Physics 622

Problem set 6 *(due March 25)*

Sakurai and Napolitano problems (each problem is 10 points):

4.5,4.12

A1. **Parity non-conservation (PNC) in hydrogen**.

Show that the weak-interaction Hamiltonian (see problem 4.5) does not violates the time-reversal invariance.

A2. **Parity measurements**: A quantum system has only two energy eigestates, $|1\rangle, |2\rangle$, corresponding to the energy eigenvalues E_1, E_2. Apart from the energy, the system is also characterized by a physical observable whose operator $\hat{\pi}$ acts on the energy eigenstates as follows:

$$\hat{\pi}|1\rangle = |2\rangle;\ \hat{\pi}|2\rangle = |1\rangle$$

The operator $\hat{\pi}$ can be regarded as a parity operator.

(a) Assuming that the system is initially in a positive-parity eigenstate, find the state of the system at any time.
(b) At a particular time *T* a parity measurement is made on the system. What is the probability of finding the system with positive parity?
(c) **Quantum Zeno effect**: Imagine that instead of a single measurement at time *T* you make a series of *N* parity measurements at the times $\Delta t, 2\Delta t, \dots N\Delta t = T$. Assuming that *N* is very large and $\Delta t \ll (E_1 - E_2)/\hbar$, what is the probability of finding the system with positive parity at time *T*? Compare this probability with the probability to find the system in the positive parity state with a single measurement at $t=T$. The "freezing" of the system in the initial state for a repeated series of measurements has been called the *quantum Zeno effect*.

A3. **Permanent electric dipole moment (EDM)**.

Let's assume that an electron has an intrinsic dipole moment $\vec{d}_a$. In this case its interaction with an external electric field $\vec{\mathcal{E}}$ will be described by the Hamiltonian $\hat{H}_{edm} = -\vec{d}_a \cdot \vec{\mathcal{E}}$. Show that the existence of such a dipole moment would violate both parity and time-reversal invariance. (*Hint*: what vectors are available for $\vec{d}_a$ to point along?)

Q1. **Triangular potential well** A particle of mass *m*, moving in one dimension, is localized inside a symmetric triangular potential well $V(x) = V_0 \cdot |x|$. Consider a trial wave function $\psi(x) \propto e^{-\alpha|x|}$ and estimate the ground-state energy by minimizing the expectation value of the total energy of the particle.