PHYS 622

Problem set # 7 *(due April 1)*

Each problem is 10 points.

A1 Consider a pair of free identical particles of mass m. For simplicity, suppose that they are moving in one dimension and neglect their spin variables. Each particle is described in terms of a real wave function, well-localized around points $+a$ and $-a$ respectively. For definiteness, assume that $\psi_{\pm}(x) = (\beta/\pi)^{1/4} exp\{-\frac{\beta}{2}(x \pm a)^2\}$. A well-localized state corresponds to $\beta \gg 1/a^2$. Write down the wave function of the system and calculate the expectation value of the energy. Show that if the two particles are fermions then there is an *effective repulsion* between them. Compare with the case of two identical bosons. *Hint: One can find the effective force by evaluating the change in energy of the system, resulting from a variation in the distance of the two particles.*

A2 Consider two particles, each with orbital angular momentum quantum number $\ell = 1, m_\ell = 0$. What are the possible values of the total orbital angular momentum? What is the probability that a measurement will find each of these values? Consider the case where the two particles are spin-1/2 fermions. Neglect their interaction and assume that they both have the same radial wave function. What are the total spin and the total angular momentum of the system?

A3 Consider N identical particles. Assume that their interactions can be neglected and that the Hamiltonian of the system is the sum of N identical one-particle Hamiltonians with known eigenvalues E_i:
$\hat{H} = \sum_{a=1}^{N} \hat{H}_a,\ \hat{H}_a|i\rangle_a = E_i|i\rangle_a$
(a) What is the energy of the ground state if these particles are spin-0 bosons? What if they are spin-1/2 fermions?
(b) Consider the case of three such particles and write down the corresponding ground-state wave functions.

A4 Three identical spin-1 bosots are in the same orbital state, described by the wavefunction $\phi(\vec{r})$. Write down all possible normalized spin wavefunctions of the system. How many distinct states the system have? What are possible values of the total spin?

A5 What values are possible for the total spin S of two identical spin-s bosons in the state with total orbital angular momentum L? Repeat the arguments for the system of two identical fermions.
Hint: It may be convenient to consider the relationship between permutation and parity operators for a two-particle system to figure out the permutation symmetry of the states with various value of the angular momentum.