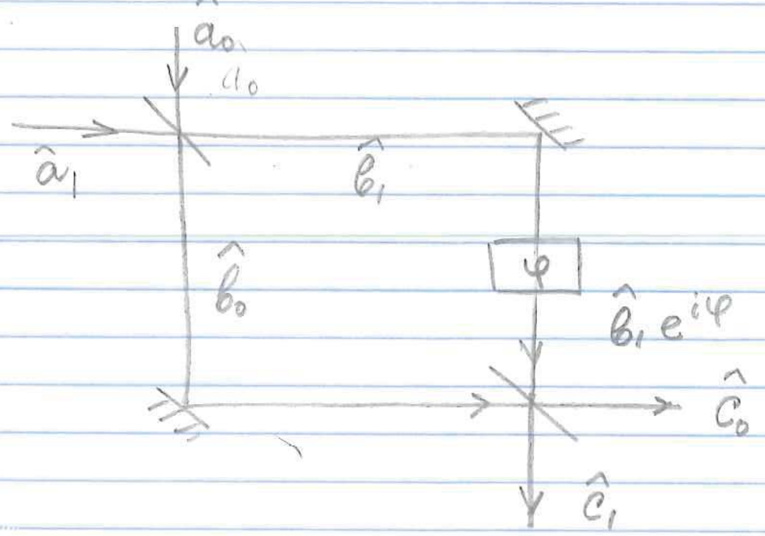



First, we are going to express the ~~output~~ modes $\hat{c}_0$ and $\hat{c}_1$ in terms of input modes $\hat{a}_0$ and $\hat{a}_1$

$$\hat{b}_0 = \frac{1}{\sqrt{2}}(\hat{a}_0 + i\hat{a}_1)$$

$$\hat{b}_1 = \frac{1}{\sqrt{2}}(i\hat{a}_0 + \hat{a}_1)$$

and

$$\hat{c}_0 = \frac{1}{\sqrt{2}}(\hat{b}_0 + i\hat{b}_1 e^{i\varphi}) = \frac{1}{2}\left[(\hat{a}_0 + i\hat{a}_1) + ie^{i\varphi}(i\hat{a}_0 + \hat{a}_1)\right]$$

$$= \frac{1}{2}\left[(1 - e^{i\varphi})\hat{a}_0 + i(1 + e^{i\varphi})\hat{a}_1\right]$$

$$\hat{c}_1 = \frac{1}{\sqrt{2}}(i\hat{b}_0 + \hat{b}_1 e^{i\varphi}) = \frac{1}{2}\left[i(1 + e^{i\varphi})\hat{a}_0 - (1 - e^{i\varphi})\hat{a}_1\right]$$

Measured signal $I_{ph} \propto \langle \hat{n}_{c_1} \rangle - \langle \hat{n}_{c_0} \rangle = \langle \hat{c}_1^\dagger \hat{c}_1 \rangle - \langle \hat{c}_0^\dagger \hat{c}_0 \rangle$

$$\hat{c}_1^\dagger \hat{c}_1 = \frac{1}{4}\left[(-i(1+e^{-i\varphi})\hat{a}_0^\dagger - (1-e^{-i\varphi})\hat{a}_1^\dagger)(i(1+e^{i\varphi})\hat{a}_0 - (1-e^{i\varphi})\hat{a}_1)\right] =$$

$$= \frac{1}{2}(1+\cos\varphi)\hat{a}_0^\dagger \hat{a}_0 + \frac{1}{2}(1-\cos\varphi)\hat{a}_1^\dagger \hat{a}_1 + \frac{1}{2}(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_0)\sin\varphi$$

similarly

$$\hat{c}_0^\dagger \hat{c}_0 = \frac{1}{2}(1-\cos\varphi)\hat{a}_0^\dagger \hat{a}_0 + \frac{1}{2}(1+\cos\varphi)\hat{a}_1^\dagger \hat{a}_1 - \frac{1}{2}(\hat{a}_0^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_0)\sin\varphi$$

thus $\hat{c}_1^+\hat{c}_1 - \hat{c}_0^+\hat{c}_0 = \left(\hat{a}_0^+\hat{a}_0 - \hat{a}_1^+\hat{a}_1\right)\cos\varphi - (\hat{a}_0^+\hat{a}_1 + \hat{a}_1^+\hat{a}_0)\sin\varphi$

