Search for Dark Matter in Globular Clusters

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by

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Abstract

Globular clusters are effectively approximated by using a King-Michie distribution. Sherbakov proposed that identifying the possible amount of dark matter present in globular clusters would improve understanding of their behavior. This thesis examines the properties of one-component and two-component models in order to maximize the percentage of dark matter. We find that we can obtain excellent density fits for some globular clusters (e.g., NGC 288 and NGC 6981) with up to 95% of the matter being dark. On the other hand, for at least one globular cluster (NGC 5824) we have not found solutions with more than 50% dark matter.

Acknowledgements

To the Professors of the College of William and Mary.

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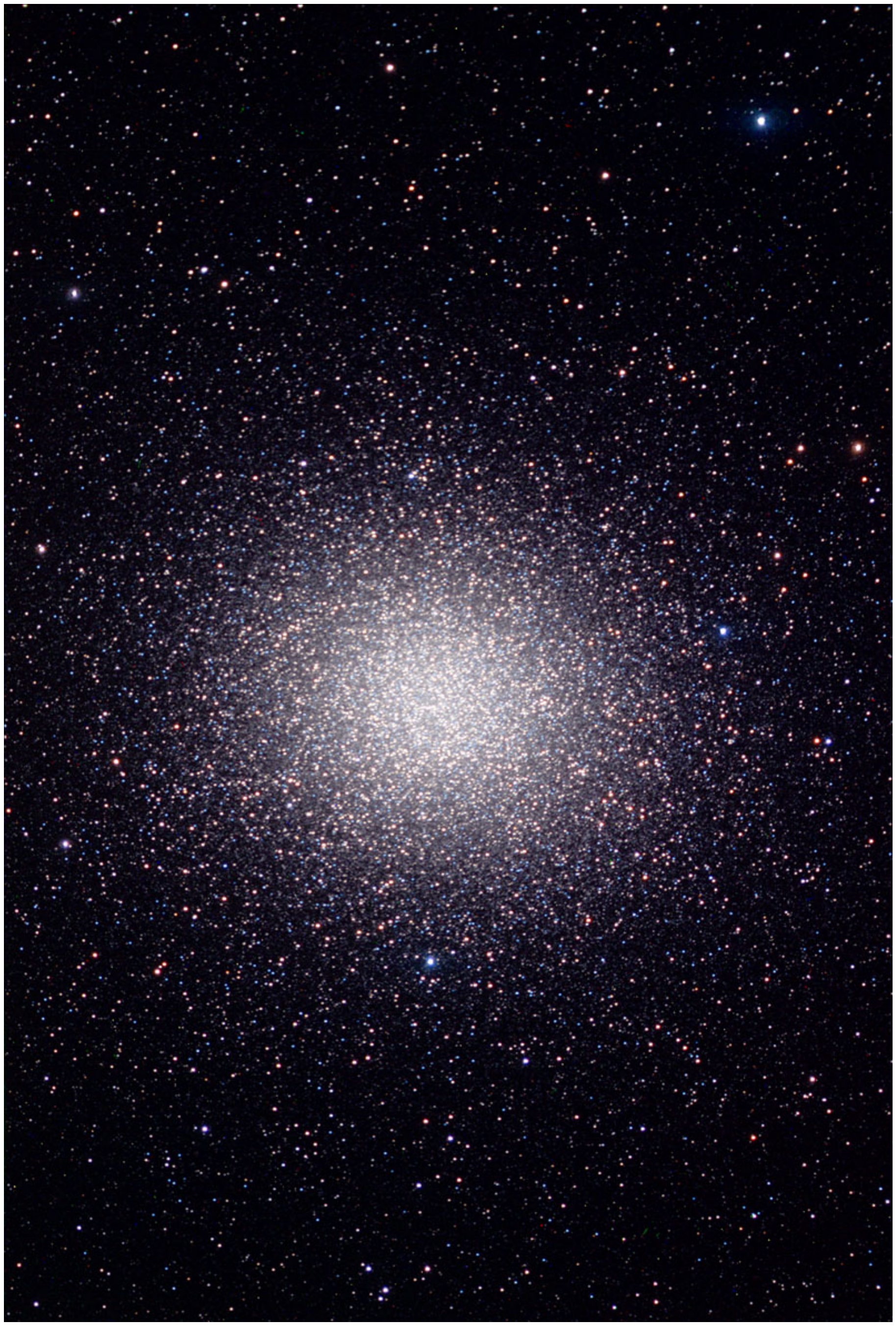


Figure 1: Omega Centauri, NGC 5139

1 Introduction

This thesis attempts to approximate the observable density distributions of globular clusters that are poorly described by current astronomical models by allowing for the presence of dark matter. By adding enough matter to create closer fits to clusters that are inadequately modeled; as well as those well-fit by current models, it is possible to suggest maximum densities of dark matter within globular clusters. An extension of this thesis could be to compare these models with those suggested by more complicated force laws.

