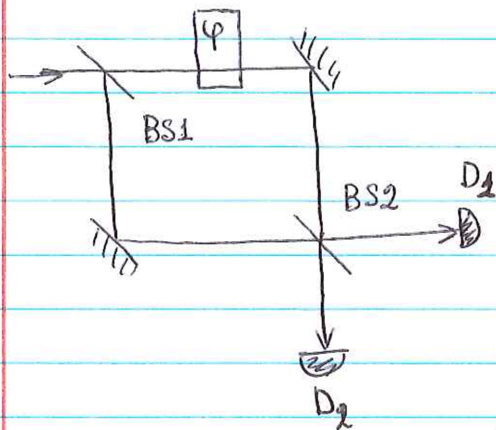
Classical interferometer



Input $E = E_0 \underbrace{e^{ikz-i\omega t}}_{\text{same for all fields, will not write}}$

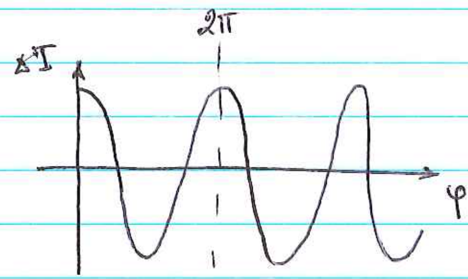
Input	E_0	
After BS1	$\frac{1}{\sqrt{2}}E_0$	$\frac{i}{\sqrt{2}}E_0$
After phase shifter	$\frac{1}{\sqrt{2}}E_0 e^{i\varphi}$	$\frac{i}{\sqrt{2}}E_0$
After BS2	$\frac{i}{2}E_0(1+e^{i\varphi})$	$\frac{1}{2}E_0(1-e^{i\varphi})$

Light intensity $I_{1,2} \propto |E_{1,2}|^2$

$I_1 = \frac{1}{2} I_0 \cos^2 \varphi/2$

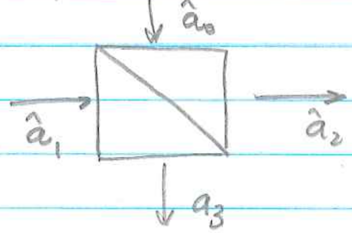
$I_2 = \frac{1}{2} I_0 \sin^2 \varphi/2$

Differential intensity $\Delta I = I_2 - I_1 = \frac{1}{2} I_0 (\cos^2 \frac{\varphi}{2} - \sin^2 \frac{\varphi}{2}) =$
$= I_0 \cos\varphi$



Interferometry with quantum states

Recall a "rules" for beam-splitter operation



50/50 Beam splitter

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \hat{U}_{BS}^{+} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} \hat{U}_{BS}$$

$$U_{BS} = e^{i\pi/4 (\hat{a}_0^+ \hat{a}_1 + \hat{a}_0 \hat{a}_1^+)}$$

$$\left.\begin{aligned} \hat{a}_2 &= \frac{1}{\sqrt{2}} (\hat{a}_0 + i\hat{a}_1) \\ \hat{a}_3 &= \frac{1}{\sqrt{2}} (i\hat{a}_0 + \hat{a}_1) \end{aligned}\right\} \Longleftrightarrow \left\{\begin{aligned} \hat{a}_0 &= \frac{1}{\sqrt{2}} (\hat{a}_2 - i\hat{a}_3) \\ \hat{a}_1 &= \frac{1}{\sqrt{2}} (-i\hat{a}_2 + \hat{a}_3) \end{aligned}\right.$$

Another important element - phase shifter

$$U_\theta = e^{i\hat{a}^+\hat{a}.}$$

$$U_\theta |\alpha\rangle = e^{i\hat{a}^+\hat{a}\varphi} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n e^{in\varphi}}{\sqrt{n}} |n\rangle$$

