Using Type Ia Supernovae and CMB Observations to Constrain Cosmological Parameters

A thesis submitted in partial fulfillment of the requirement for the
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by

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Abstract

A union compilation of 398 Type Ia supernovae (SNe Ia) observations taken from the recent and older Supernovae Legacy Surveys, the ESSENCE survey, and the HST survey datasets is compared with theoretical predictions. Theoretical luminosities as functions of redshift are compared to observed luminosities so as to place constraints on the cosmological parameters (Ω_m,w_Λ). These constraints are then superimposed with constraints based on the cosmic microwave background data, generating a global fit. We assume a flat universe in our analysis.

Acknowledgements

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1 Introduction

The standard cosmological model gives a complete description of the universe in terms of different component densities, denoted Ω_i, where the subscript stands for mass, dark energy, radiation and curvature, and a Hubble parameter, H_0, which is an expression of the rate of expansion of the universe. An important task for cosmologists is determining these parameter values; unfortunately however, such values are not directly measureable. Therefore, our investigation relies on the statistical inferences based on astronomical observation. The utility of this methodology has recently been tested; the past two decades has marked the era of precision cosmology in which technological advances in astrophysical observation and measurement have provided large and precise datasets. Studies of observables such as luminosity distance, redshift, the cosmic microwave background, baryonic acoustic oscillations, and large scale structure formation have yielded consistent bounds on Ω_m and Ω_Λ (matter density and dark energy density, respectively). Currently, the widely accepted concordance model lists values of ($H_0 \sim 66$ km/s/Mpc, $\Omega_m \sim 0.3$, $\Omega_\Lambda \sim 0.7$) [1]. Moreover, the existence of a dark energy component, *w*, has been inferred from the current accelerated expansion of the universe. These results are a product of Type Ia supernovae (SNe Ia) studies; a discussion of their importance in cosmology is provided in section 2.

The purpose of this paper is to provide a statistical analysis of the recently published In section 5, Type Ia supernovae union compilation dataset in order to place constraints on a (Ω_i, w) cosmology in accordance with the standard model. Further, this analysis is combined with similar studies [1] done on the cosmic microwave background and baryon acoustic oscillations in an attempt to generate more precise constraints. To account for the theory behind the standard model, a brief overview of general relativity and some relevant results is given in section 3. Finally, conclusions and suggestions for further study are given in section 6, including brief overviews of some other interesting cosmological quantities as well as a possible theoretical interpretation of our results.

2 The Importance of Type Ia Supernovae and the Cosmic Microwave Background in Cosmology

Type Ia Supernovae: It is believed that a SNe Ia begins life as a slowly rotating White Dwarf star in a binary system [2]. The dwarf slowly accretes mass from its binary companion until it nearly approaches the Chandrasekhar limit of approximately 1.38 solar masses, at which point the quantum degeneracy pressure of the star cannot withstand its own weight and the star implodes. Within 1% of this limit, the increased density and pressure of the star raise the temperature of the stellar core to a point conducive to nuclear fusion[1] which converts carbon into oxygen. When this ignition temperature is reached, nuclear fusion takes place and a chain reaction occurs which releases enough energy to cause the star to explode into a Type Ia supernovae. This unique process, which takes place at a standard mass, is the motivation for the use of SNe Ia in cosmological studies.

1 The exact mechanics of nuclear fusion ignition are not yet totally understood

Type Ia Supernovae are extremely useful phenomena in cosmology as their use as "standard candles." [3] In extragalactic observation, it is difficult to measure luminosity or distance to a supernova without ambiguity, since this quantity depends on both the elapsed time since explosion as well as mass. However, due to the characteristic mass at which they are created, SNe Ia exhibit characteristic light curves as a function of time after explosion which describe flux of light from the supernovae as a function of time after explosion. The Phillips curve relates the shape of a light curve to the intrinsic luminosity of the supernova. Because the intrinsic luminosity of a SNe Ia can be determined, Type Ia SNe are good standard candles, making them a good tool to probe the expansion history of the universe.

In 1998, measurements of luminosity distance vs. redshift (called the Hubble Curve) of different Type 1a supernovae showed that highly redshifted SNe Ia had dimmed luminosity magnitudes. This implied an outward accelerating universe. This dimming agent was theorized to be dark energy, which acts as a repulsive gravitational field and hence causes the outward acceleration of the universe. Theoretically, this is a manifestation of the cosmological constant in Einstein's equations, which describe how the fluid-like energy-momentum distribution of matter determines the large scale evolution of the universe (see section 3). Intuitively, a universe full of baryonic matter would tend to decelerate or even collapse onto itself; these recent results, however, have shown otherwise, and thus are the most direct evidence for the existence of dark energy.

