Fringing Fields from Frozen-in Vortices of a Superconductor

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Abstract

Type II superconductors may have locked-in magnetization when cooled below their critical temperature in a magnetic field. We examined the fringing magnetic fields resulting from this magnetization in two ways. First, we generated a computer model to predict the fields we expect to see around a circular magnetized sample. Second, we tested this model with experimental measurements of field-cooled $YBa_2Cu_3O_7$, a type II superconductor. Measurements were made on sintered samples and a single-crystal sample. We found that the fringing field from single-crystal samples is lower than expected by a factor of three, and the fringing field from sintered samples are lower than expected by a factor of nearly a thousand. We expect that in future muon spin rotation experiments investigating superconductors, fringing fields from sintered $YBa_2Cu_3O_7$ samples will not significantly affect the results, and fringing fields from single-crystal samples will have to be measured separately to be discounted because prediction of their fringing fields may not be reliable.

1 Introduction

This year was spent examining flux vortex pinning and fringing fields in high-temperature superconductors. This work was done in preparation for a proposed series of muon spin rotation (μSR) experiments to be carried out next year at the Tri-University Meson Facility in British Columbia (TRIUMF).The work this year was done in three parts. The first part consisted of a literature search and examination of the existing theoretical and experimental framework of vortex pinning to aid the proposal to the NSF for the TRIUMF experiments. The second part consisted of writing a program in FORTRAN to model the fringing fields we expect to see when the superconducting material is in the presence of a perpendicular external magnetic field. The third part was to experimentally measure the magnetic field resulting from locked-in magnetization of high-temperature superconducting samples and compare these results to predictions from our model.

1.1 Superconductor properties

Superconductors are a class of materials whose properties below a critical temperature T_c include resistance-free transmission of current and perfect or near-perfect diamagnetism, the repulsion of magnetic fields from the interior of the sample. Because of these unusual properties, superconductivity is one of the most heavily-studied fields in modern physics. In type II superconductors, which include nearly all known high-temperature superconductors, magnetic fields can penetrate into the superconductor in the form of magnetic flux vortices. These vortices, also known as fluxons, are small lines of non-superconducting phase surrounded by superconducting phase. Magnetic field lines can penetrate through the non-diamagnetic normal phase, causing shielding currents to circulate in the surrounding superconducting phase. Repulsive forces between the vortices result in their arrangement in a regular lattice across the superconducting material. Figure 1 shows this lattice in superconducting $NbSe_2$. The geometry of the lattice can be described by a repeating isocoles triangle in which ψ is the angle between the basis vectors. A correct understanding of the vortex lattice geometry in a superconductor is crucial. Calculations of other important parameters including the magnetic penetration depth λ, and the coherence length between electron pairs ξ, implicitly depend upon the lattice geometry.

1.2 Conflicting models

There is still much about the underlying mechanisms of superconductivity that remains unknown. Theory predicts that as the temperature in a superconductor decreases, the number of superconducting electron pairs will increase. The rate of increase of the number of these

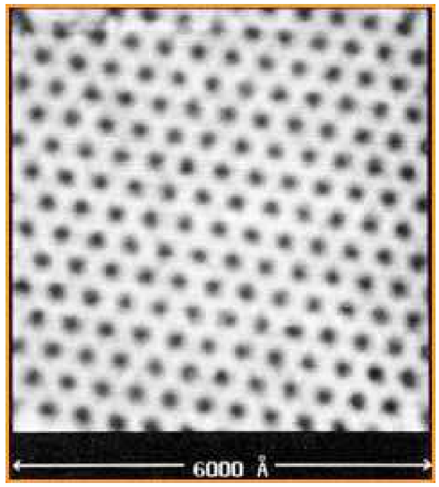

Figure 1: Superconducting vortex lattice in $NbSe_2$. Imaged with STM by H.F. Hess et al.[1]

pairs will approach zero as the temperature approaches absolute zero if the superconducting gap is nowhere zero. But recent research by Sonier et al.[2, 3, 4] has instead found that in these conditions the rate of increase of the number of superconducting pairs remains positive and approximately linear with temperature at low temperatures approaching absolute zero for certain field values.

These researches have proposed that their results are due to intrinsic effects, a result of the nature of the superconducting order parameter. By this account the results arise because the order parameter has a symmetry of $d_{x^2-y^2}$. However, Professor Kossler and other researchers [5] have suggested that there could be a systematic explanation. High-temperature superconducting materials form a planar lattice. But in any real-world sample there will always be a certain amount of defects and impurities in the lattice of the material. As temperature decreases, the defects more closely align with the rest of the lattice. If these pinning effects aid in allowing more superconducting pairs to form, they could be responsible for the observed lack of saturation at very low temperatures.

