Fitting High-T_c Superconductor Data to Theories of Vortex Pinning

A thesis submitted in partial fulfillment of the requirement
for the degree of Bachelor of Science
Physics from the College of William and Mary in Virginia,

by

Amanda L.Goehring

Accepted for: BS in Physics

Advisor: W. J. Kossler

Williamsburg, Virginia
May 2008

Abstract

Building on previous analysis, data gathered in μ-SR experiments on high-T_c superconductors of both YBCO and BSCCO varieties at TRIUMF were analyzed in an attempt to determine the theoretical model that best suited the experimental results. Four models were compared: two-fluid, BCS, D-wave, and empirical. Second moments for YBCO were obtained by fitting the μ-SR precession data to a London model distribution convoluted with Gaussian smearing. The second moments were corrected in quadrature for a Gaussian background depolarization. For BSCCO a back-to-back Gaussian form was fit to the precision data and then the second moments obtained. By analyzing the second moments of the magnetic field distribution, theoretical fits were obtained and analyzed.

Acknowledgements

I would first like to thank Dr. W. J. Kossler for all of his guidance and support this past year as my research advisor. In the same way, I would like to thank the rest of my professors in the William and Mary Physics department for their patience and their help, but especially for their friendliness and sense of humor. Finally, I'd like to express gratitude to my predecessors on this experiment, Abagail Shockley and Mannix Shinn, for creating a solid base on which to base my work.

Contents

1 Introduction **1**

1.1 Overview 1

1.2 Theory 1

2 The Experiment **5**

3 Conclusions **7**

4 Tables of Results **8**

1 Introduction

1.1 Overview

Superconductors are unusual materials because they have no electrical resistance should a current be run through them. Typically, most superconductors only obtain their fantastic properties below a certain critical temperature, T_c [1]. In 1956, Cooper theorized that inside superconductors electrons would pair off and become bound to one another. These entities, now known as Cooper pairs, were the origin of the effects seen in superconductors [2]. Generally, they are divided into Type I and Type II superconductors, with type II containing high T_c varieties; both BSCCO and YBCO are Type II.

In Vancouver, British Colombia, Canada, muon spin rotation (μ-SR) experiments were done with both YBCO and BSCCO superconductors using the Belle (high-field) spectrometer at the TRIUMF cyclotron facility. In μ-SR experiments, positive muons all with perfectly the same polarization are sent into samples. Eventually the muons stop inside the sample and emit a positron along the final spin-polarization direction to be detected by one of four detectors placed above, below, and to the sides of the sample [3]. The time between entrance of the muon into the sample and detection of the positron is recorded, and the process is repeated to eventually gain an average of the time evolution of their spins. This average is used to learn about the magnetism, for example, the variance in the local magnetic field of a sample.

1.2 Theory

Various ideas have been proposed to best explain high T_c superconductivity, and they have been applied to the TRIUMF data. Of interest is the description of the temperature dependence of the superconducting pair density, $n_s(T)$. Four models to describe $n_s(T)$ were considered. These models are: the S-Wave BCS, the Empirical, the Two Fluid, and the D-Wave, although the D-Wave is different because it presents an effective n_s by using the London equation to find the effective penetration depth from the second moments. The penetration depth, a characteristic property of superconductors, is related to the observed field distribution.

The London equation is:

$$\nabla^2 \mathbf{b} - \frac{4\pi n e^2}{mc^2}\mathbf{B} = 0 \tag{1}$$

and was originally proposed by F. and H. London [1].

The penetration depth is defined as

$$\lambda_L = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{\frac{1}{2}} \tag{2}$$

where n_s is the number density of superconducting electrons. Using the penetration depth, the London equation can be expressed

$$\nabla^2 \mathbf{b} - \frac{1}{\lambda_L^2}\mathbf{B} = 0 \tag{3}$$

Of the four models, the Empirical model is the simplest, and can be expressed as

$$\left(\frac{\lambda}{\lambda_0}\right)^2 = \frac{1}{1 - \left(\frac{T}{T_c}\right)^{\gamma}} \tag{4}$$

By setting $\gamma = 2.0$, a description of the penetration depth can be achieved [4].

