PHYS 622
Problem set # 11 *(due April 29)*
Each problem is 10 points.

Sakurai and Napolitano problems: 8.1, 8.10, 8.12

A1 At some instant of time (say, $t = 0$), the normalized Dirac wave function for a free electron is known to be:

$$\psi(x,0) = \frac{1}{\sqrt{V}} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} e^{ip_z z}$$

where a, b, c and d are independent of the space-time coordinates and satisfy: $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$.
(a) Find the probabilities for observing the electron with

- $E > 0$, spin up
- $E > 0$, spin down
- $E < 0$, spin up
- $E < 0$, spin down

A2 Construct the normalized Dirac wave functions for $E > 0$ plane waves that are eigenstates of the helicity operator $h = \vec{\Sigma} \cdot \hat{p}$, where $\vec{\Sigma}$ is the spin operator [see SN Eq.(8.2.21)] and $\hat{p} = \vec{p}/|\vec{p}|$ is the momentum direction. Evaluate the expectation values of $\vec{\Sigma} \cdot \hat{p}$ and $\gamma^0 \vec{\Sigma} \cdot \hat{p} = -\gamma^5 \gamma \cdot \hat{p}$.
Q1 Consider one-dimensional delta function potential $V(x) = \hbar^2 \lambda/(2m)\delta(x)$.
*(a)*Solve the energy eigenvalue problem for both signs of the coupling λ. In the case of the continuum (scattering states), write the eigenfunctions in terms of the scattering amplitude. Examine the analytic properties of the scattering amplitude in the k-plane. Are there poles? What do they correspond to?
*(b)*Determine the one-dimensional Green's function:

$$\left(\frac{d^2}{dx^2} + k^2\right) G_k(x,x') = -4\pi\delta(x-x') \quad (1)$$

and solve the above eigenvalue problem with the help of the *scattering integral equation*

$$\psi_k(x) = \phi_x^{(0)}(x) - \frac{m}{2\pi\hbar^2} \int dx' G_k(x,x') V(x') \psi_k(x). \quad (2)$$