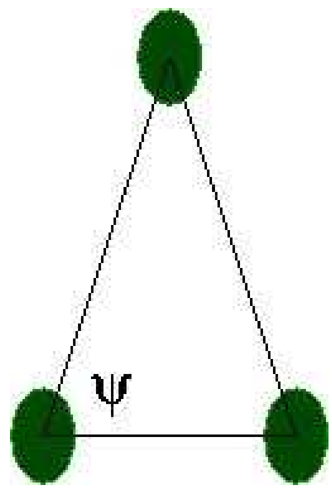

Figure 2: The angle ψ describes the lattice geometry.

1.3 TRIUMF experiment

The proposed TRIUMF experiment aims to shed new light on the problem raised by Sonier and Kossler as well as other questions in superconductivity. These μSR experiments can be used to determine the flux vortex lattice geometry in superconducting materials. In these experiments, a high-quality sample of superconducting $YBa_2Cu_3O_7$ will be cooled to below T_c in the presence of an applied magnetic field. This magnetic field will give rise to a flux vortex lattice and current resistancelessly flowing within the sample. The flux vortex lattice and the magnetization of the sample will persist indefinitely even after the applied field is removed, so long as the sample remains below the critical temperature T_c. To study the lattice state, the applied field is removed and a beam of nearly completely polarized μ^+ muons is directed towards the sample. Scintillators on all sides of the sample measure the muons and their decay product positrons after the muons interact with the sample. However, some muons will interact instead with the material of the container that holds the sample. We want to be able to discount their effects to produce a cleaner measurement of the sample itself. But to do this, we will need to know the magnetic field at the location of the container backing. This wass the goal of our FORTRAN model and experimental measurements.

Polarized positive muons will decay into a positron, electron neutrino, and antimuon neutrino in the reaction $\mu^+ \rightarrow e^+ + \nu_e + \nu_{\bar{\mu}}$. These muons have a half-life of only 2.2 μs, so many muons in the beam will decay during their interactions with the superconducting sample. Parity is not conserved in this decay, instead positrons tend to have the same spin as the incoming polarized muons. The asymmetry between positron spins can be measured by looking at the measurements from the top and bottom scintillators and taking their difference. This asymmetry function will vary with time as the muons in the incoming beam precess. The asymmetry function can be used with a cosine transform to calculate the gradient of the localized magnetic field gradient near a fluxon in the superconducting sample. The gradient information is then used to get the relative positions of vortices in the lattice and find the characteristic angle ψ.

In addition to vortex geometry, the proposed TRIUMF experiments may be able to tell us something about flux vortex pinning. Pinning phenomena tend to destroy the regular periodicity of the idealized fluxon lattice. They are generally a result of defects in the crystal structures of superconducting specimens. These defects can cause individual fluxons in a vortex or vortices themselves to lie off site and to have self energies that are position-dependent. Pinning phenomena are especially important in the debate over the nature of the superconducting order parameter for reasons listed above. The proposed TRIUMF experiments will include measurements at field angles of 0, 45, and 90 degrees. If the claim that unusual activity near absolute zero is due to pinning effects is to be supported for each case, any pinning dependence on field angle will have to be known. The first part of the work this year involved a literature search to see if any researchers have satisfactorally resolved this question. This literature search was inconclusive. Many articles of tangential importance to the proposal were found, but none directly addressed the question of angle dependence of flux pinning phenomena. Thus this is now one of the questions that we hope to address in the TRIUMF experiments themselves.

2 Computer model

We developed a computer model of the fringing effects of the superconducting magnetic field. The program to model the fringing fields was written from scratch in FORTRAN. In this model, the superconducting material was approximated as a flat cylinder of radius a. After a perpendicular external magnetic field is removed, the superconductor retains a proportional locked-in magnetization along the crystalline c-axis. This magnetization is the result of the circulating superconducting current, which is not expected to attenuate over time. In the TRIUMF experiment, the superconducting material will be held in a container with a backing a distance h away from the center of the material along the c-axis. The fringe field interferes with the muons at this backing, producing undesirable background noise. By modeling the fringe fields and calculating their effects, we can subtract this noise from the data and produce a cleaner final result.

2.1 Equations and algorithms

The equations to calculate the fringe fields are adapted from a textbook by J.D. Jackson [6]. The localized field distribution on a small scale at the surface of the superconducting material is dependent upon vortex position. Magnetic fields only pass through the fluxon lines, and so thefield peaks above the lattice points. However, at larger distances and larger scales the contributions from individual vortices sum together to form a much smoother distribution. On these scales the magnetization of the superconductor is approximately equivalent to that produced by a circular current of radius a and magnitude I. Then the components of the magnetic field in spherical coordinates, where θ is the angle from the c-axis, ϕ is the angle in the x-y plane, and r is the distance from the center of the sample, are as follows:

$$B_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\phi) \tag{1}$$

$$B_\theta = -\frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) \quad (2)$$

$$B_\phi = 0 \quad (3)$$

In the above equations the A_ϕ component of the vector potential is defined as:

$$A_\phi(r,\theta) = \frac{Ia}{c}\int_0^{2\pi} \frac{\cos\phi' d\phi'}{(a^2 + r^2 - 2ar\sin\theta\cos\phi')^{1/2}} \quad (4)$$

FORTRAN does not include prepackaged integral or differential functions, so the adaptation of these equations was a particular challenge. To approximate the integral, the expression was evaluated at one hundred values of ϕ' between 0 and 2π and multiplied by $d\phi' = \frac{2\pi}{100}$. Likewise, the definition of a derivative was used for the partial differentials. The partials were calculated by finding the value of the expression just above and just below the point of interest, taking their difference, and dividing by the distance covered.