1.1 History

Many scientists have tried to discover patterns behind the behavior of globular clusters, since they have been resolved. The first clusters were mistaken at first, for luminous spots and patches by the great English scientist, Edmund Halley, in the Royal Societys Philosophical Transactions in 1715. It was not until the German-born English astronomer, William Herschel, with sufficient time and equipment, that these luminous spots could be discerned into individual stars.

As technology improved, some 150 globular clusters were discovered in the Milky Way, but more interestingly, similar formations were found around other galaxies, usually in amounts proportional to the galaxies size. Around the nearby Andromeda for instance, there are some 250 known clusters; while larger galaxies, such as M87, have tens of thousands of globular clusters in orbit. [7]

Naturally, scientists have attempted to apply physical principles to form models that explain the distribution of globular clusters. An early model, suggested by H.C. Plummer in 1911 is based on the assumption that the potential is:

$$V = -\frac{GM}{\sqrt{r^2 + a^2}} \tag{1}$$

and by applying Poissons equation, the density profile is:

$$\rho(r) = \frac{3M}{4\pi a^3}(1 + \frac{r^2}{a^2})^{-5/2} \quad (2)$$

But while the Plummer Model appeared accurate enough in 1911, new data provided by more powerful telescopes proved the model ineffective near the center and tail of most distributions.

While many more models were presented which proposed unusual force laws or potentials, King and Michie, in the 1960s advocated different methods to describe the organization of globular clusters. Their idea was to focus on the observable density profile, by creating a reasonable numerical distribution based on principles of statistical and classical mechanics. Then, by solving Poissons equation self-consistently, it was possible to find the approximate potential for the cluster, and then the actual stellar density.

The disadvantage of this method is that there are several initially unknown parameters within the integration and solution of the differential equation, which must be matched to observational data.

2 The King-Michie Method

The King-Michie method, soon to be described, received support more recently from the works of Kent and Gunn [5] and Grillmair et al. [3], who found that such models could match closely with experimental evidence. While Kent and Gunn found that the King model was adequate for many globular clusters, Grillmair et al. noted that several clusters were poorly mapped with the King Method. But as the Michie model is a more versatile approach, which uses an extra parameter not accounted for by Grillmair, it seems reasonable that a model that takes into account only visible matter may be able to successfully describe the stellar distribution of globular clusters.

This approach was attempted by Sherbakov [4] with some success.

The general method used to solve explicitly for models of this type is to begin by identifying certain observable conditions, which can be exploited for the solution. The model can be solved by imposing several such restrictions, including: the tidal radius, a limit to the size of the cluster, beyond which all stars are considered to be separated from the cluster; the core radius, the point at which the surface density is estimated to fall by half; and the anisotropy radius, the point at which the radial component begins to dominate the velocities of stars in the globular cluster. As the later King Model does not account for anisotropy, the King model can be considered a Michie Model as the anisotropy radius approaches infinity.

The distribution for the Michie model, presented in 1961, is

$$f(\vec{r},\vec{v}) = ke^{-j^2L^2/r_a^2}\left(e^{-j^2\left(2V(r)+v^2\right)}-1\right)\theta(-2V(r)-v^2) \tag{3}$$

where k is a normalization constant, j is a constant proportional to the velocity dispersion (Sherby), L is the angular momentum, ra is the anisotropy radius, V(r) is the gravitational potential, and v is the velocity. The step function is imposed so that any star with a velocity greater than the classical escape velocity is not included.