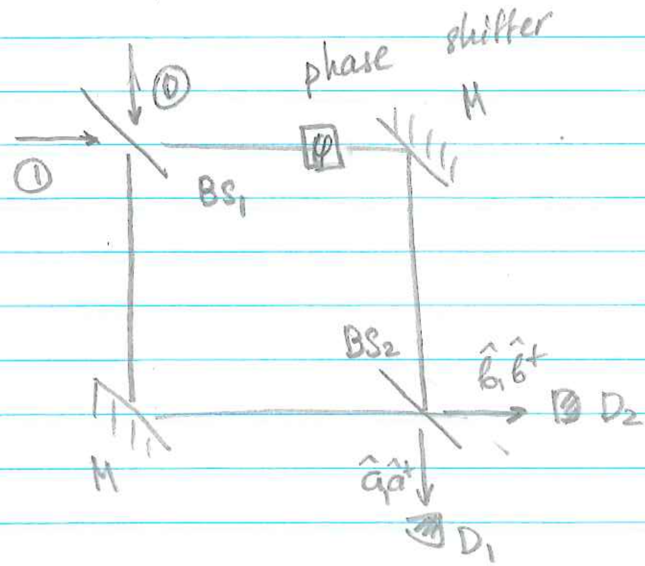
$$= \sum_{n=0}^{\infty} \frac{(\alpha e^{i\varphi})^n}{\sqrt{n}} |n\rangle = |\alpha e^{i\varphi}\rangle = |\alpha'\rangle$$

As expected from classical E&M, the phase shifter changes the phase of the EM field by θ.

However $U_\theta |n\rangle = \underline{\underline{e^{in\varphi}}} |n\rangle$

n-times larger phase-shift

Simple Mach-Zender interferometer



Coherent state as an input.
Question: how accurately one can measure the phase φ?

Input $\psi_i = |0\rangle|\alpha\rangle$
After the first BS: $\rightarrow \left|\frac{i\alpha}{\sqrt{2}}\right\rangle \left|\frac{\alpha}{\sqrt{2}}\right\rangle$

Phase-shifter acts on one of the arms

$$\rightarrow \left|\frac{i\alpha}{\sqrt{2}} e^{i\varphi}\right\rangle \left|\frac{\alpha}{\sqrt{2}}\right\rangle$$

Second beamsplitter

$$\psi_f \Rightarrow \left|\frac{i(e^{i\varphi}+1)\alpha}{\sqrt{2}}\right\rangle \left|\frac{(e^{i\varphi}-1)\alpha}{\sqrt{2}}\right\rangle$$

Signal detected on the photodetectors proportional to the mean values of photons. Measured difference

$$\hat{O} = \hat{a}^+\hat{a} - \hat{b}^+\hat{b}$$

$$\langle\psi_f|\hat{O}|\psi_f\rangle = \frac{|(e^{i\varphi}+1)|^2}{2}|\alpha|^2 + \frac{|e^{i\varphi}-1|^2}{2}|\alpha|^2 =$$

$= |\alpha|^2\cos\varphi$ same as classical

There is always an uncertainty

$$\Delta O = \sqrt{\langle O^2\rangle - \langle\hat{O}\rangle^2}$$

For a coherent state $\Delta n = \bar{n}^{1/2} = |\alpha|$

Uncertainty of the phase measurements

$$\Delta\varphi = \frac{\Delta O}{\left(\frac{\partial\langle O\rangle}{\partial\theta}\right)} = \frac{1}{|\alpha||\sin\varphi|} = \frac{1}{\sqrt{\bar{n}}|\sin\varphi|}$$

Max sensitibity $|\sin\varphi| \sim 1$

$\Delta\varphi_{min} = \frac{1}{\sqrt{\bar{n}}}$ ← shot noise level

This is the fundamental limit for measurements with coherent inputs state

Fundamental limit : Heisenberg uncertainty

$\Delta\varphi \sim \frac{1}{\bar{n}}$ ← Holy grail of quantum measurements

Single photon interferometer

Input state $|0\rangle|1\rangle$

After the first BS: $\frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + i|1\rangle|0\rangle\right)$

After the phase shifter $\frac{1}{\sqrt{2}}\left(e^{i\varphi}|0\rangle|1\rangle + i|1\rangle|0\rangle\right)$

After the second BS:

$$\left[\begin{array}{l} |0\rangle|1\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + i|1\rangle|0\rangle\right) \\ |1\rangle|0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|1\rangle|0\rangle + i|0\rangle|1\rangle\right) \end{array}\right]$$ each component transforms