Cosmic Microwave Background: The standard model of cosmology is explained by the Big Bang theory. This theory states that the current universe has evolved from an initially much smaller and hotter universe. The young universe was filled with a hot hydrogen gas which cooled down as the universe expanded. As the temperature cooled with expansion, stable atoms formed which couldn't absorb radiation at all frequencies during a period called recombination. This radiation has itself lost energy with the expansion of the universe and hence, its wavelength has become longer and is now predominantly in the microwave spectrum. Although this radiation is mostly homogeneous at 2.7K, there are small anisotropies defined by slightly more dense, hotter regions and less dense, cooler regions. These temperature differences are a result of the quantum mechanical density fluctuations which were magnified during inflation.

The Wilkinson-Microwave Anisotropy Probe, launched in 2001, is a joint project between NASA and Princeton which measures temperature differences in the Cosmic Microwave Background. The program CMBFAST analyzes this data by computing anisotropy, polarization and matter power spectra of the CMB. Since these properties are inherently functions of cosmological parameters, this CMB analysis is useful in constraining the composition of the universe.

3 The Theory behind the Standard Model

In general relativity, the cosmological principle is assumed which states that the universe is homogeneous and isotropic, meaning that all points in space are considered equivalent at any particular time (essentially, coordinate origins are ambiguous). The metric which incorporates the cosmological principle and describes a line element on the spacetime hypersurfaces is the Friedmann- Robertson-Walker metric [4]:

$$ds^2 = c^2dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\sin^2\theta 2d\varphi^2\right] \qquad (1)$$

where k takes on the values of -1, 0, and 1 in the cases of negative, zero, or positive spatial curvature respectively and

$$R(t) = a(t)R_0 \qquad (2)$$

where $a(t)$ is a dimensionless scale factor which refers to the expansion or contraction of the universe and It is related to the redshift z of an emitted light source by

$$1 + z = \frac{a(t_e)}{a(t_r)} \qquad (3)$$

where t_e is the time a photon is emitted and t_r is the time received. Hence, if the universe is expanding and the scale factor is increasing with cosmic time, then the photon is redshifted by an amount z. This redshift is caused by the expansion of the space through which light propagates. The present day scale factor is taken to equal 1 so that the redshift z of an event here and now is zero.

Einstein's equations assume that the distinct components of the universe take the form of a "cosmological fluid." These fluids are described by a linear equation of state:

$$p_i = w_i\rho_i c^2 \qquad (4)$$

which relates pressure, p_i to energy density, ρ_i, by an equation of state parameter w_i. The fluids which make up the universe are categorized as matter, radiation, and vacuum. Matter is assumed to behave as a pressureless dust described by $w_m = 0$, and radiation (photons, neutrinos, etc) has a traceless stress tensor, corresponding to $w_r = 1/3$. The energy of the vacuum is uniquely characterized by a *negative* pressure, $w = -1$. It is this contribution which accounts for the accelerated expansion of the universe. It is more convenient to work with dimensionless "densities," so let us define:

$$\Omega_i(t) \equiv \frac{8\pi G}{3H^2(t)}\rho_i \tag{5}$$

.

Where G is the gravitational constant and

$$\mathrm{H} = \frac{\dot{\mathrm{a}}(\mathrm{t})}{\mathrm{a}(\mathrm{t})} \tag{6}$$

and H is the Hubble constant as previously defined. This allows us to write the elegant equation:

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1 \tag{7}$$

which relates the energy densities of matter (*m*), including both dark and baryonic matter, and dark energy (Λ). The contribution of radiation is on the order of 10^{-5}, and so is neglected for the purposes of this paper.[2] spatial curvature (k) is defined differently, but this paper takes $k = 0$, and hence $\Omega_k = 0$, i.e., a flat universe. This assumption is well motivated by many studies, including the WMAP five year observations [1] and is consistently accepted in the literature. These are the specific parameters which will be the interest of our analysis, including the dark energy equation of state parameter w_Λ.