And now we can average the final photon difference using known initial states.

$$\bar{n}_{cd} = \langle \hat{c}_1^+\hat{c}_1 - \hat{c}_0^+\hat{c}_0\rangle = \left(\langle \hat{a}_0^+\hat{a}_0\rangle - \langle \hat{a}_1^+\hat{a}_1\rangle\right)\cos\varphi - \left(\langle\hat{a}_0^+\rangle\langle\hat{a}_1\rangle + \langle\hat{a}_1^+\rangle \times \right.$$
$$\left. \times \langle \hat{a}_0\rangle\right)\sin\varphi$$

where $\langle\ldots\rangle = \langle i|_{\ldots}|i\rangle$

$$|i\rangle = |\alpha\rangle_{\hat{a}_0}\ |\xi\rangle_{\hat{a}_1}$$

coherent state   squeezed vacuum

$$\bar{n}_{cd} = \left(\langle\alpha|\hat{a}_0^+\hat{a}_0|\alpha\rangle - \langle\xi|\hat{a}_1^+\hat{a}_1|\xi\rangle\right)\cos\varphi -$$
$$- \left(\langle\alpha|\hat{a}_0^+|\alpha\rangle\langle\xi|\hat{a}_1|\xi\rangle + \langle\alpha|\hat{a}_1^+|\alpha\rangle\langle\xi|\hat{a}_1^+|\xi\rangle\right)\sin\varphi =$$
$$= (|\alpha|^2 - \sinh^2 r)\cos\varphi \qquad \left|\frac{\partial\bar{n}_{cd}}{\partial\varphi}\right| = (|\alpha|^2 - \sinh^2 r)\sin\varphi$$

max sensitivity $\varphi = \pi/2$

For $\varphi = \pi/2$ $\qquad \bar{n}_{cd} = 0$

$$-\hat{n}_{cd} = \hat{a}_0^+\hat{a}_1 + \hat{a}_1^+\hat{a}_0$$

$$\hat{n}_{cd}^2 = (\hat{a}_0^+\hat{a}_1 + \hat{a}_1^+\hat{a}_0)^2 = (\hat{a}_0^+)^2(\hat{a}_1)^2 + (\hat{a}_1^+)^2(\hat{a}_0)^2 + \hat{a}_0^+\hat{a}_0\underbrace{\hat{a}_1\hat{a}_1^+}_{(\hat{a}_1^+\hat{a}_1+1)} +$$
$$+ \underbrace{\hat{a}_0\hat{a}_0^+}_{(\hat{a}_0^+\hat{a}_0+1)}\hat{a}_1^+\hat{a}_1$$

$$\langle\Delta n_{cd}\rangle^2 = \langle\hat{n}_{cd}^2\rangle = \langle\alpha|\hat{a}_0^{+2}|\alpha\rangle\langle\xi|\hat{a}_1^2|\xi\rangle + \langle\alpha|\hat{a}_0^2|\alpha\rangle\langle\xi|\hat{a}_1^{+2}|\xi\rangle +$$
$$+ 2\langle\alpha|\hat{a}_1^+\hat{a}_1|\alpha\rangle\langle\xi|\hat{a}_0^+\hat{a}_0|\xi\rangle + \langle\alpha|\hat{a}_0^+\hat{a}_0|\alpha\rangle + \langle\xi|\hat{a}_1^+\hat{a}_1|\xi\rangle =$$
$$= (\alpha^*)^2(-e^{i\theta})\sinh r\cosh r + \alpha^2(-e^{-i\theta})\sinh r\cosh r + 2|\alpha|^2\sinh^2 r + |\alpha|^2 + \sinh^2 r$$

$$= -|\alpha|^2 \cos(\theta - \varphi_\alpha) \sinh r \cosh r + 2|\alpha|^2 \sinh^2 r + |\alpha|^2 + \sinh^2 r$$

