Refractive index

From the very first lecture:
if $\chi = \chi' + i\chi''$ is a susceptibility of the atomic medium, then for the steady-state situation $\left(\frac{\partial \vec{E}}{\partial t} = 0\right)$

$$E_z(z) = \underbrace{\left[E_0 \, e^{-k\chi''/2 \cdot z} \right]}_{\text{amplitude}(z)} e^{\overbrace{}^{}\underbrace{ik(1+\frac{1}{2}\chi')z - i\omega t + i\varphi(0)}_{\text{phase}(z)}}$$

Absorption coefficient $\left(E(z) = E(0)\, e^{-\alpha z}\right)$ $\alpha = \frac{k\chi''}{2}$

Refractive index $n = 1 + \chi'/2$

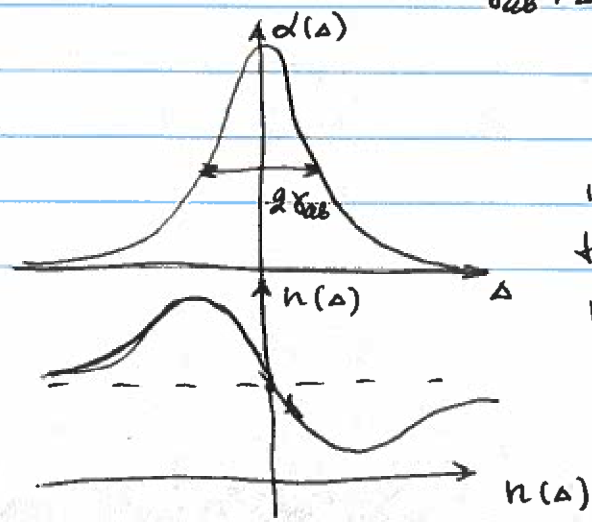
Real part of susceptibility affects the phase variation of the propagating e-m field

Unsaturated two-level system

$$\chi(\Delta) = -i \frac{\wp_{ab}^2}{\varepsilon_0 \hbar} \frac{\Delta\rho_0}{\gamma_{ab} - i\Delta} = -i \frac{\wp_{ab}^2}{\varepsilon_0 \hbar} \Delta\rho_0 \frac{\gamma_{ab} + i\Delta}{\gamma_{ab}^2 + \Delta^2}$$

$$n(\Delta) = 1 + \frac{\chi'}{2} = 1 - \frac{\wp_{ab}^2}{2\varepsilon_0 \hbar} \underbrace{(\rho_{bb} - \rho_{aa})}_{-\Delta\rho_0} \frac{\Delta}{\gamma_{ab}^2 + \Delta^2}$$

$$\alpha(\Delta) = \frac{\wp_{ab}^2 k}{2\varepsilon_0 \hbar} (\rho_{bb} - \rho_{aa}) \frac{\gamma_{ab}}{\gamma_{ab}^2 + \Delta^2}$$



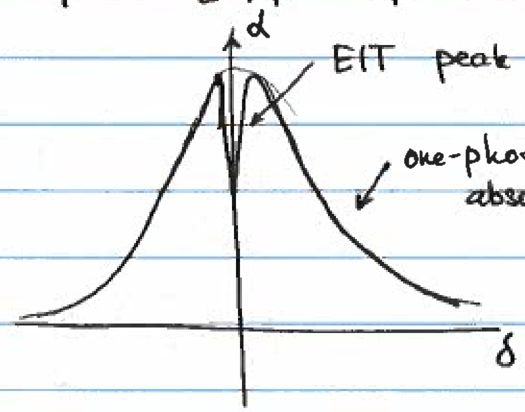
For $\Delta \gtrsim \gamma_{ab}$ the absorption is small $(\alpha \sim 1/\Delta^2)$, but the refractive index falls much slower $(n-1 \sim 1/\Delta)$, so the medium can be almost transparent but refractive