In this case, the Einstein's field equations take the form of the *Friedman-Lemaître* equations:

2 However, the radiation component was not always negligible, as the primordial universe was predominantly filled with radiation.

$$H^2 = \frac{8\pi G}{3}\left(\sum_i \rho_i\right) - \frac{c^2 k}{a^2 R_0^2} \tag{8}$$

along with

$$\dot{\rho}_i + 3(\rho_i + p_i)H = 0 \tag{9}$$

Integrating the previous equation with respect to time gives an expression for ρ_i in terms of the scale factor a and equation of state parameter w_i:

$$\rho_i = \rho_{0i}(1+z)^{3(1+w_i)} \tag{10}$$

Upon substituting equations (8) and (10), the Friedman-Lemaître equation can be rewritten:

$$H^2 = H_0^2(1+z)^2[1 + \sum_i \Omega_i((1+z)^{1+3w_i} - 1)] \tag{11}$$

SNe Ia are characterized by Hubble diagrams, which plot luminosity distance as a function of redshift. If a source of absolute luminosity L is observed a distance d away from us in a regular, static Euclidean universe, then the flux we measure is simply $F = L/(4\pi d_L^2)$, where d_L is the luminosity distance and is a function of redshift z and the cosmological parameters. Operationally,

$$L = 4\pi(d_L)^2 F \tag{12}$$

In order to make inferences on the SNe Ia data, it is necessary for us to define this quantity in an expanding universe described by the FRW metric. This is equivalent to imagining a source at a comoving distance r_s away which emits photons at some time s. These are measured by a detector at a later time at $r = 0$ at time t_0. Upon arrival, each photon has been redshifted in frequency by an amount $a(t_e) = a(t_s)/ 1+z$. The time interval between emission and reception are also reduced by this factor. Therefore, the observed flux is

$$L = 4\pi r_s^2 R_0^2 (1+z)^2 F \tag{13}$$

We now return to the RWF metric (equation (1)) and observe that $d\theta$, $d\varphi = 0$ since the path of the photon is only radial. Also, this interval is null ($ds^2=0$) since photons always travel along lightlike geodesics in the space-time. As seen in [5], Integrating equation (1) gives

$$\int_0^{r_s} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_s}^{today} \frac{cdt}{R_0 a(t)} = \int_0^{z_s} \frac{cdz}{R_0 H(z)} \tag{14}$$

after the substitution

$$\frac{dr^2}{1-kr^2} = \frac{c^2 dt^2}{R_0^2 a^2(t)} \tag{15}$$

and the appropriate change of variables $t \longrightarrow z$. We now make the substitution for $H(z)$ from equation (11) in the denominator of the right hand side equation (14):

$$r_s(z_s) = \frac{c}{R_0 H_0} \int_0^{z_s} \frac{dz}{(1+z)\sqrt{1+\sum_i \Omega_i((1+z)^{1+3w_i}-1)}} \tag{16}$$

This integral gives us a relation between the redshift z, the density parameters Ω_i and the equation of state parameter, w_i.[3] Thus, equation (14) characterizes the properties of the SNe Ia which themselves are functions of density ratios and the equation of state parameter. It is for this analytical reason that SNe Ia are such useful tools.

The SNe Ia data analyzed in this paper was presented in the form of a magnitude as a function of redshift, defined by [4]:

$$\mu_B = m_0 + 5\log_{10}\left(\frac{H_0 d_L}{c}\right) \tag{17}$$

The parameter m_0 refers to a conventional magnitude to which the SNe Ia luminosities are compared. Note that m_0 is also dependent on the parameters of each astronomical survey

3 This integral is specific to the $k = 0$ cosmology. For open and closed universes, the integral is the argument of sinh(x), sin(x) respectively.

because luminosities are calibrated based on the frequency band accessible to each telescope. We include m_0 in our fits as a nuisance parameter.

Lastly, it is important to discuss the consequences of the values which the parameter w may take on.[4] In general relativity, the pressure of a substance contributes to its gravitational field. If a pressure is positive, it tends to add to gravitational attraction, while negative pressures generate gravitational repulsions. Depending on the parameters defining the dark energy equation of state, the associated gravitational repulsion can dominate gravitational attraction, and the expansion of the universe can accelerate. The cosmological, $w = -1$ characterizing the dark energy density component, implies a universe which will accelerate outwards exponentially at some point in time (this is true for any $w < -1/3$). It can be shown that for $w < -1$, the universe will accelerate outward faster than exponentially, a condition known as the Big Rip. The Big Rip hypothesis theorizes that at some point, gravitational attraction will be too weak to bind any particles, and the fabric of the universe will eventually be "torn" apart.

