

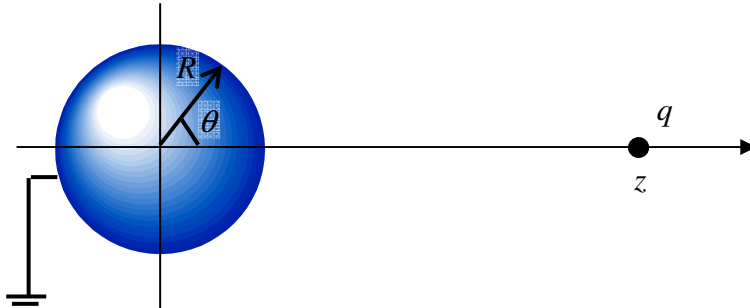
## Homework #5 (due on 03/06)

### Boas Chapter 12

2.3; 4.3; 5.4; 5.6; 5.10; 5.12; 9.5; 9.9; 23.2; 23.9

### Extra-credit problem – Induced charge density expansion

This problem deals with the charge density  $\sigma(\theta)$  induced on a metallic grounded sphere due to an external point charge  $q$ . Consider a sphere of radius  $R$  with its center at the origin. Let the external point charge be on the  $z$ -axis, at distance  $z$  ( $z > R$ ) from the origin.



from the origin. Let the external point charge be on the  $z$ -axis, at distance  $z$  ( $z > R$ ) from the origin.

The formula for a surface charge density induced on the spherical surface is given by

$$\sigma(\theta) = -\frac{qt}{4\pi R^2} \frac{1-t^2}{(1-2tx+t^2)^{3/2}}$$

Where  $t=R/z$ ,  $x=\cos\theta$  and  $\theta$  is the angle between  $z$ -axis and a position vector of a point lying on a sphere.

(a) Show that  $\frac{1-t^2}{(1-2tx+t^2)^{3/2}} = \Phi(x,t) + 2t \frac{\partial \Phi(x,t)}{\partial t}$  for the generation function  $\Phi(x,t) = (1-2tx+t^2)^{-1/2}$ .

(b) Using the result from (a), expand the charge density  $\sigma(\theta)$  in terms of Legendre polynomials.