The Two-Fluid model is similarly expressed as

$$\left(\frac{\lambda}{\lambda_0}\right)^2 = \frac{1}{1 - \left(\frac{T}{T_c}\right)^{4}} \tag{5}$$

and is derived from considering the magnetic properties of a very small sample, specifically its magnetic susceptibility [5]. It sets the γ of the Empirical model to equal 4.

The Bardeen-Cooper-Schrieffer (BCS) model is widely known, and addresses lattice vibrations caused by Cooper pairs [1]. In this model,

$$\lambda_L^{-2}(T) = \lambda_L^{-2}(0)\left[1 - 2\int_{\Delta}^{infty}\left(-\frac{\partial f}{\partial E}\right)\frac{E}{(E^2-\Delta^2)^{\frac{1}{2}}}dE\right] \tag{6}$$

where f is the Fermi function [6].

And, finally, the D-Wave model, which is a generalization of the BCS model for cases when the interaction energy has D-wave symmetry, was introduced by Anderson and Morel [7]. Unique among the four models used, it presents the second moments. Using the London equation, it can find the effective penetration depth. It can be expressed as

$$\lambda_{eff}^{-4} = C\sum_{k\neq 0}\frac{e^{-\xi^2 k^2}}{(1+\mathcal{L}_{ij}k_i k_j)^2} \approx C\sum_{k\neq 0}\frac{e^{-\xi^2}}{(\mathcal{L}_{ij}k_i k_j)^2} \tag{7}$$

where

$$\lambda_{eff}^{-4} = \lambda_0^{-4}\left(\frac{\partial \bar{B}^2}{\partial \bar{B}_0^2}\right) \tag{8}$$

k are the reciprocal lattice wave vectors, $\hat{\mathbf{Q}}(\mathrm{k})$ is the electromagnetic response tensor

$$Q_{ij}(\mathbf{k}) = \frac{4\pi T}{\lambda_0^2}\sum_{n>0} < \frac{\Delta_{\hat{p}}^2 \hat{v}_{F_i}\hat{v}_{F_j}}{(\omega_n^2+\Delta_{\hat{p}}^2)^{\frac{1}{2}}(\omega_n^2+\Delta_{\hat{p}}^2+\gamma_k^2)} > \tag{9}$$

and

$$\mathcal{L}_{ij}(\mathbf{k}) = \frac{Q_{ij}(\mathbf{k})}{det\hat{\mathbf{Q}}(\mathbf{k})} \quad , \quad C^{-1} = \sum_{k\neq 0}\frac{e^{-\xi_0^2 k^2}}{k^4} \tag{10}$$

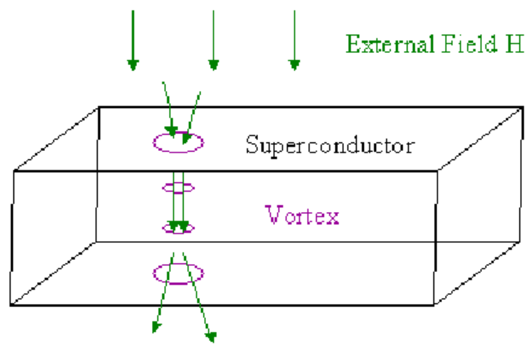

Figure 1: Vortexes in Superconductors

Also important is the concept of vortex pinning. Type II superconductors have a mixed state that appears just before total superconductivity, and there manifest vortexes composed of superconducting currents through which external magnetic fields can pass. Although in theory a vortex line should be straight, there seems to be displacement along the line. This displacement can either occur in the form of the entire vortex line being displaced, u_l, or individual vortexes being displaced, u_p.

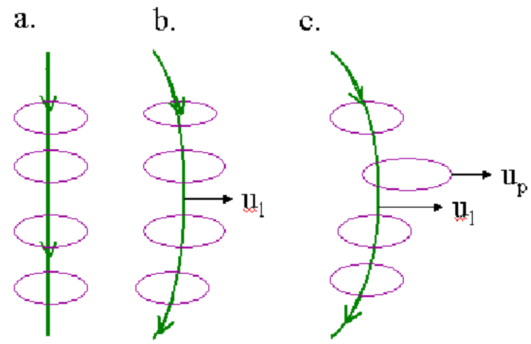

Figure 2: Deformations of the vortexes - u_p and u_l

The original fits to the data used a program written for this purpose, trix06 [8]. The fits were redone, however, with another program, fit4 [9]. Fit4 is able to fit all of the models once given either the BSCCO or YBCO second moments and display the resultant function graphically. Parameters can be adjusted either manually or by making use of the fitting portion of the program. The program fits on eight parameters: λ_0, T_c, B, ϵ_0, u_{l0}, u_{p1}, u_{p2}, and ξ_0.