The number distribution is then normalized over all positions,

$$N = \int d^3x\, d^3v\, f(\vec{r},\vec{v}) \tag{4}$$

Which is proportional to the density,

$$\begin{aligned}
\rho(r) &= m\int d^3v\, f(\vec{r},\vec{v}) \\
&= m\int_{-1}^{1} 2\pi d(\cos\theta)\int \frac{1}{2}v\, dv^2 f(\vec{r},\vec{v}) \\
&= 2\pi mkj^{-3}\,\frac{1}{2}\int_{-1}^{1} dy\int_0^W d\eta\,\eta^{1/2}\,e^{-\alpha^2\eta(1-y^2)}\left(e^{W-\eta}-1\right)
\end{aligned} \tag{5}$$

The integration is then performed by introducing several placeholder variables:

$$W = -2j^2V(r)$$

$$
\begin{aligned}
\eta &= j^2 v^2 \\
\alpha &= \frac{r}{r_a} \\
y &= \cos\theta
\end{aligned} \tag{6}
$$

The density integral becomes, then,

$$
\rho(r) = 2\pi m k j^{-3}\, e^{W} \int_0^W d\eta\, \eta^{1/2} \left(e^{-\eta} - e^{-W}\right) \int_0^1 dy\, e^{-\alpha^2 \eta (1-y^2)} \tag{7}
$$

Then, by applying Poissons equation

$$
\nabla_r^2 V(r) = 4\pi G \rho(r) \tag{8}
$$

Substituting $W(r)$, and $R = r/r_c$, where the value of $8\pi j^2 r_c^2 \rho_0 = 9$, was defined by King's observations.

$$
\begin{aligned}
\nabla_r^2 W(r) &= -8\pi j^2 G\rho \\
R &= \frac{r}{r_c} \\
\nabla_R^2 W(R) &= -8\pi j^2 r_c^2 G\rho
\end{aligned} \tag{9}
$$

and, taking the following condition, defined by King

$$
8\pi j^2 r_c^2 G \rho_0 \equiv 9 \tag{10}
$$

In spherical coordinates,

$$
\nabla_R^2 W(R) = -9\frac{\rho(r)}{\rho_0} \tag{11}
$$

This differential equation is solved with the help of Mathematicas NDSolve. The initial condition of W determines the tidal radius, R_t, while the values of R_a and r_c

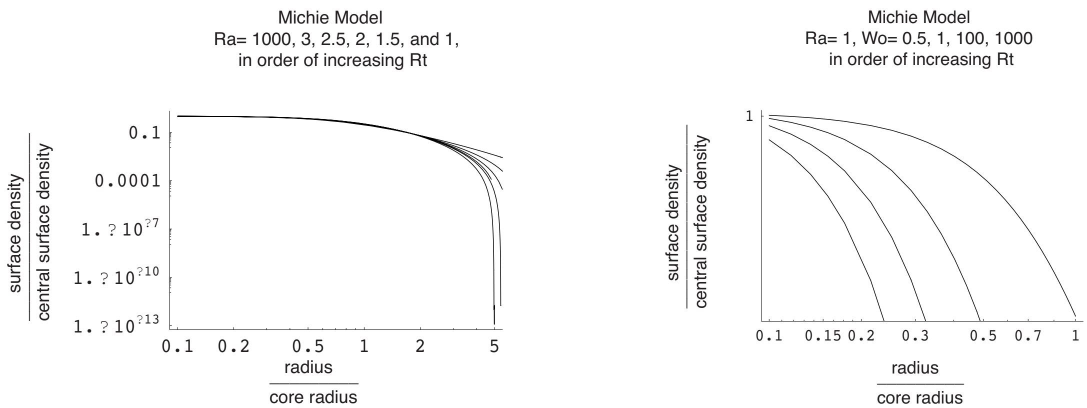


Figure 2: The behavior of the model as R_a and W_0 are increased

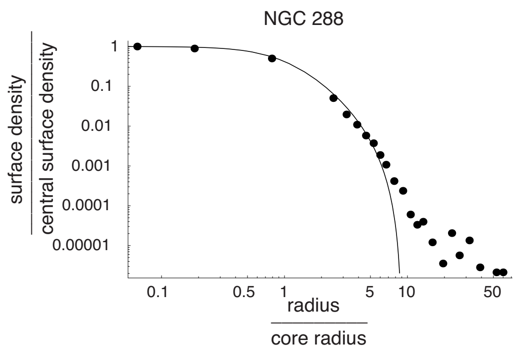
are adjusted so that the data is matched as closely as possible. Usually this method is applied so that the slope on a log-log plot of the data is identical to that of the model. The slope between two points on the cluster can be matched by varying W_0 and R_a which show the general trend of increasing R_t when W_0 is increased and when R_a is decreased. Both have the effect of flattening the model, but an increase in R_a is bound by the King model as R_a goes to infinity.

