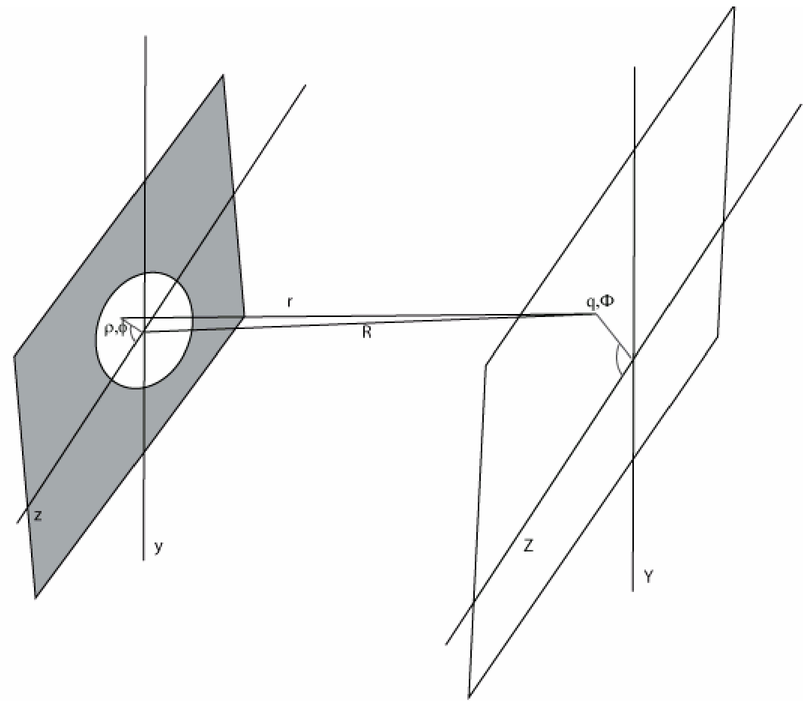
Homework #6 (due on 03/20)

Boas Chapter 12
22.9; 22.11; 22.17;
12.9; 15.2; 16.5; 18.3; 23.14; 23.16; 23.20;

Extra-credit problem – Diffraction from a Circular Aperture (**double points!**)



To find the amplitude of the electromagnetic field passed through a circular aperture of radius *a* we need to add contributions from all points of the aperture taking into account the relative phase difference due to difference in path length from different aperture points to a point on the screen. Polar coordinates ρ and ϕ describe a point on the aperture, and we will be interested in the light intensity. We will be interested in the diffraction pattern far away from the aperture, such that the distance from the aperture center to the point on the screen is $R >> a$. It is also convenient to use the distance from the center of the screen to the point of interest q, and assume that $R >> q$ as well.

In this case the total field amplitude $\vec{E}(q)$ can be obtained by integrating all the "point sources" contributions on the surface of the aperture:

$$\vec{E}(q) = \frac{\vec{E}_0}{R} e^{i(kR - \omega t)} \int_0^a \rho d\rho \int_0^{2\pi} d\phi e^{-(ik\rho q \cos\phi)/R},$$

where $k = 2\pi/\lambda = \omega/c$ is the wave-vector of the incoming light, (λ is its wavelength, and ω is its frequency), and $\vec{E}_0$ is the initial amplitude of the light before the aperture.

a) Find the amplitude and intensity ($\left|\vec{E}(q)\right|^2$) of the light on the screen in terms of Bessel functions. To do that use the following integral expressions for Bessel functions $J_n(u)$:

$$J_n(u) = \frac{1}{2\pi} \int_0^{2\pi} e^{i(n\varphi + u\cos\varphi)} d\varphi,$$

as well as the recursion relations (15.1) [Boas, ch.12].

b) Sketch or plot the intensity distribution of the light intensity on the screen as a function of distance q from the center of the screen. Your final answer will involve one Bessel function. By finding the first zero if this function, derive the radius of the first diffraction ring $q_1 = 1.22\frac{\lambda R}{(2a)}$.