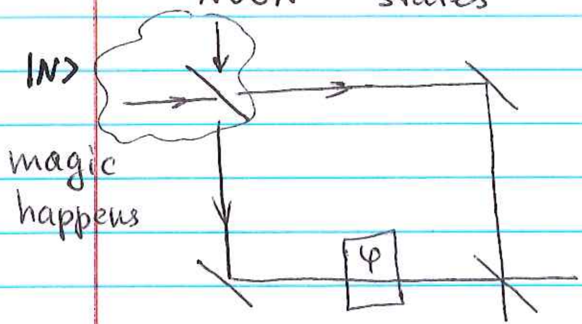
$$\psi_{fin} = \frac{1}{2}\left\{ e^{i\varphi}\left(|0\rangle|1\rangle + i|1\rangle|0\rangle\right) + i\left(|1\rangle|0\rangle + i|0\rangle|1\rangle\right)\right\} =$$

$$= \frac{1}{2}\left\{ \left(e^{i\varphi} - 1\right)|0\rangle|1\rangle + i\left(e^{i\varphi} + 1\right)|1\rangle|0\rangle \right\}$$

Each detector will click with probabilities $P_{1,2} = \frac{1}{2}(1 \pm \cos\varphi)$

$\langle \psi_{fin} | \hat{O} | \psi_{fin} \rangle = \cos\theta$ (similar to coherent case)

NOON states



After the magic beam splitter

$$\frac{1}{\sqrt{2}}\left(|N\rangle_1 |0\rangle_2 + |0\rangle_1 |N\rangle_2\right)$$

(NOON state)

After the phase-shifter

$$e^{in\varphi} |N\rangle_2 = e^{iN\varphi} |N\rangle_2$$

$$|N\rangle_1 |0\rangle_2 + e^{iN\varphi} |0\rangle_1 |N\rangle_2$$

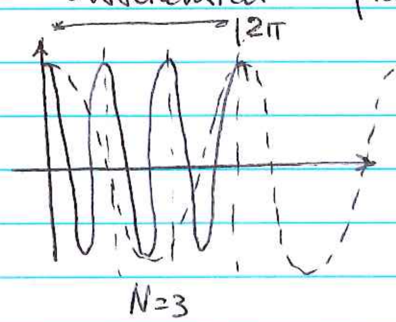
After the second beam splitter

$$\frac{1}{2}\left\{ (e^{iN\varphi} - 1) |0\rangle_1 |N\rangle_2 + i(e^{iN\varphi} + 1) |N\rangle_1 |0\rangle_2 \right\}$$

Probabilities of detection in each channel

$$P_{1,2} = \frac{1}{2}(1 \pm \cos N\varphi)$$

Differential photocount system $\Delta I \propto \cos N\varphi$

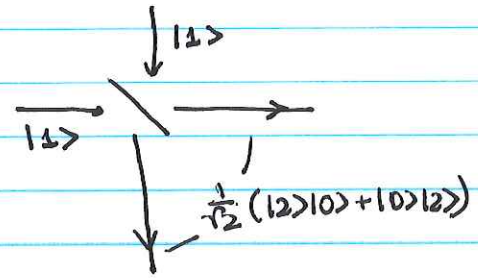


Supersensitivity

$$\Delta\varphi \propto \frac{1}{N}$$

in principle can reach the Heisenberg limit

Realistic realization for $N=2$



Input $|1\rangle|1\rangle$

After the first beamsplitter

$$\frac{1}{\sqrt{2}}\left(|2\rangle_1|0\rangle_2+|0\rangle_1|2\rangle_2\right)$$

After the beam shifter

$$\frac{1}{\sqrt{2}}\left(|2\rangle_1|0\rangle_2+e^{i2\varphi}|0\rangle|2\rangle\right)$$

After the second beam splitter

$$\frac{1}{2\sqrt{2}}\left(1-e^{2i\theta}\right)\left(|2\rangle|0\rangle-|0\rangle|2\rangle\right)+\frac{i}{2}\left(1+e^{2i\theta}\right)|1\rangle|1\rangle$$

Detection problem: one can verify that

$$\langle \hat{a}^+_{2,3}\,\hat{a}_{2,3}\rangle = 1 \quad \text{for any value of } \varphi$$

no fringes!