3 Evidence

The data provided by Grillmair, which compares the stellar distribution of seven clusters in the Milky Way galaxy with the King model, is successful for NGC 2808 and NGC 4590, but fails near R_t for NGC 288, NGC 362, NGC 1904, and NGC 3201, and misses entirely for NGC 7089. In all circumstances, particularly NGC 288, the approximation is improved when the Michie model is applied. Above is a good fit using the King Model [4]

The next best approximation method, the Michie model, improves the accuracy by adding the additional constraint, r_a. Even under the best circumstances, the Michie model does not perfectly match actual data

Interestingly, there are a variety of good fits to the data provided for every value



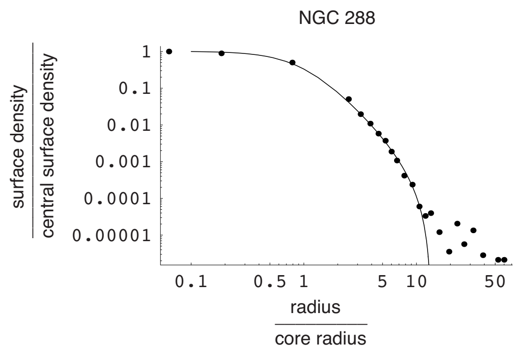


Figure 3: King Model with $R_t = 9.28$, and Michie Model with $R_a = 1.3887$ and $W_0 = 3.643$

of r_c, and since the value of r_c is uncertain, there are a very large number of possible conditions which produce equally good fits. Without more accurate determination of the stars at the boundary of the cluster, it becomes difficult to find which curve is the correct model, only a range of curves, and an even greater range of possible initial conditions.

4 Dark Matter

The remaining question, introduced in the introduction, is how the presence of dark matter, a massive but not luminous component of the globular cluster, affects the accuracy of these models. When an observed distribution continues with a strict power law dependence in the tail of the distribution, particularly in clusters like NGC 288 and NGC 5824, it becomes necessary to alter the King-Michie model to include an extra matter distribution to produce accurate fits.

The two-component model, developed by Roueff *et al.* [2], suggests that there is another dark-matter based solution to Poissons equation, which can be modeled by extending the Michie Method. In effect, it serves to increase the number of parameters to flexibly modify a one component model, while at the same time, allowing the tidal radius to be shifted outward by increasing the dark matter ratio. Since the model is nonlinear, it is not generally possible, except for special cases, to find a one-component model that matches the two-component model exactly. The added dark matter component serves to increase the number of possible solution curves. Considering the great uncertainties in observing low mass stars, and of accounting for fluctuations in the stellar background near the cluster's edge, it seems reasonable to consider the possibility of an unknown mass within the cluster. This method does not; however, allow for the density profile to exceed a power-law distribution, a possibility suggested by Kent and Gunn [5].

Rouef *et al.* [2] compared the change in the velocity dispersion along the line of sight using a two-component Michie model, while adjusting the anisotropy radius, and the light to dark matter ratio. They concluded that while a model with large anisotropy radii would allow for large changes in the tail as the dark matter ratio was increased by a factor of ten, the effect was negligible at low anisotropy radii. They suggested that either a change in the distribution function, or some new factor outside the tidal radii, such as a tidal tail, might be needed to accurately model globular clusters.

This idea was explored earlier by Moore [10], who suggested that there might be a surrounding dark-matter halo which serves to stabilize the stellar distributions of globular clusters, and through the works of Grillmair [8] [9] and Leon, Meylan, & Combes [6] who observed tails around numerous clusters, in the Milky Way and Andromeda galaxies.

The difficulties faced in matching the two-component model to observational data are similar to those encountered with the one-component, except that finding a good fit near the tail of the distribution becomes increasingly difficult. Since these tails are observed to extend far beyond the imposed tidal radius condition, a perfect model remains elusive. The question becomes then, if current models can suggest a limit to the amount of dark matter present in globular clusters.