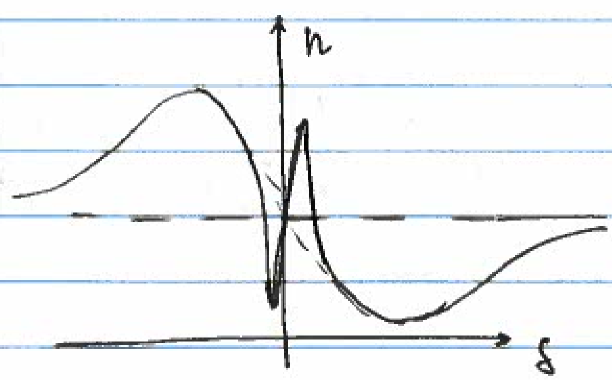
Three-level system

$$\chi_p = \frac{i\,p_{13}^2}{\varepsilon_0 \hbar} \frac{\Gamma_{12}}{\Gamma_{13}\Gamma_{12} + |\Omega_2|^2}$$

(exact calculations of the refractive index → HW 2)

$$n_p = 1 + \frac{1}{2} \mathrm{Re}(\chi_p)$$





The refractive index determines <u>phase</u> <u>velocity</u>

$$v = \frac{c}{n}$$

Phase vs group velocity

Up to now we considered only monochromatic fields, which means that its amplitude is time-independent. If $\mathcal{E}_0(t)$ changes in time, we must consider what happens to its different frequency components

$$\mathcal{E}(t,z) = \int d\omega\, \mathcal{E}_\omega(z)\, e^{-i\omega t + ik(\omega)z} = \int d\omega\, \mathcal{E}_\omega(z) \times$$

$$\times\, e^{-i\omega t + i\frac{\omega}{c}n(\omega)z}$$

We can simplify this expression considering that $\mathcal{E}(t,z)$ is nearly monochromatic, meaning that $\mathcal{E}_\omega$ has non-vanishing contributions only in the vicinity of some average frequency $\bar{\omega}$. Then we can use $n(\omega) \approx n(\bar{\omega}) + \frac{\partial n}{\partial\omega}\Big|_{\bar{\omega}} \delta\omega$, where $\delta\omega = \omega - \bar{\omega}$

Then the phase of the exponent in the integral is:

$$-i\omega t + i\frac{\omega}{c}n(\omega)z \approx -i(\bar{\omega}+\delta\omega)t + i\frac{(\bar{\omega}+\delta\omega)}{c}\left(n(\bar{\omega}) + \frac{\partial n}{\partial\omega}\delta\omega\right)z \approx$$

$$= \underbrace{\left[-i\bar{\omega}t + i\frac{\bar{\omega}}{c}n(\bar{\omega})z\right]}_{\text{carrier phase}} + \underbrace{\left[-it\delta\omega + i\frac{\delta\omega}{c}n(\bar{\omega})z + i\frac{\bar{\omega}}{c}\frac{\partial n}{\partial\omega}z\delta\omega\right]}_{-i\delta\omega\,[t - z/v_g]}$$

where $v_g = \dfrac{c}{n(\bar{\omega}) + \bar{\omega}\,\frac{\partial n}{\partial\omega}\Big|_{\bar{\omega}}}$ group velocity

$$\mathcal{E}(t,z) \approx e^{-i\bar{\omega}t + i\bar{k}z} \int \mathcal{E}_{(\bar{\omega}+\delta\omega)}(z)\, e^{-i\delta\omega\,[t - z/v_g]}\, d\delta\omega \approx$$

$$\approx \mathcal{E}(t - z/v_g,\ z)$$

The envelope propagates at v_g

Two-level system

$$n(\Delta) = 1 - \tilde{C} \cdot \frac{\Delta}{\gamma_{ab}^2 + \Delta^2} \qquad \tilde{C} = \frac{\wp_{ab}^2}{2\varepsilon_0 \hbar}(\rho_{bb} - \rho_{aa})$$

Near the center of the resonance $(\Delta = 0)$

$$n_g \approx 1 + \omega \frac{\partial n(\Delta)}{\partial \Delta} \approx$$

$$\approx 1 - C \cdot \omega \left. \frac{\gamma^2 - \Delta^2}{(\gamma^2 + \Delta^2)^2} \right|_{\Delta = 0} \approx 1 - \frac{\tilde{C}\omega}{\gamma^2}$$

$$v_g = \frac{c}{n_g} = \frac{c}{1 - \frac{\tilde{C}\omega}{\gamma^2}} > c$$ superluminal light

Since the group velocity is defined by the reshaping of the pulse (i.e. different spectral component propagating differently w/respect to each other) v_g may be smaller or larger than c without breaking Einstein postulates. It was proven that no useful information can travel faster than c — so no issues with causality.

(~~for the three-level system, see slides + HW2~~)

For the three-level system: see slides + HW2