However, additional problems were encountered with small values of r and θ. These both produce impossible results using the field components as written above. However, an alternate expansion of A_ϕ in powers of $a^2r^2\sin^2\theta/(a^2 + r + 2)^2$ can be found. When the components of the field are derived from this expansion, they become:

$$B_r = \frac{2\pi Ia^2\cos\theta}{c(a^2 + r^2)^{3/2}}[1 + \frac{15a^2r^2\sin^2\theta}{4(a^2 + r^2)^2} + \ldots] \quad (5)$$

$$B_\theta = -\frac{\pi Ia^2\sin\theta}{c(a^2 + r^2)^{5/2}}[2a^2 - r^2 + \frac{15a^2r^2\sin^2\theta(4a^2 - 3r^2)}{8(a^2 + r^2)^2} + \ldots] \quad (6)$$

With these new expressions, the small angle approximations $\sin\theta \approxeq \theta$ and $\cos\theta \approxeq (1 - \frac{\theta^2}{2})$ can be employed. Under these conditions the field components can be accurately approximated for $\theta \ll 1$:

$$B_r = \frac{2\pi I a^2 (1 - \frac{\theta^2}{2})}{c(a^2 + r^2)^{3/2}} \tag{7}$$

$$B_\theta = -\frac{\pi I a^2 \theta}{c(a^2 + r^2)^{5/2}} (2a^2 - r^2) \tag{8}$$

Likewise, approximations can be found for $r \ll a$ and the special case where both $r \ll a$ and $\theta \ll 1$. These were included in the FORTRAN model, but will not be presented here. These small values of r correspond to a point that would experimentally be inside the superconducting sample. This model cannot give appropriate interior calculations because it assumes the field to be continous rather than channeled through quantum flux vortices in the material.

The FORTRAN model was adapted to find the field across a range of points, in cartesian coordinates, in the x-c plane. Because the model is symmetric with respect to ϕ, the choice of x was arbitrary - rotating the plane at any angle about the c-axis would give the same result. For the purposes of the TRIUMF experiment we are interested in finding the fringing field at a distance h away from the sample along the c-axis, to correspond to the distance of the backing of the container material that will hold the sample. This distance has not yet been specified, but is expected to be on the order of 1cm or less. This modeling situation is described graphically in Figure 3.

2.2 Model results

The results of this model were obtained and graphed for backing distances of 0.5cm and 1cm away from the sample. We expect to have a sample radius of approximately 1cm, and although the experiment will be run at multiple applied field intensities, a fringing model equivalent to an applied field of 30 gauss was chosen for these runs. At other intensities we expect the fringe fields to only differ in magnitude, not in the shape of the distribution. The results of this modeling situation can be seen in Figure 4.

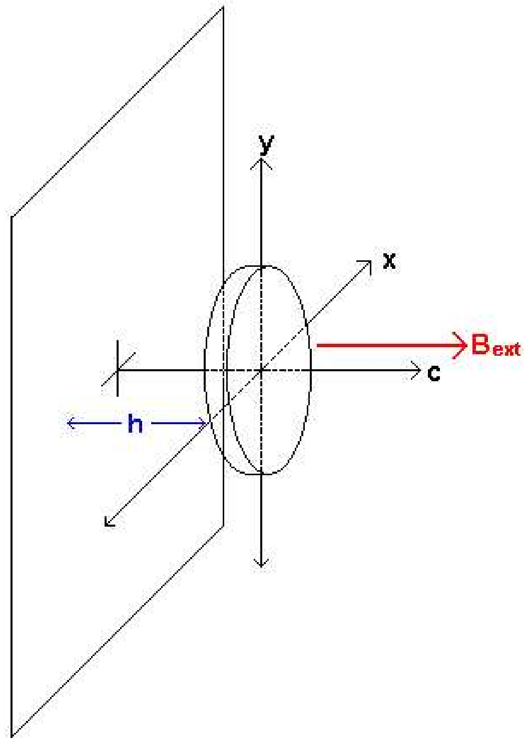

Figure 3: Model diagram of the sample, container backing material, and applied magnetic field.

We see that at shorter distances the fringing field is expected to be bimodal, with peak magnitudes directly above the edges of the superconducting sample. With symmetry about the angle ϕ, we can envision the field magnitude distribution across the entire plane of the backing material to form a volcano-like shape about the c-axis. However, at a distance of 1cm the overall field magnitude is lower and the bimodal peaks are no longer visible.

