Homework #9 (due on 04/21)

Boas Chapter 14
6.18'(includes 6.18); 6.35'(includes 6.35); 7.4; 7.10; 7.16; 7.28; 7.35;
Boas Chapter 1
16.26
Boas Chapter 12
23.8; 23.29;

Extra-credit problem – Kramers-Kronig relations

In optics Kramers-Kronig relations provide a relation between the real and imaginary parts of the linear susceptibility* $\chi(\omega)$ of a substance for an electromagnetic wave with frequency ω.

$$\mathrm{Re}(\chi(\omega)) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\chi(\omega')d\omega'}{\omega'-\omega}$$

$$\mathrm{Im}(\chi(\omega)) = -\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{\chi(\omega')d\omega'}{\omega'-\omega}$$

These are very general expressions valid for majority of optical media. They only assume that $\chi(\omega)$ is analytic and finite in a upper half plane.
Prove the Kramers-Kronig relations, using the residue theorem, using the following integral:

$$Int = \int_{-\infty}^{+\infty}\frac{\chi(\omega')d\omega'}{\omega'-\omega},$$

and then turning in into a contour integral in the upper half plane.

* Optical susceptibility $\chi(\omega)$ characterizes the absorption and the refractive index of the optical medium. In particular, the absorption per unit length is proportional to Im{χ(ω)}, and the refractive index is n=1+4πRe{χ(ω)}.