z)$. For the Standard Model, the result for $\Delta\alpha_3^{-1}$ is $+50.8\sigma$ where $\sigma = .1148$. The Lee-Wick Standard Model has a similar deviation, $+50.6\sigma$, and clearly does not unify (see Figure 2). Although, both the Lee-Wick Standard Model and the Standard Model don't unify, the Lee-Wick model is more appealing to work with, given that it solves the hierarchy problem. Now we determine if adding particles to the Lee-Wick Standard Model improves unification.

Added Particle	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
None	$(-19/2, -2, 61/5)$	4×10^7	14.03	$+50.6\sigma$
$(1, 1, 1/2)$	$(-19/2, -2, 123/10)$	4×10^7	14.14	$+51.6\sigma$
$(1, 2, 1/2)$	$(-19/2, -5/3, 62/5)$	5×10^7	13.18	$+43.3\sigma$
$(1, 3, 1/2)$	$(-19/2, -2/3, 25/2)$	1×10^8	9.83	$+14.0\sigma$
$(3, 1, 1/2)$	$(-55/6, -2, 25/2)$	3×10^7	15.02	$+59.0\sigma$
$(3, 2, 1/2)$	$(-53/6, -1, 64/5)$	6×10^7	12.87	$+40.5\sigma$
$(3, 3, 1/2)$	$(-17/2, 2, 131/10)$	2×10^9	1.73	-56.5σ
$(8, 1, 1/2)$	$(-15/2, -2, 13)$	2×10^7	18.78	$+92.0\sigma$
$(8, 2, 1/2)$	$(-11/2, 2/3, 69/5)$	2×10^8	15.75	$+65.6\sigma$
$(8, 3, 1/2)$	$(-7/2, 26/3, 73/5)$	3×10^{15}	-30.78	-340σ

Table 4: Scalars with $U(1)$ hypercharge 1/2 in the fundamental and adjoint representations. Scalars that transform as octets under $SU(3)$ don't unify well so we don't consider them in later combinations.

3 Results

Using the methods outlined above to determine the degree to which a theory unifies, we add particles to the Lee-Wick Standard Model. The tables contain quantum numbers for the added particles, adjusted beta functions, energy scale of unification, predicted coupling constant, and $\Delta\alpha_3^{-1}$. Since $SU(N)$ has an adjoint representation, particles can be represented in an 8-dimensional representation in $SU(3)$ and a 3-dimensional one in $SU(2)$. Taking into account the 1, N, and $N^2 - 1$ dimensional representations of $SU(N)$, Table 2 contains possible combinations of these $SU(2)$ and $SU(3)$ representations with hypercharge $-1/2$ for fermions. We use a $U(1)$ hypercharge of $-1/2$ because this charge appears in the Standard Model for leptons; however, since the α_1^{-1} gauge coupling depends on the square of the $U(1)$ hypercharge, the unification for a particle with hypercharge $+1/2$ would be the same. Note that

# : Added Particle	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
None	$(-19/2, -2, 61/5)$	4×10^7	14.03	$+50.6\sigma$
1: $(1,2,1/2)$	$(-19/2, -5/3, 62/5)$	5×10^7	13.18	$+43.3\sigma$
6: $(1,2,1/2)$	$(-19/2, 0, 67/5)$	9×10^7	8.71	$+4.26\sigma$
7: $(1,2,1/2)$	$(-19/2, 1/3, 68/5)$	1×10^8	7.76	-4.01σ
1: $(1,2,1/3)$	$(-19/2, -5/3, 553/45)$	5×10^7	13.05	$+42.1\sigma$
1: $(1,2,1/4)$	$(-19/2, -5/3, 49/4)$	5×10^7	13.01	$+41.7\sigma$
5: $(1,2,1/4)$	$(-19/2, -1/3, 249/20)$	2×10^8	8.47	$+2.18\sigma$
5: $(1,2,1/6)$	$(-19/2, -1/3, 554/45)$	2×10^8	8.24	$+0.157\sigma$

Table 5: Adding scalar $SU(2)$ doublets to LWSM. The number scalar doublets with hypercharge $+1/2$ that improves unification is seven, which is similar to the number needed to improve unification in the Standard Model [5]. Dropping the stipulation that the $U(1)$ hypercharge be 1/2, we can improve unification even more. Unification is greatly improved by adding five doublets with hypercharge $+1/6$.

the fermions with quantum numbers $(3,3,-1/2)_L$, $(3,3,-1/2)_R$ and $(8,3,-1/2)_L$, $(8,3,-1/2)_R$ have $SU(2)$ and $U(1)$ gauge couplings that do not meet.