4 Statistical Methods

The data which this analysis was performed on was taken from a "union" compilation which provided data on 398 SNe Ia collected from the old and new Supernovae Legacy Survey (SNLS) and ESSENCE Survey, as well as the recently published extended dataset for distant SNe Ia observed with HST [6]. This compilation provided SNe Ia in the redshift range $0< z <1.5$. Spectroscopic analysis was performed to determine redshift. The initial observed

4 For the rest of the paper, w always refers to w_Λ.

magnitudes were corrected for differences in intrinsic luminosities between SNe Ia (for example, the Phillips curve), and this paper uses the corrected magnitudes μ_B. All analysis code was run in Mathematica vr 5.0.

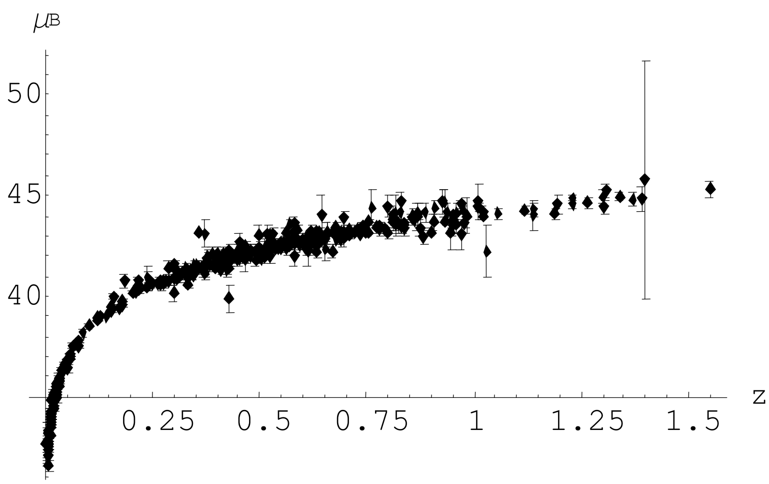

Figure 1: Magnitude μ_B and redshift z of 398 SNe Ia with error bars

The significance of our model compared to our data was determined by performing a chi-squared analysis, which takes the theoretical form:

$$\chi^2 = \sum_i^n \frac{(\theta_{th} - \theta_m)^2}{\sigma_m^2} \tag{18}$$

.

Here, θ_m and θ_{th} are measured and theoretical values respectively and σ_i refers to the corresponding uncertainties. This took the form, specifically [7],

$$\chi^2 = \sum_i^n \frac{(m_0 + 5\log_{10}((1+z)r(z)) - \mu_B)^2}{\sigma_{i,B}^2} \tag{19}$$

where *r(z)* is as defined in equation (16). This is a sum over all SNe Ia. Note that with each iteration of *i* corresponding to a different SNe Ia, the upper limit of equation (16) was replaced by the specific redshift z_i, and the integral computed. The error in the denominator is simply the error associated with each SNe Ia as given in [6].

The idea is to next minimize chi-squared with respect to certain parameters. Originally there were three free parameters associated with equation [17]: m_0, Ω_m, and w. Due to equation (7), Ω_Λ can be replaced by (1- Ω_m). In order to be able to fit a theoretical line over the data, m_0 was minimized with respect to the values $\Omega_m = 0.3$, $w = -1$. With the fitted magnitude shift, the next step was to minimize chi-squared again with respect to both Ω_m and w. This was done by providing constraints on the parameters before the minimization; these were: $0 < \Omega_m < 1$ and $-2 < w < 0$. These values were chosen so that the minimization would be within the range predicted by theory; also, these are the constraints placed on the minimizations from the WMAP and BAO analysis. A statistically good chi-squared fit obeys the relation χ^2 /(degrees of freedom) ~ 1. These values were then used to define a function

$$\Delta\chi^2 = \chi^2 - \chi^2_{min} \tag{20}$$

which was used to make confidence plots from contours at 68%, 90% and 95% confidences [7], constraining the (Ω_m, w) and (Ω_Λ, w) cosmologies.