3 Measurement of fringing fields in YBCO

To test our model, we conducted small-scale experimental measurements of the magnetic field near the surface of several medium-quality samples of superconducting $YBa_2Cu_3O_7$.

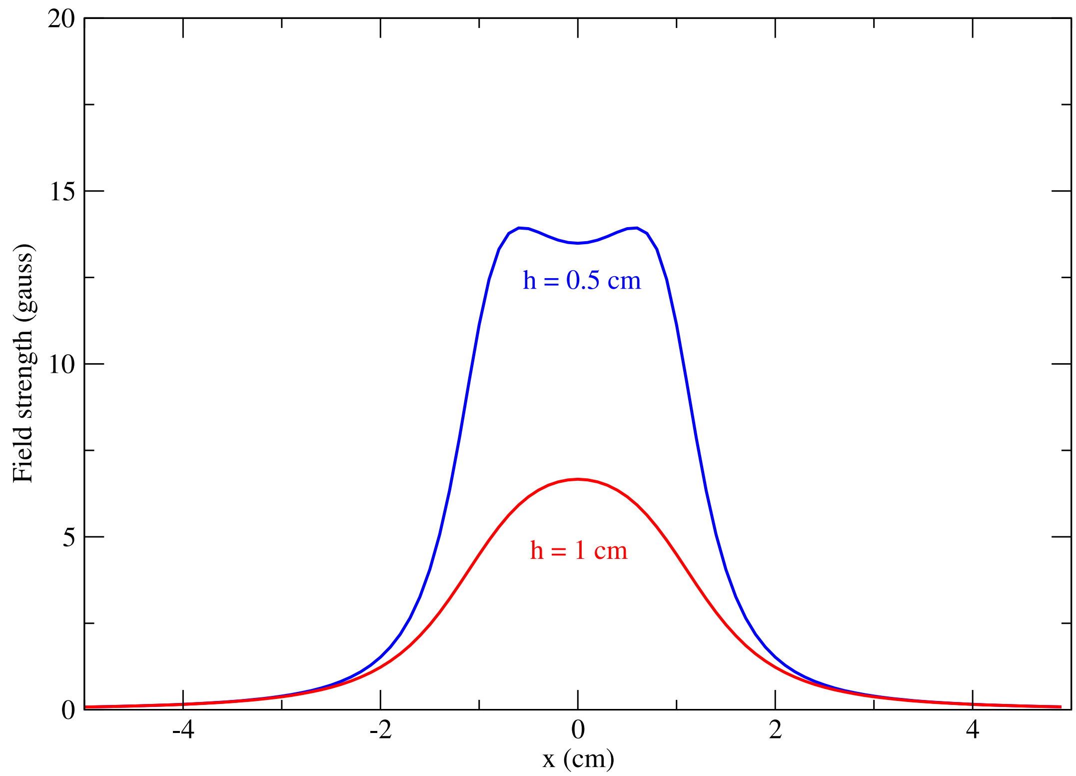

Figure 4: Fringing fields from a superconducting sample with a radius of 1 cm magnetized in the presence of a perpendicular external magnetic field of 30 gauss, modeled at distances of 0.5 cm and 1cm from the sample along the c-axis.

These measurements were done with samples magnetized in fields parallel to the crystalline c-axis, and on samples magnetized perpendicular to the c-axis. Measurements were completed for several sintered samples, which are made by heating powdered $YBa_2Cu_3O_7$ below the melting point until the grains adhere, and for one single-crystal sample.

3.1 Experimental setup

In our experimental apparatus, superconducting samples were held in place with cardboard and electric tape to a long nonmetallic stick. This holder was lowered into a tapered dewar that is long and thin at the bottom. The holder was fitted to the dewar in a way that would place the sample directly between the poles of a 220 mT permanent ferromagnet placed around the bottom of the dewar. In $YBa_2Cu_3O_7$ the critical temperature below which the sample will superconduct is 92K. The sample was cooled to below T_c by filling the dewar with liquid nitrogen at 77K.

After ten minutes of cooling to ensure that the sample was well below T_c, the holder was removed from the dewar and the sample was immediately transferred from the holder to a secondary container of liquid nitrogen. The sample was placed on top of a conducting copper stand inside the container, adjusted to a height that would put set the top surface of the sample flush with the rim of the container. A hall probe on a moveable stage was set up to measure the surface magnetic field in a line directly across the center of the sample. A strip of tape beneath the stage was marked with distances in millimeters to show the position of the probe. Measurement would begin at 25mm behind the center of the sample and continue forward to 25mm in front of the center of the sample. During measurement the container would be periodically topped off to the brim with additional liquid nitrogen whenever the level fell below the bottom of the sample. However there was very little risk of falling below T_c as the nitrogen level fell, due to the conductivity of the copper stand.