5 The Two-Component Method

The two-component method was developed by Roueff *et al.* specifically to better describe the velocity dispersion profiles within globular clusters, and includes the addition of a second, possibly non-luminous, stellar density profile, with an unknown mass.

The Michie profile, defined above, can be conveniently written in terms of the function,

$$H(W,\alpha) = e^W \int_0^\infty d\eta\, \eta^{1/2} \left(e^{-\eta} - e^{-W}\right) \int_0^1 dy\, e^{-\alpha^2 \eta (1-y^2)} \tag{12}$$

Where the constant of the first model has been absorbed by the integral, and the stellar density is derived from the number distribution.

$$f(r,v) = k e^{-j^2 L^2/r_a^2} \left(e^{-j^2(2V(r)+v^2)} - 1\right) \theta(-2V(r) - v^2) \tag{13}$$

The second mass system, with a similar form for the number distribution, has potential and kinetic energy proportional to the ratio between the two masses,

$$M = m_2/m_1 \tag{14}$$

as follows,

$$\begin{aligned} f_2(r,v) &= k_2 e^{-Mj^2 L^2/r_{a2}^2} \left(e^{-Mj^2(2V(r)+v^2)} - 1\right) \theta(-2V(r) - v^2) \\ &= k_2 e^{-Mj^2 v^2 r^2 \sin^2\theta/r_{a2}^2} \left(e^{-j^2(2V(r)+v^2)} - 1\right) \theta(-2V(r) - v^2) \end{aligned} \tag{15}$$

where we have allowed for a different anisotropy radius r_{a2} and a different normalization. The number distribution is then simplified by substituting the following placeholder variables:

$$\begin{aligned} \eta &= Mj^2 v^2 \\ W &= -2j^2 V(r) \\ \alpha_2 &= r/r_{a2} \\ y &= \cos\theta \end{aligned} \tag{16}$$

So the distribution, $f_2(r,v)$ becomes

$$f_2(r,v) = k_2 e^{-\alpha_2^2 \eta(1-y^2)} \left(e^{MW-\eta} - 1\right) \theta(MW - \eta) \tag{17}$$

And the density,

$$\begin{aligned} \rho_2(r) &\equiv m_2 \int d^3v \; f_2(r,v) \\ &= m_2 k_2 \int_{-1}^{1} 2\pi d(\cos\theta) \int_0^{v_e^2} \frac{1}{2} v dv^2 \; e^{-\alpha_2^2 \eta(1-y^2)} \left(e^{MW-\eta} - 1\right) \\ &= 2\pi m_2 k_2 M^{-3/2} j^{-3} e^{MW} \int_0^M W d\eta \; \eta^{1/2} \left(e^{-\eta} - e^{-MW}\right) \int_0^1 dy \, e^{-\alpha_2^2 \eta(1-y^2)} \end{aligned} \tag{18}$$

Using the function definition $H(W,\alpha)$, the density distribution of the second stellar mass component is given by,

$$\rho_2(r) = 2\pi k_2 m_2 \left(m_2/m_1\right)^{-3/2} j^{-3} H(\frac{m_2}{m_1} W, \alpha_2) \tag{19}$$

Imposing the scaling factor, of the central density of the cluster,

$$\rho_2(r) = \rho_{c2} \frac{H(m_2 W/m_1, \alpha_2)}{H(m_2 W_0/m_1, 0)} \tag{20}$$

Poisson's equation becomes,

$$\nabla_r^2 V(r) = 4\pi G \rho(r)$$

In terms of $W(r)$

$$\nabla_r^2 W(r) = -8\pi j^2 G \left(\rho_1(r) + \rho_2(r)\right) \tag{21}$$

And substituting the additional scaling factor,

$$R = \frac{r}{r_c} \tag{22}$$

The equation becomes

$$\nabla_R^2 W(R) = -8\pi j^2 G r_c^2 (\rho_1 + \rho_2) \tag{23}$$

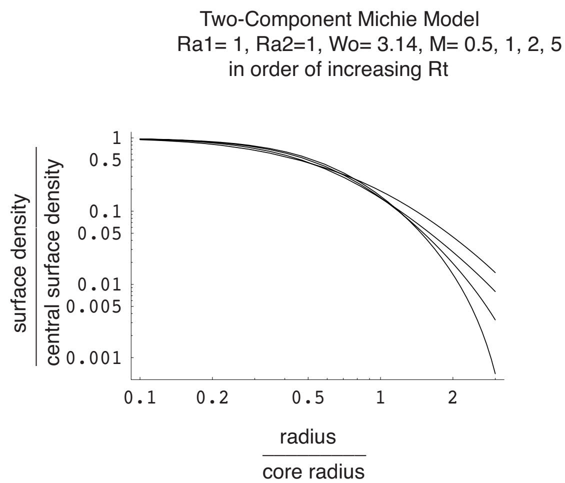


Figure 4: The behavior of the model as the proportion of dark matter is increased

