$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}dx$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}dx$$

$$c_n = \frac{1}{2l}\int_{-l}^{l} f(x)e^{-\frac{in\pi x}{l}}dx$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\cos mx\cos nx dx = \delta_{mn}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\sin mx\sin nx dx = \delta_{mn}$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\cos mx\sin nx dx = 0$$

$$|z| = \sqrt{x^2+y^2}; \qquad \tan\theta = y/x$$

$$\ln z = Ln|z| + i\theta + i2\pi n \qquad w^z = e^{z\ln w}$$

$$\sinh z = \frac{e^z - e^{-z}}{2}; \qquad \cosh z = \frac{e^z + e^{-z}}{2}$$

$$e^z = e^x(\cos y + i\sin y)$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi x}{l}}$$

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{l}$$

$$\mathrm{B}(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$$

$$\Gamma(p) = \int_0^{\infty} t^{p-1}e^{-t}dt$$

$\Gamma(p+1) = p!$ for integer $p>0$,

$\Gamma(p+1) = p\Gamma(p)$ for all real p.

$$f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin\pi p}.$$

$$n! = \Gamma(n+1) = n^n e^{-n}\sqrt{2\pi n}$$

for $p>0, q>0$:

$$\mathrm{B}(p,q) = 2\int_0^{\pi/2}(\sin\theta)^{2p-1}(\cos\theta)^{2q-1}d\theta,$$

$$\mathrm{B}(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx,$$

$$\mathrm{B}(p,q) = \int_0^{\infty}\frac{y^{p-1}}{(1+y)^{p+q}}dy$$

Legendre polynomials

Differential equation:

$$(1-x^2)y'' - 2xy' + l(l+1)y = 0, \qquad l = 0,1,2\ldots$$

Normalization and orthogonality

$$\int_{-1}^{1} P_l P_{l'} dx = \frac{2}{2l+1}\delta_{ll'}.$$

$$P_l(1) = 1$$

$$P_l(x) = (-1)^l P_l(-x)$$

Rodriges formula

$$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$$

Generation function

$$\Phi(x,h) = (1-2xh+h^2)^{-1/2} = \sum_{l=0}^{\infty} P_l h^l.$$

Recurrence relations

$$lP_l = (2l-1)xP_{l-1} - (l-1)P_{l-2}$$

$$xP_l' - P_{l-1}' = lP_l,$$

$$P_l' - xP_{l-1}' = lP_{l-1},$$

$$(1-x^2)P_l' = lP_{l-1} - lxP_l,$$

$$(2l+1)P_l = P_{l+1}' - P_{l-1}'.$$

Associated Legendre polynomials

Differential equation

$$(1-x^2)y'' - 2xy' + \left\{l(l+1) - \frac{m^2}{1-x^2}\right\}y = 0, \qquad l = 0,1,2\ldots \quad m = 0,\pm1,\ldots\pm l$$

$$P_l(x) = P_{l0}(x),$$

$$P_{lm}(-x) = (-1)^{l+m}P_{lm}(x).$$

$$P_{lm}(x) = (1-x^2)^{m/2}\frac{d^m}{dx^m}P_l(x),$$

$$\int_{-1}^{1} dx P_{lm}(x)P_{l'm}(x) = \frac{2}{2l+1}\frac{(l+m)!}{(l-m)!}\delta_{ll'}.$$

Spherical harmonics

$$Y_{lm}(\theta,\phi)=(-1)^m\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_{lm}(\cos\theta)e^{im\phi},$$

Bessel functions

Differential equation

$$x^2y''+xy'+(p^2-x^2)y=0$$

Bessel functions of the first kind

$$J_{\pm p}(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n+1)\Gamma(n\pm p+1)}\left(\frac{x}{2}\right)^{2n\pm p}$$

$$J_{-m}(x)=(-1)^m J_m(x)$$

Bessel functions of the second kind

$$N_p(x)=Y_p(x)=\frac{\cos(\pi p)J_p(x)-J_{-p}(x)}{\sin(\pi p)}$$

Generation function

$$e^{\frac{x}{2}(h-\frac{1}{h})}=\sum_{n=-\infty}^{\infty}J_n(x)h^n$$

Recurrence relations

$$J_{p-1}(x)+J_{p+1}(x)=\frac{2p}{x}J_p(x)$$

$$J_{p-1}(x)-J_{p+1}(x)=2J_p(x)$$

$$\frac{d}{dx}\left[x^pJ_p(x)\right]=x^pJ_{p-1}(x)$$

$$\frac{d}{dx}\left[x^{-p}J_p(x)\right]=-x^{-p}J_{p+1}(x)$$

Normalization and orthogonality

$$\int_0^1 xJ_p\left(\alpha_n^{(p)}x\right)J_p\left(\alpha_m^{(p)}x\right)dx=\frac{\delta_{nm}}{2}J_{p\pm1}^2\left(\alpha_n^{(p)}\right)=$$

$$=\frac{\delta_{nm}}{2}J_p'^2\left(\alpha_n^{(p)}\right)$$

Hermite polynomials

Differential equation

$$y''-2xy'+2ny=0,\qquad n=0,1,2...$$

Rodriges formula

$$H_n(x)=(-1)^ne^{x^2}\frac{d^n}{dx^n}e^{-x^2}$$

Normalization and orthogonality

$$\int_{-\infty}^{\infty}e^{-x^2}H_n(x)H_m(x)dx=\sqrt{\pi}2^nn!\delta_{nm}$$

Generation function

$$\Phi(x,h)=e^{2xh-h^2}=\sum_{n=0}^{\infty}H_n(x)\frac{h^n}{n!}$$

Recurrence relations

$$H_n'(x)=2nH_{n-1}(x)$$

$$H_{n+1}(x)=2xH_n(x)-2nH_{n-1}(x)$$

Laguerre polynomials

Differential equation

$$xy''+(1-x)y'+ny=0,\qquad n=0,1,2...$$

Rodriges formula

$$L_n(x)=\frac{1}{n!}e^{x^2}\frac{d^n}{dx^n}(x^ne^{-x})$$

Normalization and orthogonality

$$\int_{-\infty}^{\infty}e^{-x}L_n(x)L_m(x)dx=\delta_{nm}$$

Generation function

$$\Phi(x,h)=\frac{e^{-xh/1-h}}{1-h}=\sum_{n=0}^{\infty}L_n(x)h^n$$

Recurrence relations

$$L_{n+1}'(x)-L_n'(x)+L_n(x)=0$$

$$(n+1)L_{n+1}(x)-(2n+1-x)L_n(x)+nL_{n-1}(x)=0$$

$$xL_n'(x)-nL_n(x)+nL_{n-1}(x)=0$$