Obviously, the value of fluctuations will depend on the relative phase b/w the squeezed vacuum and the coherent field (since the latter plays a role of a local oscillator)
Min value corresponds to $\theta = \varphi_\alpha + 2\pi$

negligeable for $|\alpha| \gg 1$

$$(\Delta n_{cd})^2 = |\alpha|^2 (1 + 2\sinh^2 r - 2\sinh r \cosh r) + \cancel{\sinh^2 r} =$$

$$= |\alpha|^2 (\sinh^2 r + \cos^2 r - 2\sinh r \cosh r) = |\alpha|^2 (\cosh r - \sinh r)^2 = |\alpha|^2 e^{-2r}$$

Thus $$(\Delta n_{cd})_{\substack{sqz\\vac}} = (\Delta n_{cd})_{\substack{coh\\vac}} \cdot e^{-r}$$

and $$\Delta\varphi\Big|_{\substack{sqz\\vac}} = e^{-r} \, \Delta\varphi_{\substack{coh\\vac}}$$

Problem 3

$\hat{a}\hat{b}|y\rangle = y|y\rangle$

$(\hat{a}^+\hat{a} - \hat{b}^+\hat{b})|y\rangle = (\hat{n}_a - \hat{n}_b)|y\rangle = 0$

Pair state $|y\rangle = \sum_{n,m} C_{nm} |n\rangle_a |m\rangle_b$

$(\hat{a}^+\hat{a} - \hat{b}^+\hat{b}) \sum_{n,m} C_{nm} |n\rangle_a |m\rangle_b =$

$= \sum_{n,m} (n-m) C_{nm} |n\rangle |m\rangle = 0 \quad \Rightarrow \quad C_{nm} = 0$ for any $n \neq m$

$|y\rangle = \sum_n C_n |n\rangle_a |n_b\rangle$

$\hat{a}\hat{b}|y\rangle = \sum_n C_n \hat{a}|n\rangle_a \hat{b}|n\rangle_b = \sum_n C_n \cdot n \, |n-1\rangle_a |n-1\rangle_b$

$= y|y\rangle \quad \Rightarrow \quad C_{n+1}(n+1) = yC_n$

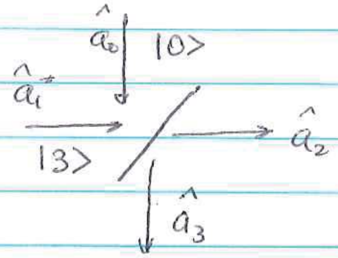
$C_n = \frac{y^{n}}{n!} C_0$

$|y\rangle = C_0 \sum_n \frac{y^n}{n!} |n\rangle_a |n\rangle_b \qquad C_0 = \frac{1}{\sqrt{\sum_n \frac{y^{2n}}{(n!)^2}}}$

Clearly, this state closely rescembles the product of two coherent states in each channel with average photon number $\sqrt{y}$. However, the ~~numb~~ number of photons in two channels is completely correlated, i.e. if $n$ photons is detected in channel "a", the exact same number will be detected in channel "b"

Problem 2

a) $|3\rangle = \frac{1}{\sqrt{6}}\hat{a}^{+3}|0\rangle \qquad |2\rangle = \frac{1}{\sqrt{2}}\hat{a}^{+2}|0\rangle \qquad |1\rangle = \hat{a}^{+}|0\rangle$




$$\hat{a}_0^+ = (i\hat{a}_2^+ + \hat{a}_3^+)$$
$$\hat{a}_1^+ = (\hat{a}_2^+ + i\hat{a}_3^+)$$

