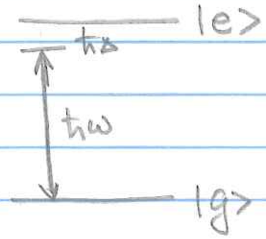
Near-resonant two-level system (Rabi model)



$$\hbar\omega_0 = E_e - E_g$$
$$\Delta = \omega_0 - \omega$$

$$|\psi(t)\rangle = C_g(t)\, e^{-iE_g t/\hbar}|g\rangle + C_e(t)\, e^{-iE_e t/\hbar}|e\rangle$$

[recall $\dot{c}_e(t) = -\frac{i}{\hbar}\sum_k c_k(t)\langle e|\hat{H}_I|k\rangle e^{i\omega_{ek}t}$]

$$\langle g|\hat{H}_I|g\rangle = \langle e|\hat{H}_I|e\rangle = 0 \qquad \langle e|\hat{H}_I|g\rangle = \wp_{eg}E_0\cos\omega t$$

$$\dot{c}_g(t) = -\frac{i}{\hbar}c_e\,\wp_{eg}^*E_0^*\cos\omega t\; e^{-i\omega_0 t}$$

$$\dot{c}_e(t) = -\frac{i}{\hbar}c_g\,\wp_{eg}E_0\cos\omega t\; e^{i\omega_0 t}$$

Using RWA (keeping only terms $\sim \omega - \omega_0$)

$$\dot{c}_g(t) = -\frac{i}{2\hbar}\wp_{eg}^*E_0^*\, e^{-i\Delta t}\, c_e(t)$$

$$\dot{c}_e(t) = -\frac{i}{2\hbar}\wp_{eg}E_0\, e^{i\Delta\cdot t}\, c_g(t)$$

$$\frac{d}{dt}\left(\dot{c}_e e^{-i\Delta\cdot t}\right) = -\frac{i}{2\hbar}\wp_{eg}E_0\,\dot{c}_g(t) = -\frac{|\wp_{eg}E_0|^2}{4\hbar^2}\underbrace{e^{-i\Delta\cdot t}}\cdot c_e(t)$$

$$\parallel$$

$$\ddot{c}_e\underbrace{e^{-i\Delta\cdot t}} - i\Delta\dot{c}_e\underbrace{e^{-i\Delta t}}$$

$$\ddot{c}_e - i\Delta\dot{c}_e + \frac{|\wp_{eg}E_0|^2}{4\hbar^2}c_e = 0 \qquad \text{oscillator eqn}$$

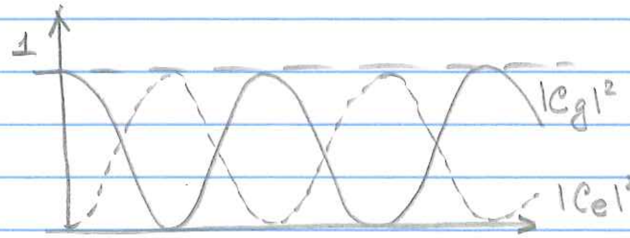
Rabi frequency $\Omega_R = \dfrac{\wp_{eg} E_0}{\hbar}$

$\underline{\Delta = 0}$

$$\left.\begin{aligned} \ddot{C}_e + \tfrac{1}{4}\Omega_R^2\, C_e &= 0 \\ \ddot{C}_g + \tfrac{1}{4}\Omega_R^2\, C_g &= 0 \end{aligned}\right] \text{ oscillating}$$

assuming $C_g(t=0) = 1$, $C_e(t=0) = 0$

$$C_g(t) = \cos\tfrac{1}{2}\Omega_R t \qquad C_e(t) = i\sin\tfrac{1}{2}\Omega_R t$$



$$P_e - P_g = \cos^2\frac{\Omega t}{2} - \sin^2\frac{\Omega t}{2} = \cos\Omega_R t$$

For $\Delta \neq 0$

$$C_e(t) = i\frac{\Omega_R}{\sqrt{\Delta^2 + \Omega_R^2}}\, e^{\frac{i\Delta\cdot t}{2}} \sin\left(\tfrac{1}{2}\sqrt{\Omega_R^2 + \Delta^2}\cdot t\right)$$

$$C_g(t) = e^{i\Delta\cdot t/2}\left\{\cos\left(\tfrac{1}{2}\sqrt{\Omega_R^2+\Delta^2}\cdot t\right) - i\frac{\Delta}{\sqrt{\Delta^2+\Omega_R^2}}\sin\left(\tfrac{1}{2}\sqrt{\Omega_R^2+\Delta^2}\, t\right)\right\}$$

Often $\tilde{\Omega}_R = \sqrt{\Omega_R^2 + \Delta^2}$ is called generalized Rabi frequency.

Notice that Rabi oscillations are fully coherent, since at any moment of time we can define atomic quantum state, i.e, for $\Delta = 0$

$$\psi(t) = \cos\tfrac{1}{2}\Omega_R t \cdot |g\rangle + i\sin\tfrac{1}{2}\Omega_R t\, |e\rangle$$

$$P_g = \cos^2\frac{\Omega_R t}{2} \qquad P_e = \sin^2\frac{\Omega_R t}{2}$$

time - dependent population

(electron oscillates (flops) b/w these two states)