An attempt was made to place more stringent constraints on these parameters by combining them with the complementary WMAP+BAO combined constraint contours as seen in [6]. Due to time constraints and the complexity of the CMB Fast analysis program, a manual approximation was made by assuming the contours shown were perfect ellipses (although they are not).

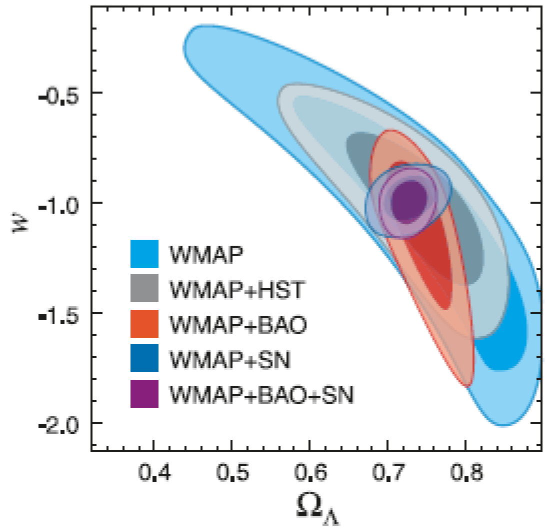

Figure 2 [1]: Combined confidence plots for different datasets at the 68% and 95% confidence levels. The data approximated in this paper is the WMAP+BAO contour at the 95% confidence level.

5 Analysis Results

Minimizing chi-squared with the reasonable (Ω_m,w) = (0.3, -1) with respect to m_0 yielded a value of $m_0 \rightarrow 43.2$. This gave good theoretical approximations to the data (see Figure 2). Using this value and minimizing with respect to (Ω_m,w) with the constraints as previously mentioned, we obtained ($\Omega_m \rightarrow 0.410$, $w \rightarrow -1.37$) with a value of $\chi^2 = 1308$. This is a rather high value for the degrees of freedom for this dataset of 398 points; $(\chi^2)/398 \sim 3$. This relatively large χ^2 is likely due to our imprecise fit of the parameter, m_0, which was fit independently with the previously mentioned (Ω_m,w) = (0.3, -1). The theoretical plots on the data in figure 2 correspond to a (Ω_m,w) cosmology = (0.3,-1), (0.5,-2) and (1,0) for the red, blue and green lines, respectively. The (0.3,-1), (0.5,-2) cosmologies correspond to decent fits with χ^2 values of 1313 and 1316 (with respect to χ^2 minimized). Although the $\Omega_m = 1$ cosmology looks to correspond to a decent fit, its associated χ^2 value is 2569, almost twice the minimum value. This result reaffirms the existence of a dark energy component; the fitted value of $w < -1$ suggests the cosmological "Big Rip" hypothesis.

The WMAP contour data used included baryon acoustic oscillations (BAO) [1]. These contours were approximated as perfect ellipses and then used the further constrain the SNe Ia data. A χ^2 minimization using both datasets more rigorously constrained the data, giving a value of 1311 with a best fit of $(\Omega_\Lambda, w) = (0.687, -1.05)$. Figure 5 shows the superimposed confidence plots for the SNe Ia and WMAP data at the 95% confidence level.

