Physics 611, Fall 2014

Problem set #3 (due October 27)

1. A He-Ne laser with wavelength λ=633nm nominally L=1m long is to be designed with a hemispherical cavity, i.e. one flat mirror and one curved mirror with R= 1m. A micrometer screw is to be used to vary the exact cavity length over a small range, so that the cavity length will be L=R-ΔL, where ΔL<<L. In this way, the spot size w_2 at the curved-mirror end can be varied to fill the 5mm radius of an aperture placed there.
    a. Write down the simple expression for w_2 as a function of ΔL.
    b. Over what range of ΔL must the micrometer screw move the curved mirror if w_2 is to vary from 5mm to all larger values?
    c. When w_2 = 5mm, what is the value of the beam size w_1 at the flat mirror end of the laser?
2. If we consider a higher-order (n,m) Hermit-Gaussian mode in a cavity, consisting of two spherical mirrors with radii R_1 and R_2 and separated by the distance L, the expression for its Guoy phase is $\varphi(z) = (n+m+1)\tan^{-1}(\frac{z}{z_R})$, where z_R is the Rayleigh range, l is axial mode index, and n and m are the transverse mode indices. Using this expression, show that the corresponding frequencies for axial and transverse modes are
$f_{lmn} = c/p\left(l + \frac{(n+m+1)}{\pi} cos^{-1}\sqrt{g_1 g_2}\right)$,
where p is the longitudinal optical path in the cavity, and g_i=1-L/R_i.
3. For the same cavity as in Problem 1 (a hemispherical geometry, L=R-ΔL, where ΔL<<L,R), assume that the laser is allowed to oscillate in several axial and transverse modes, and that the beat frequency f_{beat} is the lowers intermode beat frequency that is observed in the laser output. Verify that $w_2^2 * f_{beat}$ = cλ/π^2, independent of L or R, as is varied with ΔL<<L.
4. A collimated Gaussian beam of a fixed spot size w is to be focused to the absolute minimum possible spot size (not necessarily a beam waist) of a work piece, using a single lens located a fixed distance L from the work puece. What should be the exact focal length f of this lens, and what will be the exact spot size of the focused spot?
5. Jackson 8.2
6. Jackson 8.3
7. Jackson 8.6
8. Jackson 8.19