Recently, new μ-SR research with YBCO superconductors was published suggesting perhaps a combination of S-wave and D-wave symmetry was being seen. While this model was not considered in this paper, it may be important in future study [10].

2 The Experiment

The work done was in the form of analysis of data from μ-SR at TRIUMF, in addition to analysis of previous work done by Abagail Shockley and Mannix Shinn. Shockley worked with the BSCCO data, and was able to create usable second moments from the raw data. Shinn worked the the YBCO data doing largely the same for that set. Both created their own fits, using their own programs, and came to their own conclusions. Although this was the case, a new program, fit4, was written in the hopes of even more accurately analyzing the data [9]. fit4 used the data converted to the second moment (σ) to make new fits.

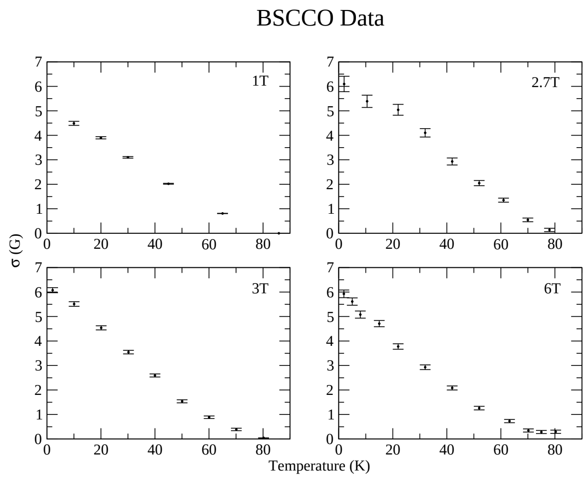

Figure 3: BSCCO Data

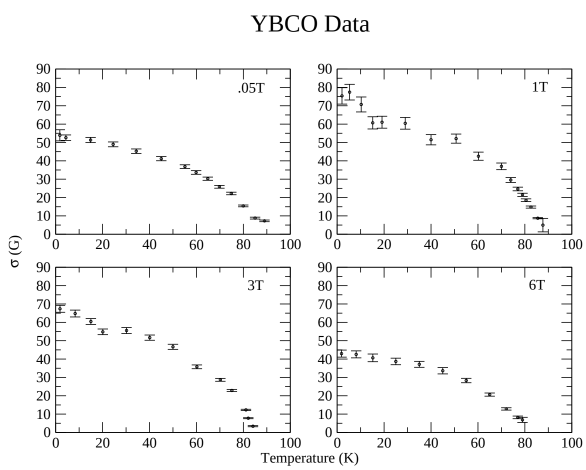

Figure 4: YBCO Data

As mentioned above, fit4 is able to fit any model on eight parameters: λ_0, T_c, B, ϵ_0, u_{l0}, u_{p1}, u_{p2}, and ξ_0. The program also calculates χ^2 values for fits and allowed for manual tweaking. To date, almost all of the BSCCO fits and four of YBCO have been completed. All of the parameters and fits, as well as χ^2 values, are included in the Tables of Results section at the end of this document.

The analysis included a bit of difficultly for the models in fitting the turning point present in most of the data sets that created a sideways s-shape. The models are in general best suited to a single sloping curve, so the change in direction was a problem for them and, as can be seen, some created better results in that area than others.

3 Conclusions

The results using fit4 are very encouraging, because many of them work extremely well. As can be observed in the fits, unfortunately, there is no clear best model in this analysis. The D-Wave model consistently provides marginally better χ^2 values than the others, but it is such a small difference that it would be unreasonable to proclaim D-Wave the best at this point. Of course, the unfinished YB CO fits may provide more insight, but it seems that adapting the program to analyze while allowing for field variation at T=0 should also be an avenue explored in the future.