3.2 Calibration

To ensure that magnetic fields were fully penetrating into the dewar, the field inside the dewar between the poles of the ferromagnet was measured with a hall probe while the dewar was filled with liquid nitrogen. The field strength was measured as 220mT, the same as the dry measurement of the magnet without the dewar.

To test whether the hall probe measurements would be affected by temperature, a ferromagnetic kitchen magnet was measured in our apparatus first at room temperature and then again when the measurement container was filled with liquid nitrogen. In the liquid nitrogen run, the magnet and the tip of the hall probe were allowed to cool for several minutes before measurement began. The results of these two runs, shown in Figure 5, are nearly identical. Thus we believe that temperature has no effect on the reliability of our measurements.

Before any measurement, the hall probe was zeroed at the -25mm position in the absence of the sample. No position-dependent background field was observed as the probe was moved in the absence of the sample.

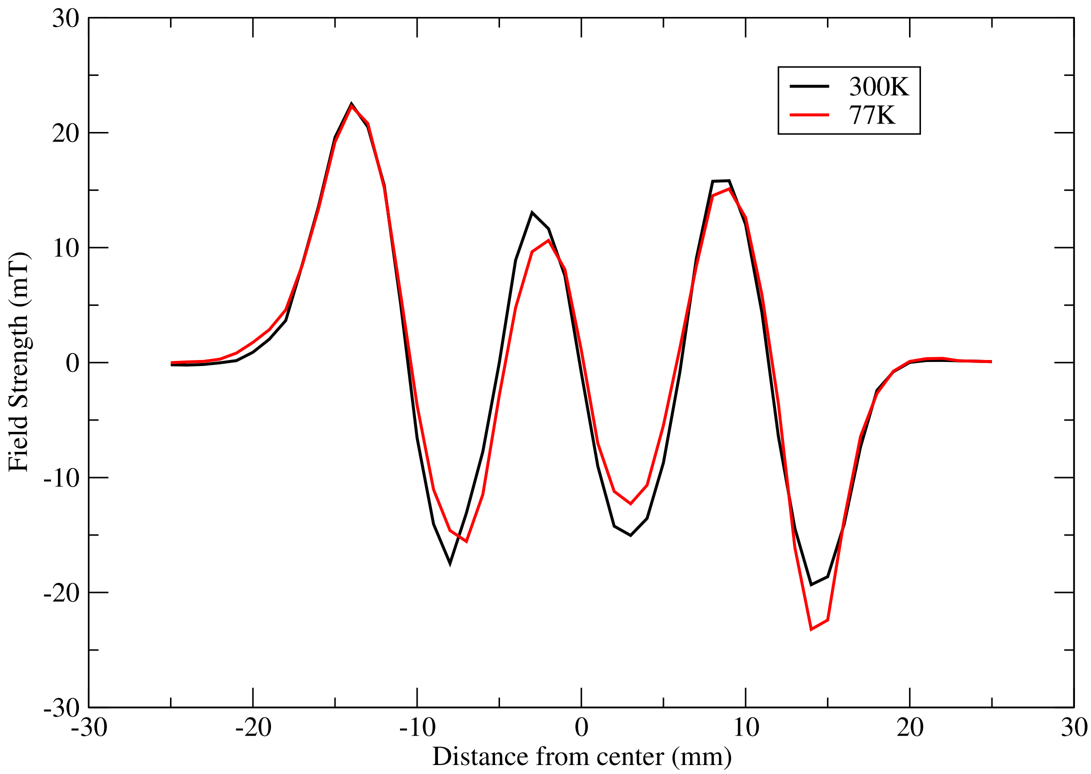

Figure 5: Surface fields of a circular kitchen magnet of 31mm diameter, measured at 300K and 77K.

3.3 Measurement results

Measurement began on a circular sintered sample of $YBa_2Cu_3O_7$ with a diameter of 23.56mm and a depth of 4.41mm. This sample was field-cooled in 220mT as explained in Section 3.1 above. This sample was magnetized with the field perpendicular to the circular plane. The measured field at the surface was an approximate bell curve or bimodal bell curve with an amplitude of only 0.12 mT, as shown in Figure 6. The hall probe can only resolve to 0.01mT, and the lack of smoothness in the results may be due in part to the low sensitivity of the probe to very small fields.