And defining r_c from the requirement

$$8\pi j^2 r_c^2 G\rho_{c1} \equiv 9 \tag{24}$$

With the result,

$$\nabla_R^2 W(R) = -9\frac{\rho_1 + \rho_2}{\rho_{c1}} = -9\left(\frac{H(W,\alpha_1)}{H(W_0,0)} + \frac{\rho_{c2}}{\rho_{c1}}\frac{H(m_2W/m_1,\alpha_2)}{H(m_2W_0/m_1,0)}\right) \tag{25}$$

Which can be solved using Mathematica's NDSolve.

The improved model shows a regular change, as the ratio of dark matter to light matter is increased, which parallels the trend observed for an increase in W_0.

When $M = 0$ the two-component model returns to the one-component, and as M is increased, solutions become increasingly difficult to determine using NDSolve. For example when R_{a1} and R_{a2} are set equal to one, the Mathematica code is limited to considering solution curves to Poisson's equation until $M = 9$. While this ratio can be increased further by changing the anisotropy radius of either the visible or dark matter component, it does not allow for the overall percentage of dark matter to increase. It becomes more difficult to accurately fit real distributions when this percentage is high, and so perhaps, the region beyond Mathematica's range can be considered past the boundary of a realistic fit.

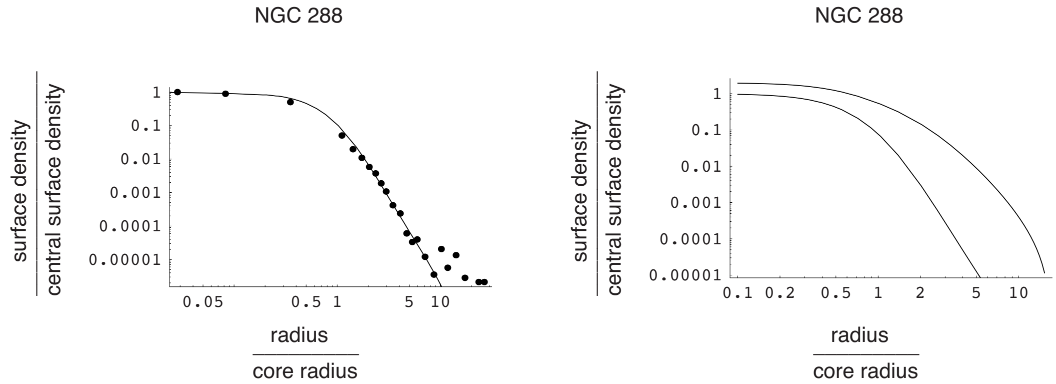


Figure 5: Modeled with $r_c = 3.2, M = 0.1, W_0 = 11, R_{a1} = R_{a2} = 1$

It is not usually necessary to consider a high dark matter in order to find a good fit, as shown by Sherbakov [4] who was successful in modeling many clusters, including NGC 288. Above is her two-component fit, and the resulting density profile, where the outer curve represents the total matter distribution. Such a distribution represents a cluster that is 95.5% dark matter.