4 Tables of Results

Table 1: Parameters for 1T fit for BSCCO

Model	λ_0	T_c (K)	B (G)	ϵ_0	u_{l0}	u_{p1}	u_{p2}	ξ_0	χ^2	χ^2_N
2-Fluid	2.997	92.0	10000	63.646	-1.85e-2	0.137	0.279	2.0e-2	46.948	7.825
D-Wave	3.135	92.0	10000	126.801	-2.02e-2	0.131	0.281	2.0e-2	35.354	5.892

Table 2: Parameters for 2.7T fit for BSCCO

Model	λ_0	T_c (K)	B (T)	ϵ_0	u_{l0}	u_{p1}	u_{p2}	ξ_0	χ^2	χ^2_N
Empirical	2.911	92.0	2.7	123.281	1.81e-2	0.237	0.524	2.0e-2	5.960	0.662
2-Fluid	2.870	92.0	2.7	52.346	1.85e-2	0.209	-2.12e-2	2.0e-2	12.469	1.385
BCS	2.415	92.0	2.7	110.183	2.10e-2	1.185	2.203	2.0e-2	7.462	0.829
D-Wave	2.894	92.0	2.7	107.893	1.73e-2	0.171	0.222	2.0e-2	5.232	0.581

Table 3: Parameters for 3T fit for BSCCO

Model	λ_0	T_c (K)	B (T)	ϵ_0	u_{l0}	u_{p1}	u_{p2}	ξ_0	χ^2	χ^2_N
Empirical	2.453	92.0	3	86.210	1.94e-2	1.019	-1.926	2.0e-2	21.725	2.414
2-Fluid	2.584	92.0	3	43.557	1.90e-2	0.739	0.313	2.0e-2	26.542	2.949
BCS	2.233	92.0	3	77.789	2.02e-2	1.409	2.514	2.0e-2	29.720	3.302
D-Wave	2.540	92.0	3	83.167	1.91e-2	0.864	1.465	2.0e-2	17.811	1.979

Table 4: Parameters for 6T fit for BSCCO

Model	λ_0	T_c (K)	B (T)	ϵ_0	u_{l0}	u_{p1}	u_{p2}	ξ_0	χ^2	χ^2_N
Empirical	2.424	92.0	6	129.183	1.84e-2	1.465	-3.455	2.0e-2	33.112	2.729
2-Fluid	2.992	92.0	6	65.141	3.33e-2	2.292	-3.832	2.0e-2	32.424	2.702
BCS	3.203	92.0	6	121.134	3.26e-2	1.953	-4.879	2.0e-2	33.087	2.757
D-Wave	3.381	92.0	6	108.204	3.66e-2	2.634	-5.208	2.0e-2	29.767	2.481

Table 5: Parameters for 3T fit for YBCO

Model	λ_0	T_c (K)	B (T)	ϵ_0	u_{l0}	u_{p1}	u_{p2}	ξ_0	χ^2	χ^2_N
Empirical	1.260	89.915	3	14.00	-1.12e-2	2.60e-3	5.28e-2	2.0e-2	2074.37	159.567
2-Fluid	1.280	92.282	3	20.00	3.49e-2	-1.77e-3	5.87e-4	3.0e-2	125.787	9.676
BCS	1.260	89.915	3	14.00	-6.71e-3	1.67e-3	-1.54e-4	2.0e-2	624.251	48.019
D-Wave	1.261	85.797	3	12.47	1.69e-2	1.96e-5	1.00e-6	2.0e-2	85.472	6.575