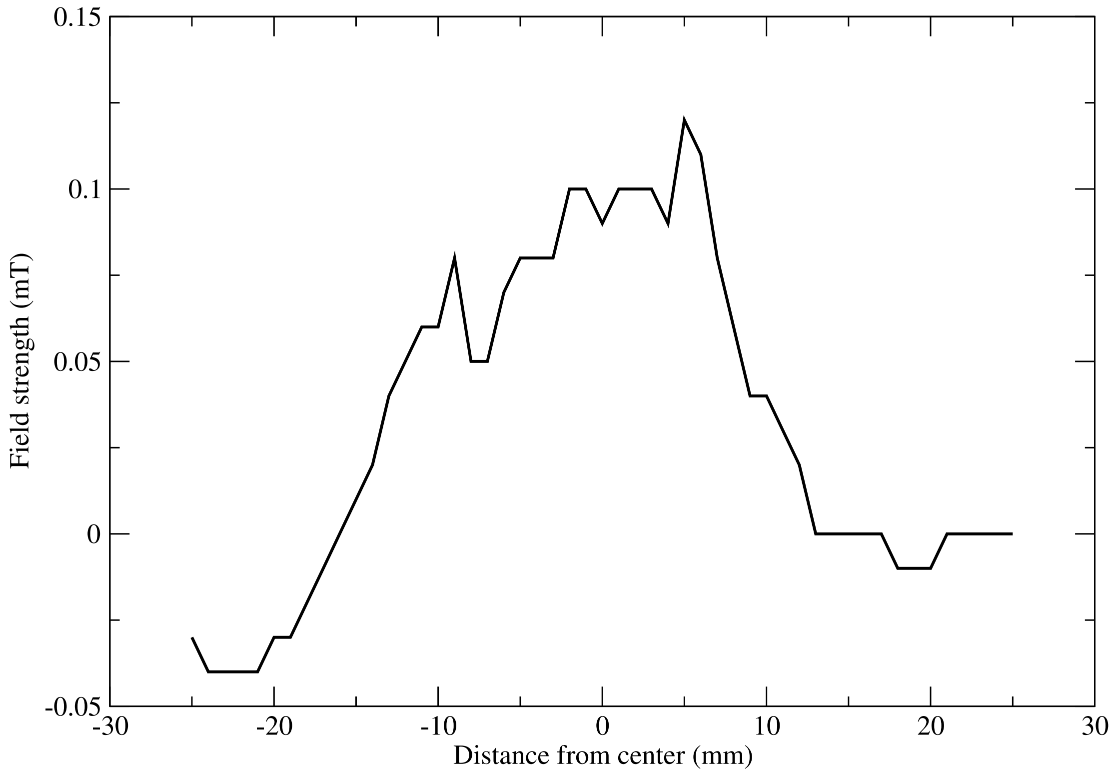

Figure 6: Surface fields of a circular sintered YBCO sample field-cooled in 220mT perpendicular to the circular plane.

These measurements were repeated with the sample magnetized parallel to the circular plane. Rather than a uniform zero field as predicted from the computer model, this resulted in a field shaped like a sine wave centered about the middle of the sample. However the fields were still very small, with a maximum amplitude of only 0.08mT.

These results were much smaller than expected, so the sample was allowed to warm to room temperature and was then field-cooled one more time. To test whether the small

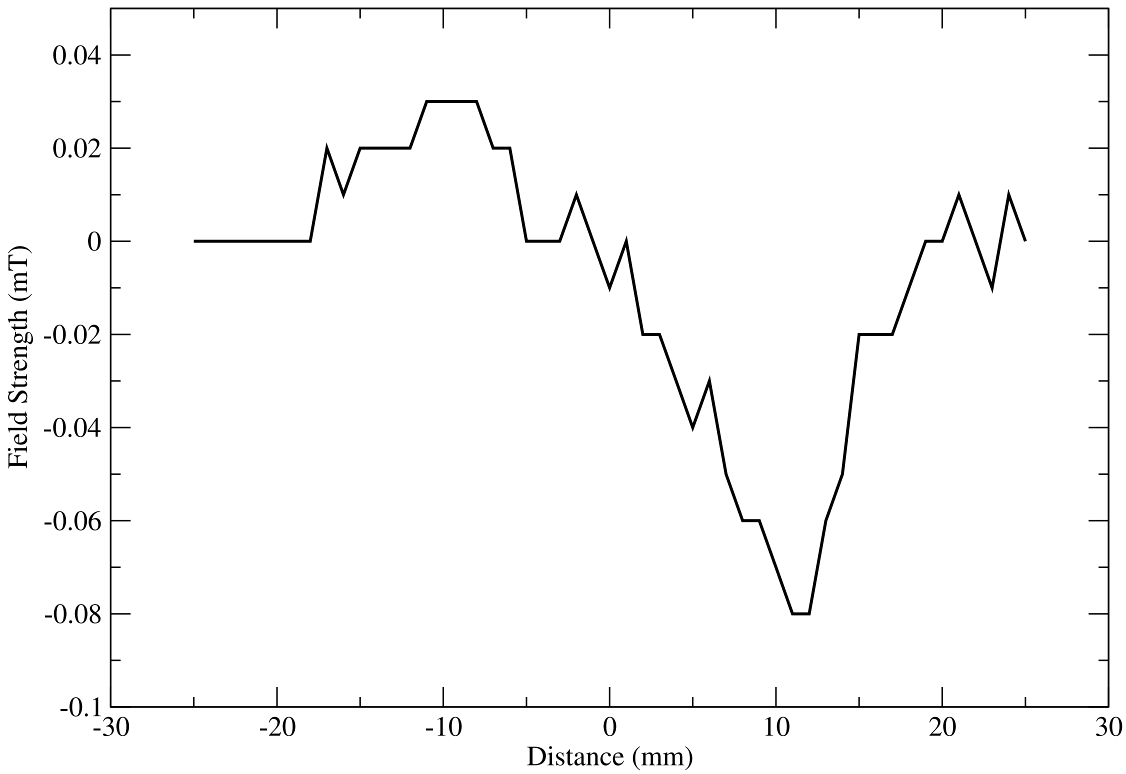

Figure 7: Surface fields of a circular sintered YBCO sample field-cooled in 220mT parallel to the circular plane.