According to Table 2, the added fermions with quantum numbers $(1,2,-1/2)$ improve unification the most the most (to within 4.26σ). This could be improved by changing the $U(1)$ hypercharge of the added $SU(2)$ doublet. However, we require that these exotic hypercharges remain rational. We see in Table 3 that changing the $U(1)$ hypercharge to $-4/9$ improves unification.

We can also add scalars to the Lee-Wick Standard Model. Once again, we consider the fundamental and adjoint representations for $SU(2)$ and $SU(3)$. The scalar particles with $U(1)$ hypercharge of $+1/2$ are located in Table 4. Here we see three scalars improving unification: $(1,2,1/2)$, $(1,3,1/2)$, $(3,2,1/2)$. The first of these has identical quantum numbers to the Higgs doublet that is part of the Standard Model.

# : Added Particle	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
None	$(-19/2, -2, 61/5)$	4×10^7	14.03	$+50.6\sigma$
1: $(1, 3, 1/2)$	$(-19/2, -2/3, 25/2)$	1×10^8	9.83	$+14.01\sigma$
2: $(1, 3, 1)$	$(-19/2, 2/3, 73/5)$	5×10^7	8.09	-1.05σ
1: $(3, 2, 1/2)$	$(-53/6, -1, 64/5)$	6×10^7	12.87	$+40.5\sigma$
5: $(3, 2, 1/2)$	$(-37/6, 3, 76/5)$	3×10^8	7.46	-6.61σ
1: $(3, 2, 1/3)$	$(-53/6, -1, 187/15)$	8×10^7	12.45	$+36.9\sigma$
3: $(3, 2, 1/3)$	$(-15/2, 1, 13)$	4×10^8	8.72	$+4.42\sigma$
3: $(3, 2, 103/384)$	$(-15/2, 1, 52093/4096)$	7×10^8	8.22	$+0.0485\sigma$

Table 6: Varied multiplicity and hypercharge for scalars with quantum numbers $(1, 3, q)$ and $(3, 2, q)$ where q is the $U(1)$ hypercharge. The first corresponds to scalar triplet under the $SU(2)$ transformation and is color invariant, while the latter is an $SU(2)$ doublet with color. We find lowest $\Delta\alpha_3^{-1}$ occurs with three doublets with color and $U(1)$ hypercharge of $+103/384$. Although, this case satisfies our requirement that the hypercharge is rational, it is not a liely theory given that it has a complicated hypercharge. Since the particles in the Standard Model have much simpler hypercharges, a result with a higher magnitude $\Delta\alpha_3^{-1}$ but simpler particle content is preferred, i.e. two particles with quantum numbers $(1, 3, 1)$.

Since adding more Higgs doublets improves unification in the Standard Model, it is not surprising that adding Higgs doublets improves unification in the Lee-Wick Standard Model. A model with six or seven Higgs doublets both unify within 5σ. These results, along with scalar doublets with exotic $U(1)$ hypercharge are included in Table 5. Varying both charges and multiplicities, the best result is five scalar doublets with hypercharge 1/6. We consider the other two scalars that unified well in Table 6: the $SU(2)$ triplet and the doublet with color. Unification was improved by varying both the number of these scalars added to the model and by changing the $U(1)$ hypercharge. Here we acheive a nice result adding two $SU(2)$ triplets with $U(1)$

# : Added Particle	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
None	$(-19/2, -2, 61/5)$	4×10^7	14.03	$+50.6\sigma$
$(3, 2, 1/6)_L$, $(3, 1, 2/3)_R$, $(3, 1, -1/3)_R$, $(1, 2, -1/2)_L$, $(1, 1, -1)_R$	$(-11/2, 2, 81/5)$	4×10^7	14.03	$+50.6\sigma$
$(1, 2, -1/2)_L$, $(1, 1, -1)_R$ $(1, 2, -1/2)_R$, $(1, 1, -1)_L$	$(-19/2, 0, 79/5)$	1×10^7	11.88	$+31.9\sigma$
$(1, 2, -1/3)_L$, $(1, 1, -2/3)_R$ $(1, 2, -1/3)_R$, $(1, 1, -2/3)_L$	$(-19/2, 0, 69/5)$	6×10^7	9.31	$+9.53\sigma$
$(1, 2, -1/4)_L$, $(1, 1, -1/2)_R$ $(1, 2, -1/4)_R$, $(1, 1, -1/2)_L$	$(-19/2, 0, 131/10)$	1×10^8	8.23	$+0.097\sigma$

Table 7: Added combinations of fermions. The first group corresponds to adding a generation of fermions to the Standard Model. The remaining combinations are color-singlet fermions with a variety of $U(1)$ hypercharges.

hypercharge of $+1$. The resulting theory unifies with $\Delta\alpha_3^{-1} = -1.05\sigma$.