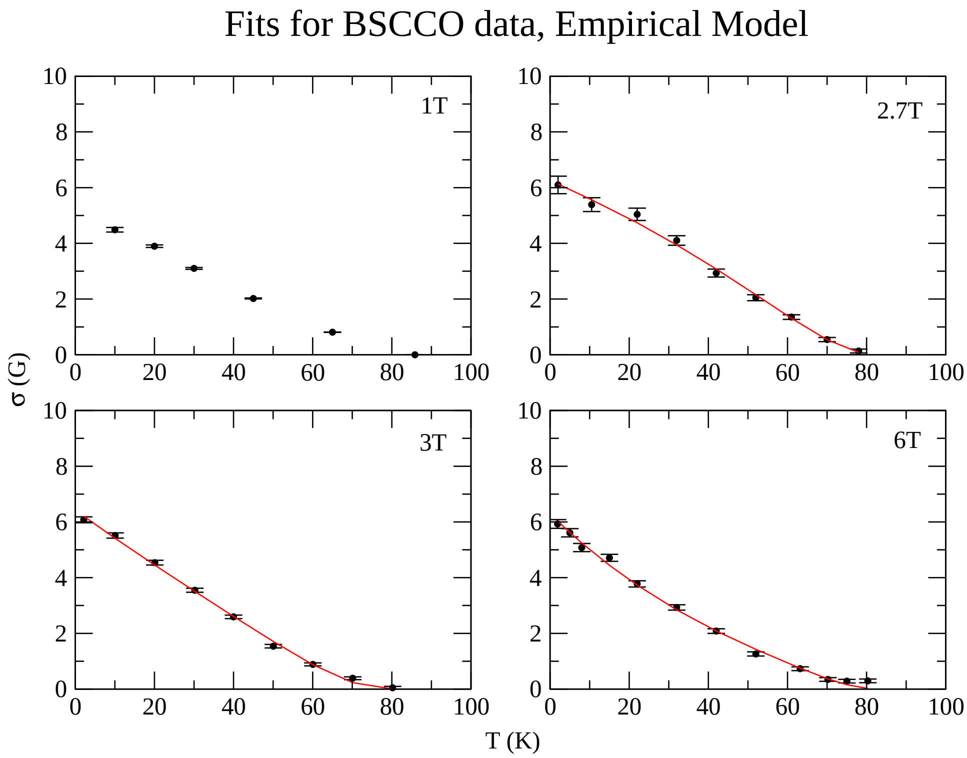

Figure 5: Empirical fits for BSCCO

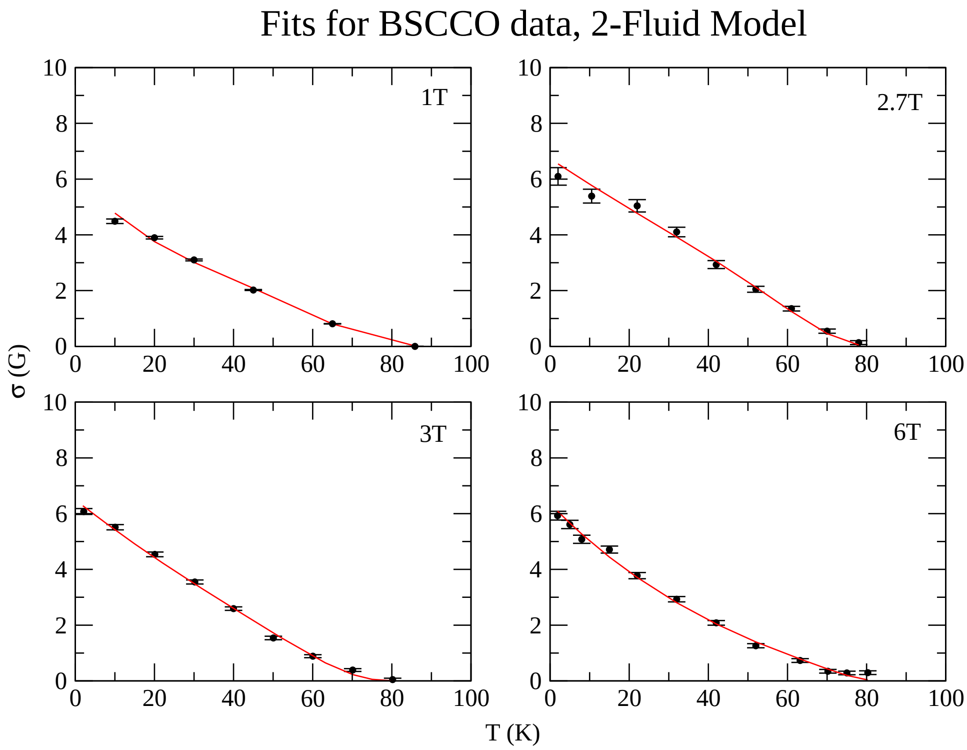

Figure 6: 2-Fluid fits for BSCCO

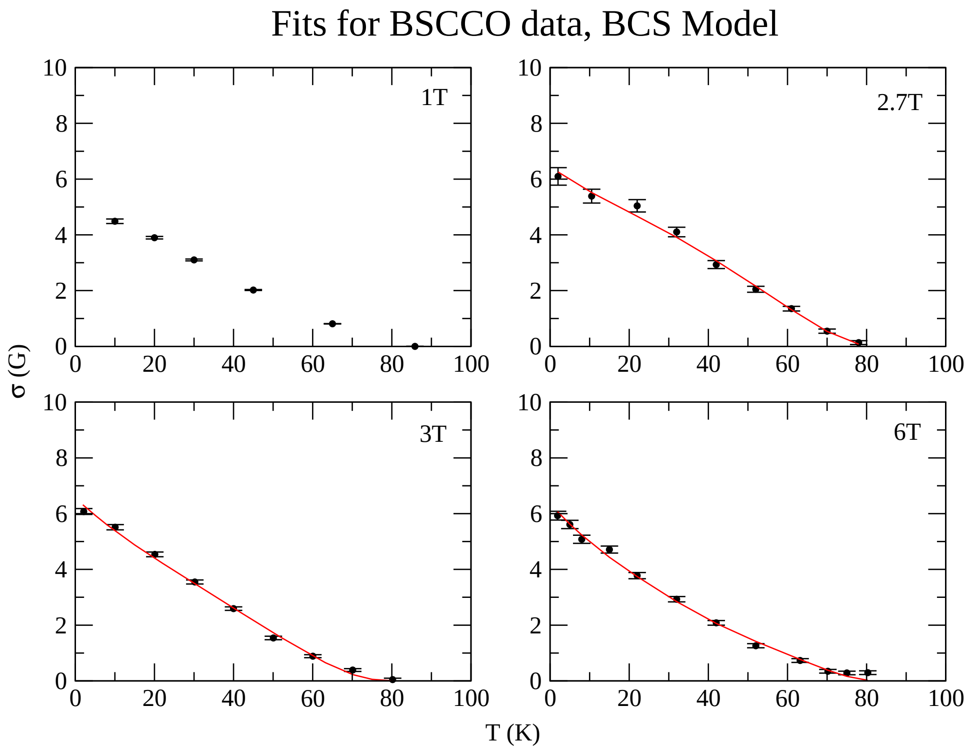

Figure 7: BCS fits for BSCCO

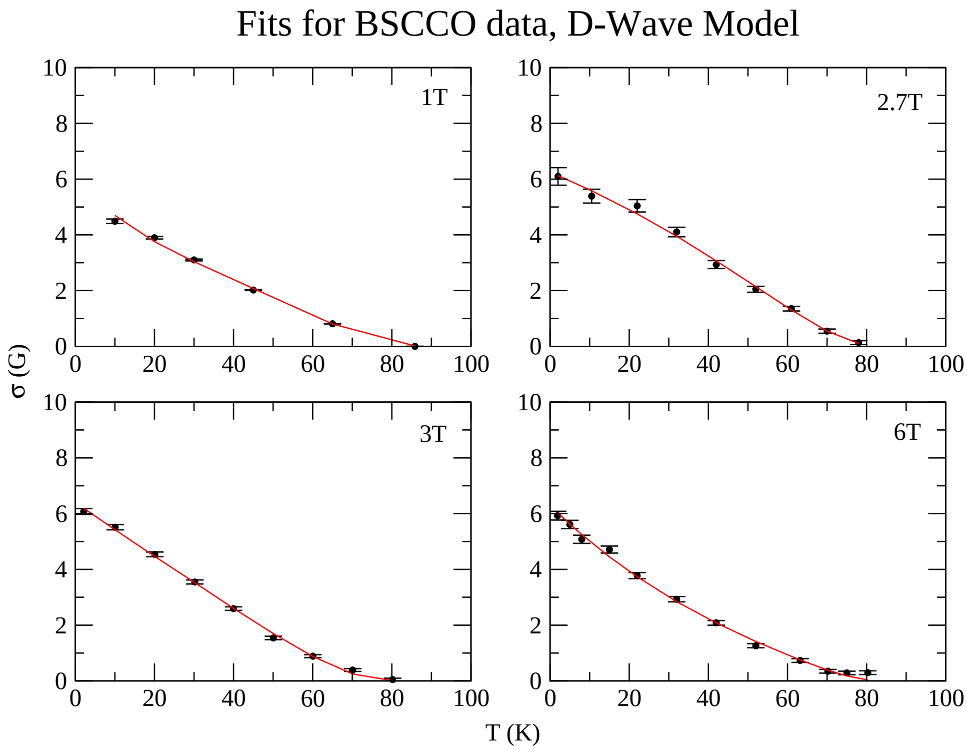

Figure 8: D-Wave fits for BSCCO

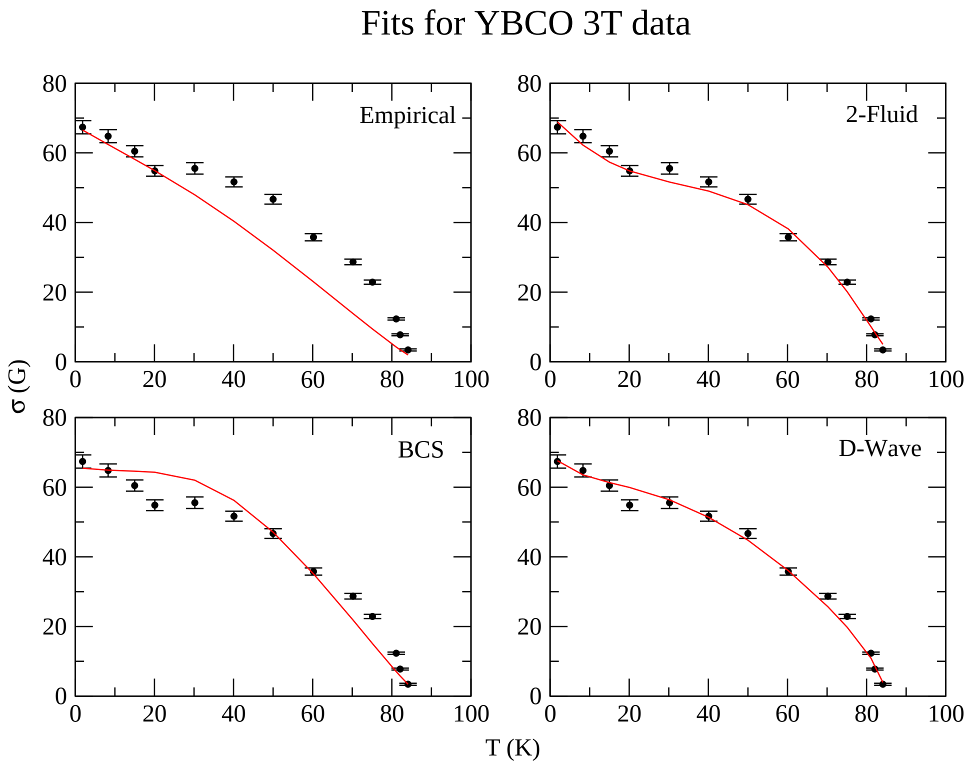

Figure 9: 3T fits for YBCO

References

[1] P. G.de Gennes, *Superconductivity of Metals and Alloys* (Addison-Wesley Publishing Company, ADDRESS, 1966).

[2] C. P. Poole, Jr., H. A. Farach, and R. J. Freswick, *Superconductivity* (Academic Press, ADDRESS, 1995).

[3] D. R. Harshman *et al.*, Phys. Rev. B. **69**, 174505 (2004).

[4] O. G. Vendik, I. B. Vendik, and D. I. Kaparkov, in *IEEE Transactions on MIcrowave Theory and Techniques* (PUBLISHER, ADDRESS, 1998), Vol. 46, p. 469.

[5] D. Shoenberg, *Superconductivity* (Cambridge University Press, ADDRESS, 1965).

[6] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, Inc., ADDRESS, 1996).

[7] Anderson and Morel, Phys. Rev. **123**, 1911 (1961).

[8] W. J. Kossler and A. J. Greer, the program trix06, 2001.

[9] W. J. Kossler, the program fit4, 2008.

[10] D. D. C. T. M. S. M. S. T. A. B.-H. R. Khasanov, S. Strassle and H. Keller, Physical Review Letters **99**, (2007).