PHYS 622
Problem set # 1 *(due January 29)*
Each problem is 10 points.

Q1 Consider an electron bound in a hydrogen atom under the influence of a homogeneous magnetic field $\vec{\mathbf{B}} = \hat{\mathbf{z}}B$. Ignore electron spin. The Hamiltonian of the system is:
$\hat{H} = \hat{H}_0 - \omega \hat{L}_z$,
where $\omega = |e|B/2\mu c$. The eigenstates $|n\,\ell\,m\rangle$ and eigenvalues $E_n^{(0)}$ of the unerturbed hydrogen Hamiltonian $\hat{H}_0$ are to be considered known. Assume that initially (at $t = 0$) the sustem is in the state:
$|\psi(0)\rangle = (|2\,1\;-1\rangle - |2\,1\,1\rangle)/\sqrt{2}$.
For each of the following state, calculate the probability of finding the system, at some later time $t > 0$, in that state:
$|2p_x\rangle = (|2\,1\;-1\rangle - |2\,1\,1\rangle)/\sqrt{2}$;
$|2p_y\rangle = (|2\,1\;-1\rangle + |2\,1\,1\rangle)/\sqrt{2}$;
$|2p_z\rangle = |2\,1\,0\rangle$.
When does each probability become equal to 1?

Q2 For a spin-1/2 particle the most general form of the spin wave function is:
$|\psi\rangle = \begin{pmatrix} cos\theta \\ e^{i\phi} sin\theta \end{pmatrix}$,
where $0 \leq \theta \leq \pi/2$ and $0 \leq \phi < 2\pi$. Find the direction in space $\hat{\mathbf{n}}$, such that this state corresponds to the $+\hbar/2$ eigenvalue of the spin operator projection on this direction $\hat{\mathbf{n}} \cdot \vec{\mathbf{S}}$. Use your solution to find the eigenvectors for the x and y components of the spin operators $\hat{S}_x$ and $\hat{S}_y$.

Q3 Consider an spinless particle of mass μ and charge q under the simultaneous influence of a uniform magnetic field and electric fields. The interaction Hamiltonian of the the two terms are:
$\hat{H}_M = -\frac{q}{2\mu c}\vec{B} \cdot \vec{L}$; $\hat{H}_E = -q\vec{E} \cdot \vec{r}$.
Show that
$|\langle \ell\, m|\hat{H}_M + \hat{H}_E|\ell'\, m'\rangle|^2 = |\langle \ell\, m|\hat{H}_M|\ell'\, m'\rangle|^2 + |\langle \ell\, m|\hat{H}_E|\ell'\, m'\rangle|^2$,
and that, always, one of the matrix elements $\langle \ell\, m|\hat{H}_M|\ell'\, m'\rangle$ or $\langle \ell\, m|\hat{H}_E|\ell'\, m'\rangle$ vanishes.

Q4 Consider a pair of particles with opposite electric charges that have a magentic-dipole-moment interaction
$\hat{H}_I = \alpha(\vec{\mu}_1 \cdot \vec{\mu}_2) = -\frac{e^2 g_1 g_2}{2m_1 m_2}\alpha\left(\vec{\mathbf{S}}^{(1)} \cdot \vec{\mathbf{S}}^{(2)}\right)$.
The system is subject to an external uniform magnetic field $\vec{\mathbf{B}}$, which introduces the interaction:
$-\vec{\mathbf{B}} \cdot (\vec{\mu}_1 + \vec{\mu}_2) = -\frac{e}{2m_1 m_2}\vec{\mathbf{B}} \cdot \left(m_2 g_1 \vec{\mathbf{S}}^{(1)} - m_1 g_2 \vec{\mathbf{S}}^{(2)}\right)$.
Determine the energy eigenvalues and eigenstates (in the basis of the spin eigenstates $|\uparrow\rangle^{(1)}|\uparrow\rangle^{(2)}$ ... $|\downarrow\rangle^{(1)}|\downarrow\rangle^{(2)}$). Express the results in terms of the parameters $a = (e^2 g_1 g_2 \alpha)/(4m_1 m_2)$ and $b_i = eBg_i/4m_i$.

Q5 Consider the state $|j_1\, j_2; j\, m\rangle$, which is a common eigenstate of the angular momentum operators $\vec{\mathbf{J}}_1^2$, $\vec{\mathbf{J}}_2^2$ and $\vec{\mathbf{J}}^2$, where $\vec{\mathbf{J}} = \vec{\mathbf{J}}_1 + \vec{\mathbf{J}}_2$. Show that this state is also an eigenstate of the inner product operator $\vec{\mathbf{J}}_1 \cdot \vec{\mathbf{J}}_2$ and find its eigenvalues. Do the same for the operators $\vec{\mathbf{J}}_1 \cdot \vec{\mathbf{J}}$ and $\vec{\mathbf{J}}_2 \cdot \vec{\mathbf{J}}$.