Instead of adding only multiplicities of the same particle, we can also add different combinations of particles. Table 7 adds combinations of fermions and their Lee-Wick partners similar to the combinations of fermions in Standard Model. We try adding an entire generation of the fermions in the Standard Model, and see that unification remains exactly the same as it does in the original Lee-Wick Standard Model. We also add combinations of color-singlet fermion doublets and singlets, because these improve unification (See Table 3). We use a doublet with half the hypercharge of the singlet because that is how the leptons appear in the Standard Model. Although adding these particles for with hypercharges $-1/2$ and -1 has little effect, adding these fermions with exotic $U(1)$ hypercharges improves the result. In fact, particles with hypercharges $-1/4$ and $-1/2$ improve unification dramatically

# : Added Particle	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
1: $(1,2,-1/2)_L$, $(1,2,-1/2)_R$				
1: $(1,2,1/2)$	$(-19/2, 1/3, 68/5)$	1×10^8	7.76	-4.01σ
1: $(1,3,1/2)$, 2: $(1,2,1/2)$	$(-19/2, 0, 129/10)$	2×10^8	7.90	-2.79σ
2:$(1,2,-1/2)_L$, $(1,1,-1)_R$				
2:$(1,2,-1/2)_L$, $(1,1,-1)_R$				
3:$(1,2,1/2)$	$(-19/2, 3, 20)$	5×10^6	7.93	-2.49σ
1: $(1,2,2/3)$, 1: $(1,3,2/3)$				
1: $(3,2,1/3)$	$(-53/6, 2/3, 619/45)$	1×10^8	8.21	-0.061σ
1: $(1,2,-1/2)_L$, $(1,2,-1/2)_R$				
1: $(1,2,3/4)$	$(-19/2, 1/3, 277/20)$	8×10^7	8.16	-0.494σ

Table 8: Combinations of fermions and scalars added to the LWSM. Many good results contain added color-singlet fermions and Higgs-like doublets.

(with $\Delta\alpha_3^{-1} = 0.097\sigma$).

Next, we add both fermions and scalars to the LWSM and list the best results in Table 8. The best combinations are multiple scalar particles and color-singelet fermions. Adding a scalar singlet without color, $(1,1,q)$, changes only the first beta function. Therefore, it has a small (positive) effect on the error. So we can add a scalar of this form with a carefully chosen $U(1)$ hypercharge to improve $\Delta\alpha_3^{-1}$ by a small amount. We add these particles to theories with small negative error from Table 7 and Table 8. These results are located in Table 9.

Although we produce extremely low values for $\Delta\alpha_3^{-1}$, quantum corrections in the running of the gauge couplings have not been considered. Therefore, we consider any theory unifying with $|\Delta\alpha_3^{-1}| \leq 2\sigma$ to unify well. There are many results satisfying these critieria, but the most appealing are those whose particle content is intuitive or simple (i.e. not having extremely exotic hypercharge). One such appealing theory

# : Added Particle	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
1: $(1,3,1/2)$, 2: $(1,2,1/2)$				
1: $(1,1,7/10)$	$(-19/2, 0, \frac{1637}{125})$	1×10^8	8.2215	$+4 \times 10^{-2}\sigma$
7: $(1,2,1/2)$, 2: $(1,1,3/5)$	$(-19/2, 1/3, 68/5)$	8×10^7	8.2201	$+3 \times 10^{-2}\sigma$
1: $(1,2,2/3)$, 1: $(1,3,2/3)$				
1: $(3,2,1/3)$, 1: $(1,1,1/10)$	$(-53/6, 2/3, \frac{30959}{2250})$	1×10^8	8.2164	$-4 \times 10^{-3}\sigma$
2: $(1,3,1)$, 1: $(1,1,4/9)$	$(-19/2, 2/3, \frac{1189}{81})$	5×10^7	8.2174	$+4 \times 10^{-3}\sigma$
1: $(1,2,-1/2)_L$, $(1,2,-1/2)_R$				
1: $(1,2,3/4)$, 1: $(1,1,3/10)$	$(-19/2, 1/3, \frac{6943}{500})$	8×10^7	8.2170	$+8 \times 10^{-4}\sigma$

Table 9: Adding a singlet scalar with an arbitrary $U(1)$ hypercharge allowed us to change α_3^{-1} by small amounts.

is the color-singlet fermion with hypercharge $-4/9$ from Table 3. This theory has a somewhat unusual hypercharge, $-4/9$, but several fermions in the Standard Model have $U(1)$ hypercharge that is a multiple of $1/3$. Another theory adding fermions is located in Table 7, which adds a pair of $SU(2)$ doublets with hypercharge $-1/4$ and a pair of $SU(2)$ singlets with hypercharge $-1/2$. These particles look similar to the leptons of the standard model, have simple hypercharges, and unifies well, making it an appealing theory to consider. Other promising theories add scalars; one adds five doublets with hypercharge $1/6$ and another adds two triplets with $U(1)$ hypercharge 1. The improved unification for the two scalars with quantum numbers $(1,3,1)$ is illustrated in Figure 3.

