

**Problem set #5 (due December 2)**

1. Jackson 10.2 (no need to compare with 10.1)
2. **Double-scattering** A long-wavelength, left circularly-polarized, monochromatic plane wave scatters into the direction  $\hat{k}_1$  from a uniform dielectric sphere with radius  $a$  and dielectric constant  $\epsilon_r$ . The scattered wave travels a distance  $r_1 \gg a$  and scatters from an identical sphere into the direction  $\hat{k}_2$ . Find the twice-scattered electric field at a distance  $r_2 \gg a$  from the second sphere. Express your answer using polarization vectors which are (i) transverse to and (ii) parallel and perpendicular to the plane of the diagram.
3. **Preservation of polarization** A linearly polarized plane wave with polarization direction  $\vec{e}_0$  is incident in a small, perfectly conducting sphere. Find the angle between the scattering wave vector  $\vec{k}$  and the incident wave vector  $\vec{k}_0$  where the radiated electric field points in the same direction as  $\vec{e}_0$ .
4. **Mie scattering from a large dielectric sphere** Find the Born approximation to the differential cross section from a uniform and lossless dielectric sphere with radius  $R$  and dielectric constant  $\epsilon$ . Plot the cross-section for an unpolarized incident beam with  $k_0 R = 15/2$ .
5. **Radiation Pressure from Scattering** An object scatters an incident plane wave with  $\vec{E}_{inc}(\vec{r}, t) = \vec{e}_0 E_0 \exp(i\vec{k}_0 \vec{r} - i\omega t)$ . Use the Maxwell stress tensor formalism to show that the time-averaged force on the object can be written in terms of the incident wave intensity  $I_{inc}$ , the total cross section  $\sigma_{tot}$  and the differential cross section for the scattering  $d\sigma_{scat}/d\Omega$  as  $\langle \vec{F} \rangle = \frac{I_{inc}}{c} \left( \sigma_{tot} \hat{k}_0 - \int d\Omega \vec{r} \frac{d\sigma_{scat}}{d\Omega} \right)$ . The projection of this force on the direction  $\hat{k}_0$  is often called the radiation pressure due to scattering. *Hint:* Integrate the stress tensor over the surface of the enormous sphere in the radiation zone.
6. Jackson 10.15