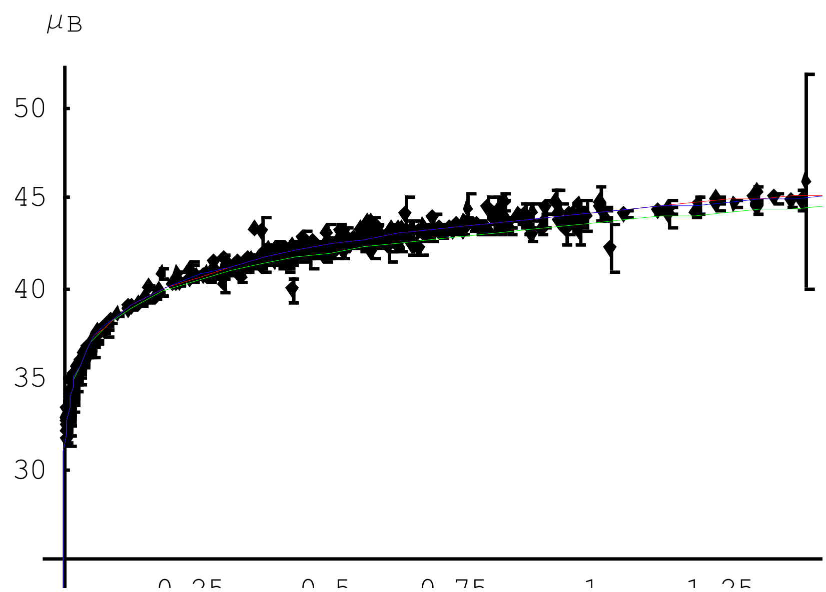

Figure 3: Three theoretical plots over the SNe Ia data: the red plot corresponds to the standard $(\Omega_m, w) = (0.3, -1)$; the green plot corresponds to $\Omega_m = 1$; the blue plot corresponds to $(\Omega_m, w) = (0.5, -2)$.

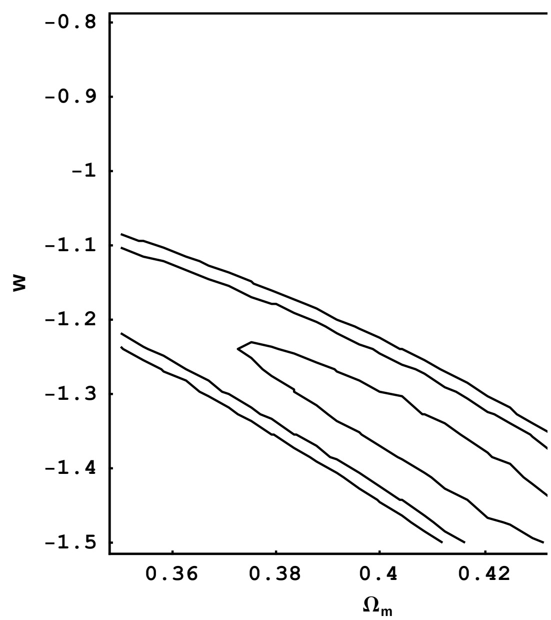

Figure 4: Confidence Plot for (Ω_m,w) at 68%, 90% and 95% confidence intervals. The best fit is (Ω_m,w) = (0.409, -1.37).

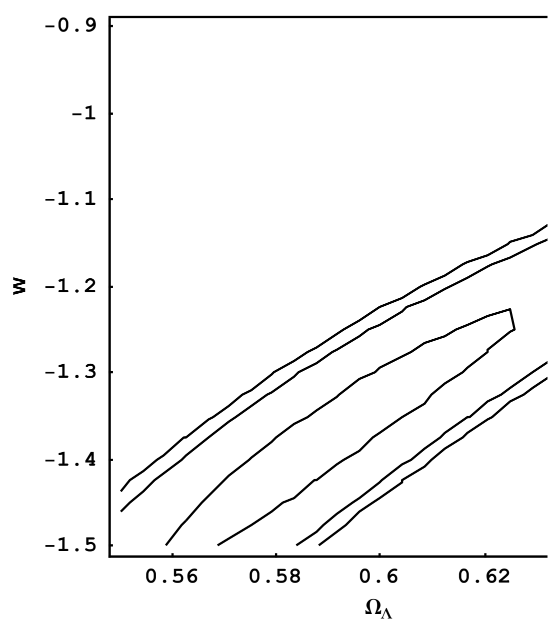

Figure 5: Confidence plot for (Ω_Λ, w) at 68%, 90% and 95% confidence intervals. The best fit is (Ω_Λ, w) = (0.590, -1.37).

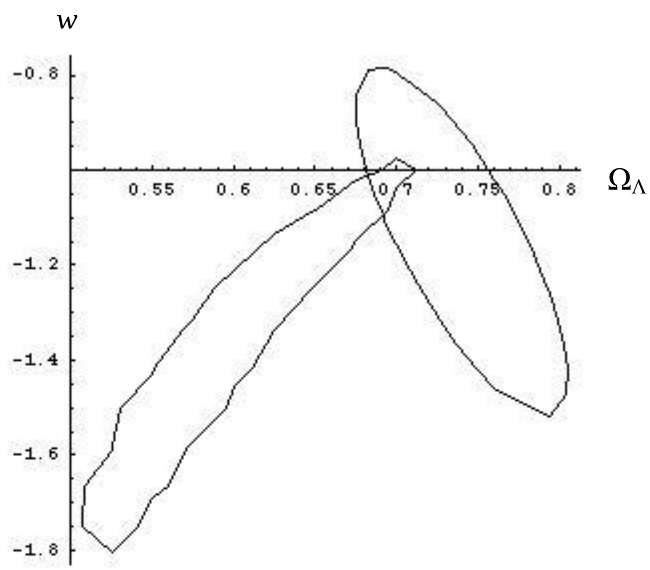

Figure 6: Superimposed confidence plot of (Ω_Λ, w) at the 95% confidence level of the SNe Ia data and the WMAP+BAO data. The best fit is (0.697, -1.05)