4 Conclusion

The Lee-Wick extension of the Standard Model introduces additional particles for each particle in the Standard Model, which originate from higher-derivative terms

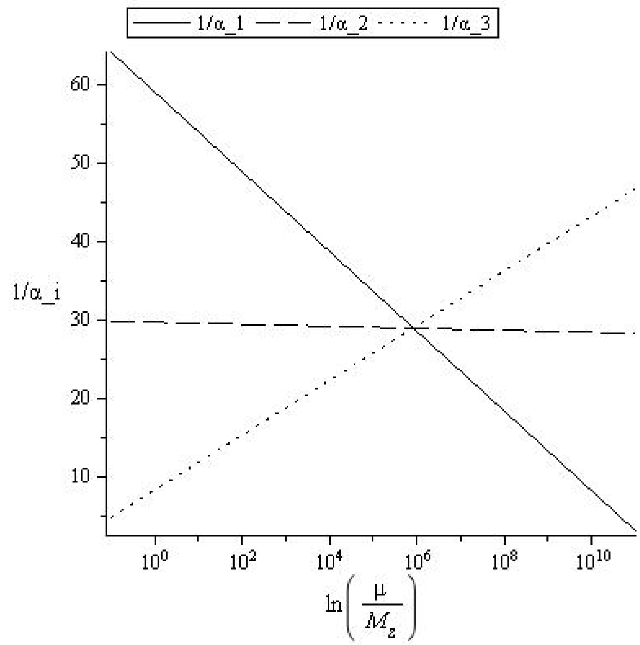

Figure 3: The running of the coupling coefficients the Lee-Wick Standard Model with added particles. The theory shown adds two scalars with quantum numbers $(1,3,1)$. This was one of several theories producing low values for $\Delta\alpha_3^{-1}$.

in the Lagrangian. Because this model solves the hierarchy problem, it is preferable to work with compared to the Standard Model. Supersymmetry is an appealing theory because, in addition to solving the hierarchy problem, it unifies. Thus, a Lee-Wick Standard Model that unifies would be an alternative to supersymmetry. By adding particles, in this case combinations of scalars and color-singlet fermions with different hypercharges, we can improve the unification of the Lee-Wick Standard Model to values better than the minimal supersymmetric model. A comparison of the unification of various models is located in Table 10. Notice that the energy of unification, M_{gut} is lower for the Lee-Wick models, a trend which we see in all Lee-Wick results producing good unification.

A variety of particle combinations produce results that unified nicely. We emphasize those theories which obtained low values for $\Delta\alpha_3^{-1}$ and which looked simple.

Theory	(b_3, b_2, b_1)	M_{GUT} (GeV)	$\alpha_3^{-1}(M_z)$	$\Delta\alpha_3^{-1}$
SM	$(-7, -19/6, 41/10)$	1×10^{13}	14.04	$+50.8\sigma$
MSSM	$(-3, 1, 33/5)$	2×10^{16}	8.55	$+2.9\sigma$
LWSM	$(-19/2, -2, 61/5)$	4×10^{7}	14.03	$+50.6\sigma$
LWSM + two $(1,3,1)$ scalars	$(-19/2, 2/3, 73/5)$	5×10^{7}	8.09	-1.05σ

Table 10: A comparison of unification among different theories [6]. Although the unification of the Standard Model can be achieved by adding more elementary particles, the fact that the LWSM solves the hierarchy problem as well, gives a compelling reason for adding particles instead to the LWSM. We have found several combinations of elementary particles which allow the LWSM to unify even better than the supersymmetric standard model. The table gives one example.

Fermions with hypercharge $-4/9$ produces a theory which unifies within 1σ. Although the $U(1)$ hypercharge is somewhat exotic, it requires only the addition of a left-handed and right-handed fermion to improve unification. Another appealing theory is shown in Figure 3. The theory adds two scalar triplets with hypercharge $+1$. This adds only a few particles, unifies nicely, and has simple hypercharge. Using Eq. (2), the particles in the $SU(2)$ triplet have electromagnetic charge 0,1, and 2, which would evade bounds on exotic stable, fractionally-charged particles, that may exculde some of the other solutions that we have resented. This result is presented in Table 10 to compare with other particle theories. There are several other theories we found which also unify better than the minimal supersymmetric model. Thus, we have presented several modifications to the particle content of the Lee-Wick Standard Model that can improve unification.

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