field could be due to warming above T_c during transfer of the sample to the measurement apparatus, the surface field was measured with a hall probe immediately upon removal from the dewar, without removal from the holding stick. The field was still measured as 0.10mT, suggesting that the small field was not due to a rise above T_c. To ensure that these small fields were not particular to this one sample, measurements were taken on several other sintered samples of $YBa_2Cu_3O_7$ field-cooled in the 220mT field perpendicular to their surface plane. One sample was an irregular polygonal fragment from a larger sample, measured across its longest point which was approximately 19mm across. Another sample was thin and circular with a diameter of 22mm and a depth of 2mm. A third sample was a new, previously unused circular sample with a diameter of 23.5mm and a depth of 5mm. The results of these three measurements are shown in Figure 8. The fields from each of these samples were still very small, not reaching an amplitude of more than 0.36 mT. The asymmetric field distribution from the thin circular sample suggests that it may have been inadvertently rotated during field-cooling.

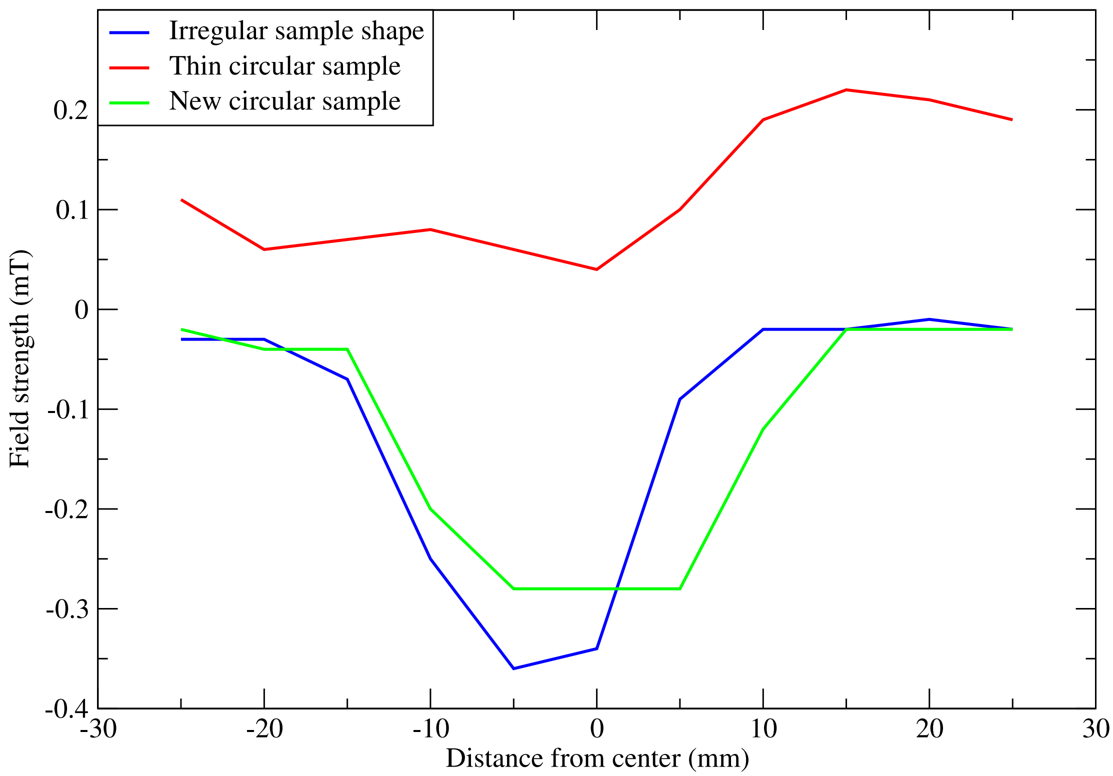

Figure 8: Surface fields of three sintered YBCO samples field-cooled in 220mT perpendicular to the circular plane.

The measurement was tried one more time with a single-crystal hexagonal $YBa_2Cu_3O_7$ sample. The sample was 23mm along its long axis between opposite corners, 21mm along its short axis between opposite sides, and had a depth of 13mm. The sample was magnetized in 220mT parallel to its crystalline c-axis, perpendicular to the crystalline plane and the surface plane. Measurements were made across the 23mm long axis, with results shown in Figure 9. The field distribution is an approximate bell curve, with a peak of 75mT. This is smaller than the magnetization field by a factor of three, but orders of magnitude larger than the field measured from the sintered samples.