The most interesting part of this model is that at sufficiently large anisotropy radii, it reduces to the original King Model, and is therefore consistent with clusters previously successfully modeled with one-component models. For NGC 6981, a cluster well-fit by the King Model, there is the following two-component fit and density profile, with 94.9% dark matter:

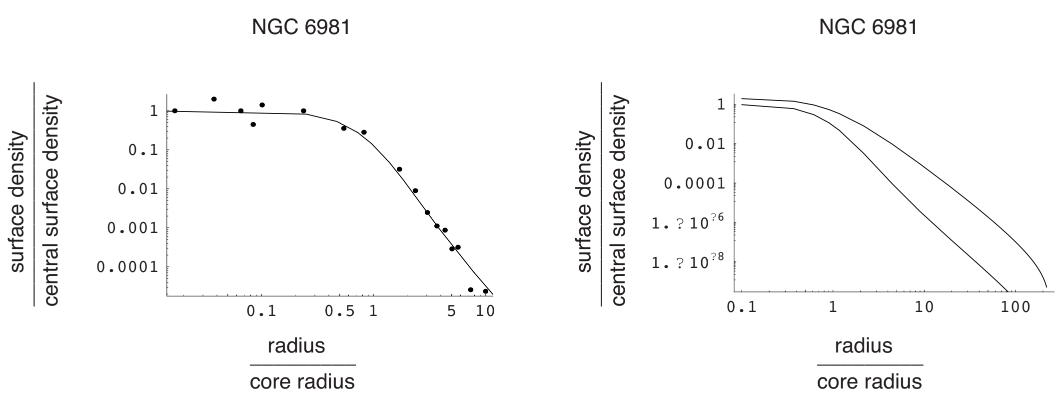


Figure 6: Modeled with $r_c = 1.5, M = 0.2, W_0 = 10.085, R_{a1} = R_{a2} = 1$

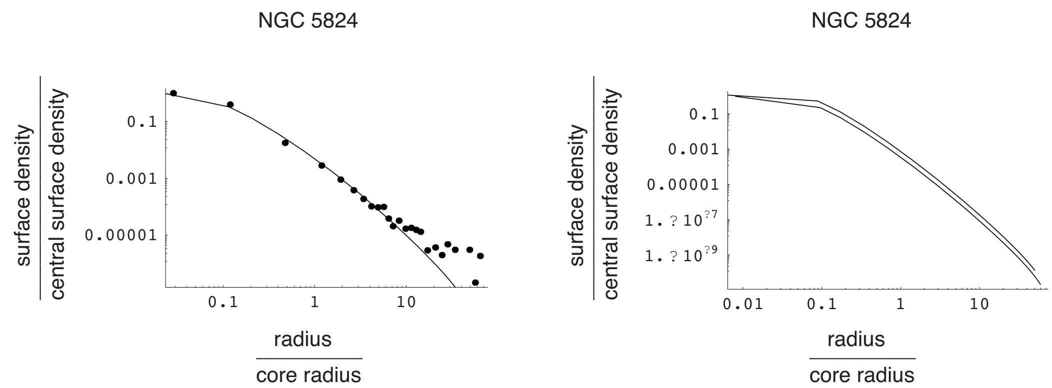


Figure 7: Modeled with $r_c = 1.33, M = 0.5, W_0 = 0.098, R_{a1} = 0.012, R_{a2} = 0.009$

It is reasonable to conclude that other clusters well-fit by King Models can contain equally large percentages of dark matter. Another cluster poorly fit by a one-component model is NGC 5824. Above is a reasonably close fit, with 53% dark matter.

Unfortunately, although the first consideration was to maximize the percentage of dark matter by setting the anisotropy radii equal, it was difficult to find a curve that would break as quickly as observation. It appears that NGC 5824, unlike every other cluster presented by Grillmair, can contain much less dark matter, unless the center of the distribution has been incorrectly identified.

6 Conclusion

The results of this analysis reveal that if several distributions are considered independently with varying anisotropy radii, there are a variety of possible selections of parameters which fit with observational stellar density profile data. More interestingly, the model allows for a great deal of dark matter, for example NGC 288 and NGC 6981, to exceed 90% of the total mass within the cluster and still provide accurate fits. However for at least one globular cluster, NGC 5824, we have not found solutions that match the density profile data that have more than 50% dark matter by mass, although we are not at the moment able to prove as a theorem that more than 50% dark matter is impossible.

To optimize approximation curves it will be necessary to construct better computer programs which can minimize the statistical fluctuations between a model and actual data. Although Mathematica is unable to solve over the entire range of possibilities, it would be useful to develop a program which can guess, check, and guess again with less human intervention. Since there are many added solutions for each added component, it would also be interesting to develop a model which considers more sub-groups and then executes the same optimization routine. With enough components, it would be possible to perfectly match any distribution derived from statistical mechanics, and when compared with new experimental results, to make better predictions about cluster behavior.

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