PHYS 690 Quantum and Nonlinear Optics

Problem set # 2 *(discussion date - October 6)*

P1 In class we have discussed a two-level system in which both levels decayed outside of the system, and both were independently repumped. Write down the block equations for the closed two-level system (that does not interact with the outside world). In this case the upper level decays only to the lower one with the rate Γ, and the lower level is the ground state (i.e., does not decay). There is also no external repumping to either level.
*(a)*Find the steady-state solution for the population difference in this system. Is it possible to achieve the steady-state population inversion?
(b) Assume that at $t = 0$ the system was in the excited state. Calculate the population inversion in this system as a function of time.

P2 Consider a three-level atom in the Λ configuration, in which both optical fields are resonant with their corresponding atomic transitions. The interaction Hamiltonian in this case is:
$\hat{H}_{int} = -\frac{\hbar}{2}\left(\Omega_1|3\rangle\langle 1| + \Omega_2|3\rangle\langle 2|\right) + c.c.$,
where Ω_1 and Ω_2 are the Rabi frequencies associated with the optical fields driving $3 \to 1$ and $3 \to 2$ transitions, respectively. Show that the eigenstates of the Hamiltonian are:

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle + \frac{\Omega_1^*}{\Omega}|1\rangle + \frac{\Omega_2^*}{\Omega}|2\rangle\right);$$
$$|\psi_0\rangle = \left(\frac{\Omega_2}{\Omega}|1\rangle - \frac{\Omega_1}{\Omega}|2\rangle\right);$$
$$|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle - \frac{\Omega_1^*}{\Omega}|1\rangle - \frac{\Omega_2^*}{\Omega}|2\rangle\right);$$

with $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2}$. Find the corresponding eigenvalues.

P3 Let's have a closer look at the optical pumping effect. We assume a single optical field with Rabi frequency Ω acting on a three-level atom, between the states $|2\rangle$ and $|3\rangle$. If there is a population exchange between the two ground states at the rate γ_0, the non-zero elements of the density matrix evolve as:

$$\dot{\rho}_{11} = \gamma\rho_{33} - \gamma_0(\rho_{11} - \rho_{22});$$
$$\dot{\rho}_{22} = \gamma\rho_{33} + \gamma_0(\rho_{11} - \rho_{22}) - i\Omega\rho_{23}/2 + i\Omega^*\rho_{32}/2;$$
$$\dot{\rho}_{32} = -(\gamma - i\Delta)\rho_{32} + i\Omega(\rho_{22} - \rho_{33})/2;$$

where γ is the population and decoherence decay rate of the state $|3\rangle$ into each of the ground states, and Δ is the optical detuning. Calculate the steady-state populations of all three levels.

P4 During the lecture we derived expression for probe field susceptibility under the EIT conditions for a resonant probe field ($\Delta_1 = 0$):

$$\chi_P = \frac{i\wp_{13}^2}{\hbar\epsilon_0}N\frac{\Gamma_{12}}{\Gamma_{12}\Gamma_{13} + |\Omega_2|^2},$$

where $\Gamma_{12} = \gamma_{12} - i\delta$, $\Gamma_{13} = \gamma_{13}$, and N is the atomic number density. *(a)* Calculate the group velocity at exact two-photon resonance ($\delta = 0$). *(b)* For some applications EIT can be used to enhance the refractive index of the material. For a given atomic parameters, what is the maximum refractive index one can achieve, and for what two-photon detuning? Is it possible to achieve it without losses, even if $\gamma_{12} = 0$?

P5 In class we also calculated the susceptibility for a far-detuned three-level system, and predicted the existance of a narrow Raman absorption resonance. Calculate the group velocity for the probe laser tuned to the bottom of this resonance. Why do you think this regime is sometimes called "superluminal"?

P6 The coherent part of interaction between a three-level system and two optical field is identical for Λ, V and ladder configurations, however, the difference in decay values results changes the parameters of the EIT resonance. Following the steps for a Λ system, calculate the EIT susceptibility for a ladder system, in which a weak probe field is applied between the ground and the first excited state, and two excited states are coupled by a strong pump field. Assume that the radiative decay rates of the excited states γ_2 and γ_3 are comparable, and there is no additional sources of decoherence in the system. What is the best absorption suppression is achievable in such system?