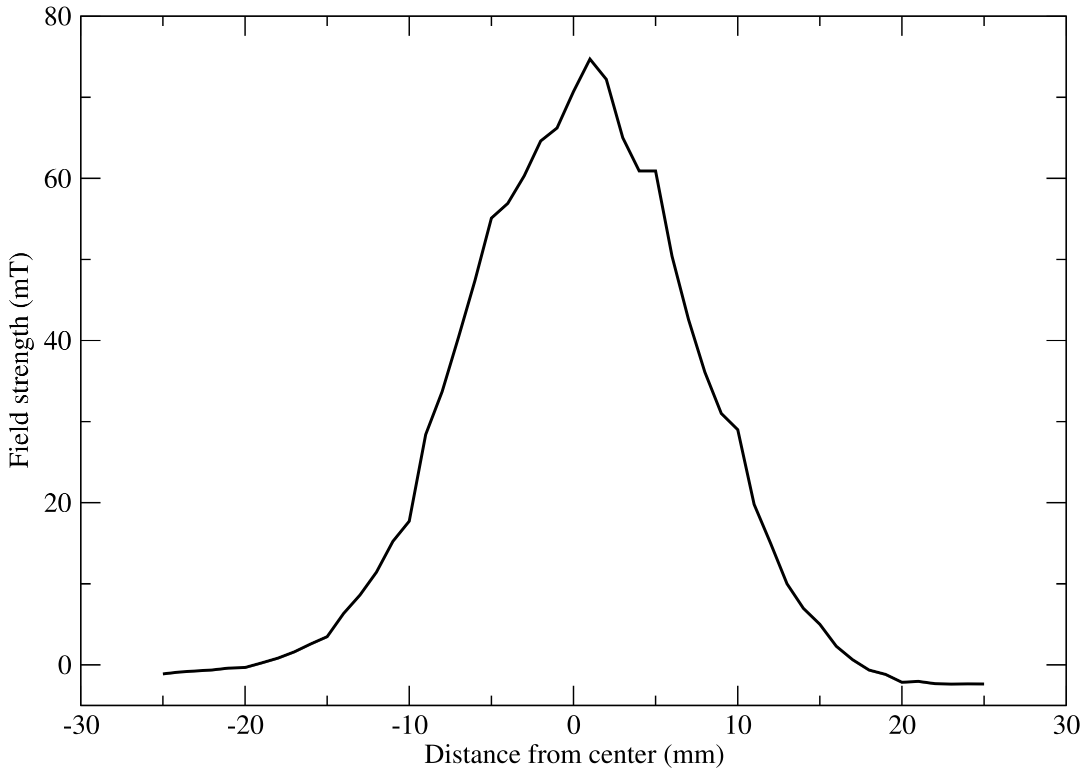

Figure 9: Surface fields of a hexagonal single-crystal YBCO sample field-cooled in 220mT perpendicular to the crystalline plane.

4 Discussion

We observe that neither sintered nor single-crystal $YBa_2Cu_3O_7$ samples appear to experimentally fit our model. We predicted that the samples would retain a locked-in magnetization equivalent to the magnetization field, so that the magnetic field at the surface of the sample would be equal to the magnetization field. However, the field at the surface of the single-crystal sample was a third of this, and the field at the surface of the sintered samples was lower than the magnetization field by a factor of nearly a thousand.

This contradiction could suggest several things about what is happening inside the superconducting samples. Perhaps vortex lines need to pass all the way through a sample to result in locked-in magnetization. If this is the case, the irregular arrangement of anisotropic superconducting grains in a sintered sample would block the passage of flux lines and result in near-perfect diamagnetism despite being made from type II superconducting material. When magnetic fields are almost completely excluded from the interior, magnetization will be much weaker.

The single-crystal results remain a puzzle. One idea is that the crystalline planes are slightly insulated from each other, meaning that even though the bottom of the thick sample was bathed in liquid nitrogen during all parts of the measurement, some crystalline planes near the top of the sample could have reached a temperature above T_c, no longer contributing to the measured magnetic field. Whenever the measurement container was topped off with more liquid nitrogen, the measured field was seen to rise by several mT before dropping down to the previous level. New measurements with an experimental apparatus that allows complete submersion of the sample in liquid nitrogen should be made to test this hypothesis.

In both cases, the magnetization of superconducting samples is not as simple as our computer model suggests. This means that fringing fields in the TRIUMF experiment might not easily be accurately predicted beforehand. If a sintered sample is to be used in the TRIUMF experiments, then our results suggest that the fringing fields may be small enough to be ignored as not having a significant effect on the results. However, for measurements of single-crystal samples at TRIUMF the locked-in magnetization of the sample may need to be separately measured to discount its effects.

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