6 Conclusions and Further Study

This project was conducted using the largest available SNe Ia union compilation to date. Theoretical luminosity curves were plotted over a Hubble Diagram for the 398 SNe Ia. A best fit was calculated for (Ω_Λ,w) which gave values (Ω_Λ,w) = (0.590, -1.37) and consequently with a χ^2 value of 1309. An approximation was made to the WMAP+BAO contours [1] and a global fit was performed yielding a best fit with (Ω_Λ,w) = (0.689, -1.05) with a χ^2 value of 1311, further constraining the density and equation of state parameters. We found our fits to be reasonably consistent with current cosmological surveys.

A more accurate study would involve the use of the CMBFAST program to analyze and produce exact confidence plots with which to combine with our data.

It is also worthwhile to examine the constraints which may be inferred from galaxy cluster surveys to produce a second global fit with the SNe Ia data. As is evident from the CMB data, the infant universe was homogeneous to one part in 10^5. It is believed that these tiny energy density fluctuations were the initial conditions which eventually spawned large-scale galaxy formation. The Press-Schechter formalism [8] provides a formal picture of structure formation by describing the distribution of large scale formations in the universe as a function of their redshift and mass. Since galaxy formation and evolution are theoretically dependent upon universal composition, such formalism and distributions are useful in constraining cosmological parameters.

In intergalactic cluster gas, hydrogen atoms collide and produce X-rays. These X-rays propagate under the influence of the gravitational potential of the cluster. These X-rays have recently been observed by the 400 square degrees ROSAT survey to detect clusters with redshift $z < 1$. Only a flux is measured, however, so procedures must be taken in order to relate the mass and luminosity as a function of flux. This is done using the M-T relation, which relates mass and temperature, and L-T relation, which relates luminosity and temperature.

Another possibility for future study includes a theoretical investigation for an explanation to the $w < -1$ fits. It is believed that there may be a mechanism which accounts for the dimness of distant SNe Ia which would not require $w < -1$. One theory hypothesizes the existence of a photon-axion dimming process called the CKT (Csáki, Kaloper and Terning) dimming mechanism [5]. In this model, which assumed a flat universe, the existence of pseudo-scalar particles called light axions is assumed which couple with photons in a randomly oriented intergalactic magnetic field. Through this coupling, photon-axion oscillation takes place which accounts for the decay of approximately one-third of highly redshift photons by the time they are detected on Earth. This interesting mechanism explains the dimming of highly redshifted SNe Ia without requiring a period of accelerated universal expansion. This interesting probe of the standard model is worthy of more detailed analysis.

References:

[1] E. Komatsu *et al*.,*Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation,* arXiv:0803.0547v1[astro-ph].

[2] Lieb, Elliot H; Yau, Horng-Tzer, "*A rigorous examination of the Chandrasekhar theory of stellar collapse,"* Astrophysical Journal, Part 1 (ISSN 0004-637X), vol. 323, Dec. 1, 1987, p. 140-144.

[3] Colgate, S.A., "*Supernovae as a Standard Candle in Cosmology*," Astrophysical Journal, Part 1, vol. 232, Sept. 1, 1979, p. 404-408.

[4] Hobson, Efstathiou, Lasenby, *General Relativity: An Introduction for Physicists,* Cambridge University Press, New York. (2006).

[5] J. Erlich and C. Grosjean, *Supernovae as a Probe of Particle Physics and Cosmology,* (2001), [//arxiv.org/abs/hep-ph/0111335].

[6] M. Kowalski, *et al., Improved Cosmological Constraints from New, Old, and Combined Supernovae Datasets*,(2008)[astro-phys/0804.4142v1].

[7] G. Cowan, *Statistics*, Physical Review D, 66, 232 (2001)

[8] J. Erlich, B. Glover, N. Weiner, *Sensitivity and Insensitivity of Galaxy Cluster Surveys to New Physics,* (2007), [arXiv:0709.3442v2].