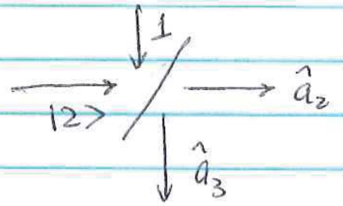
$$|3\rangle_1|0\rangle_0 = \frac{1}{\sqrt{6}}(\hat{a}_1^+)^3|0\rangle_1|0\rangle_0 \Rightarrow \frac{1}{4\sqrt{3}}(\hat{a}_2^+ + i\hat{a}_3^+)^3|0\rangle_2|0\rangle_3$$

$$= \frac{1}{4\sqrt{3}}\left((\hat{a}_2^+)^3 - i(\hat{a}_3^+)^3 + 3i(\hat{a}_2^+)^2\hat{a}_3^+ - 3\hat{a}_2^+(\hat{a}_3^+)^2\right)|0\rangle_2|0\rangle_3 =$$

$$= \frac{1}{4\sqrt{3}}\left[\sqrt{6}\left(|3\rangle_2|0\rangle_3 - i|0\rangle_2|3\rangle_3\right) + 3\sqrt{2}\left(i|2\rangle_2|1\rangle_3 - |1\rangle_2|2\rangle_3\right)\right]$$

$$= \frac{1}{2}\cdot\underbrace{\left[\frac{|3\rangle_2|0\rangle_3 - i|0\rangle_2|3\rangle_3}{\sqrt{2}}\right]}_{\text{NOON state } \varphi = -\pi/2} + \frac{\sqrt{6}}{4}\left[i|2\rangle_2|1\rangle_3 - |1\rangle_2|2\rangle_3\right]$$

Probability of the NOON state generation $P_{NOON} = \frac{1}{4}$




$$|2\rangle_1|1\rangle_0 = \frac{1}{\sqrt{2}}(\hat{a}_1^+)^2\hat{a}_0^+|0\rangle_1|0\rangle_0 \Rightarrow$$

$$= \frac{1}{4}(\hat{a}_2^+ + i\hat{a}_3^+)^2(i\hat{a}_2^+ + \hat{a}_3^+)|0\rangle_2|0\rangle_3 =$$

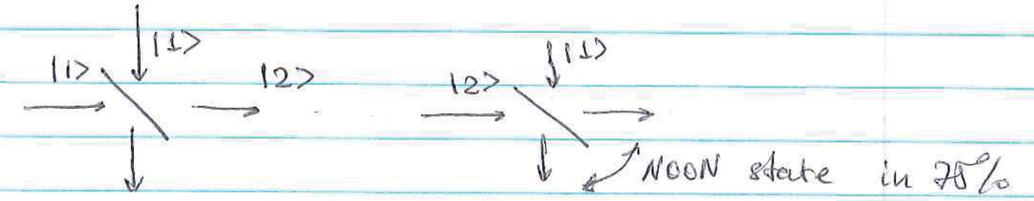
$$= \frac{1}{4}\left[i(\hat{a}_2^+)^3 - (\hat{a}_3^+)^3 - \hat{a}_2^{+2}\hat{a}_3^+ + i\hat{a}_2^+\hat{a}_3^{+2}\right]|0\rangle_2|0\rangle_3 =$$

$$= \frac{1}{4}\left[\sqrt{6}i\left[|3\rangle_2|0\rangle_3 + i|0\rangle_2|3\rangle_3\right] + \sqrt{2}\left(i|1\rangle_2|2\rangle_3 - |2\rangle_2|1\rangle_3\right)\right]$$

$$= \frac{\sqrt{3}}{2}i\underbrace{\left[\frac{|3\rangle_2|0\rangle_3 + i|0\rangle_2|3\rangle_3}{\sqrt{2}}\right]}_{\text{NOON state } \varphi = \pi/2} - \frac{1}{2\sqrt{2}}\left[|2\rangle_2|1\rangle_3 - i|1\rangle_2|2\rangle_3\right]$$

$P_{NOON} = 3/4$ ← much more efficient

b)




post-select the cases when no photons in this port

[redacted]