Possible solution: two-photon detector (detects $|2\rangle$, but not $|1\rangle$)

One possible better measurement - parity

$$\hat{\Pi}=(-1)^{\hat{a}^+\hat{a}}=e^{i\pi\hat{a}^+\hat{a}}$$

$$\langle\Psi_{out}|\hat{\Pi}|\Psi_{out}\rangle=\cos 2\varphi$$

Phase measurement uncertainty

$$\Delta\varphi=\frac{\Delta\Pi}{\left|\frac{\partial\langle\Pi\rangle}{\partial\varphi}\right|}$$

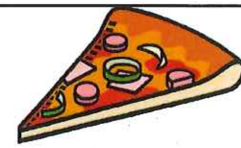
$$\Delta\Pi=\sqrt{\langle\Pi^2\rangle-\langle\Pi\rangle^2}=\sqrt{1-\langle\Pi\rangle^2}=\sin 2\varphi$$

$$\Delta\varphi=\frac{\sin 2\varphi}{2\sin 2\varphi}=\frac{1}{2}=\frac{1}{N_{photon}}$$

Heisenberg limit

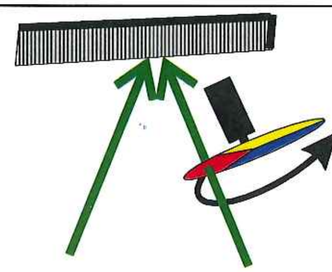
Quantum P00Per Scoopers!

H Cable, R Glasser, & JPD, quant-ph/0704.0678.



"Pizza Pie" Phase Shifter

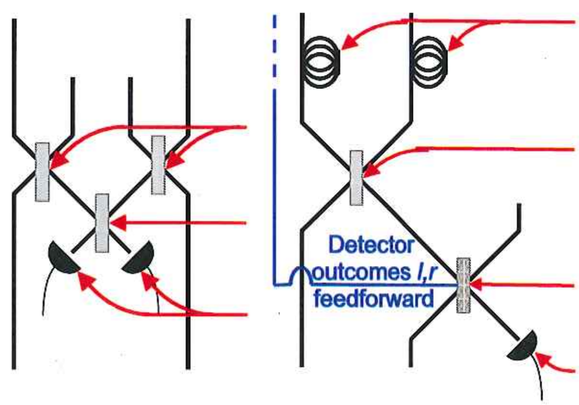
Spinning glass wheel. Each segment a different thickness.
N00N is in Decoherence-Free Subspace!



Feed Forward based circuit

Generates and manipulates special cat states for conversion to N00N states.

First theoretical scheme scalable to many particle experiments!



Quantum P00Per Scooper!

H Cable, R Glasser, & JPD, quant-ph/0704.0678.

2-mode squeezing process

X

OPO

$\sum_n p_n |n\rangle|n\rangle$

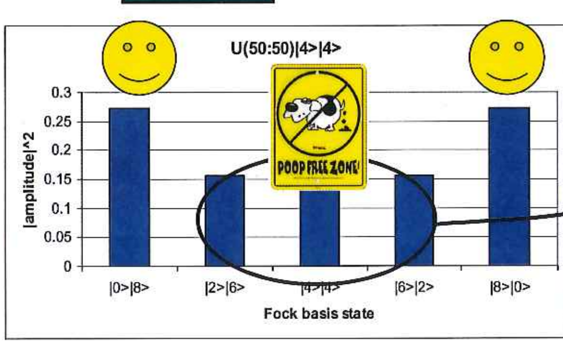
Old Scheme

linear optical processing

$\frac{|N0\rangle + |0N\rangle}{\sqrt{2}}$

beam splitter

New Scheme

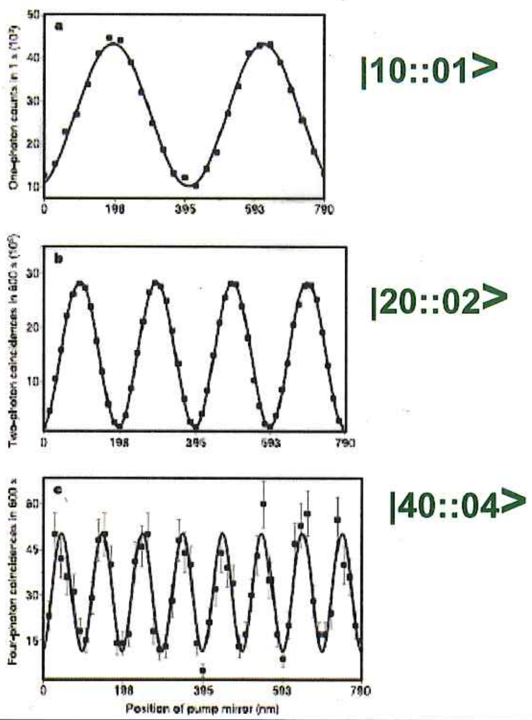


How to eliminate the “POOP”?

quant-ph/0608170
G. S. Agarwal, K. W. Chan, R. W. Boyd, H. Cable and JPD

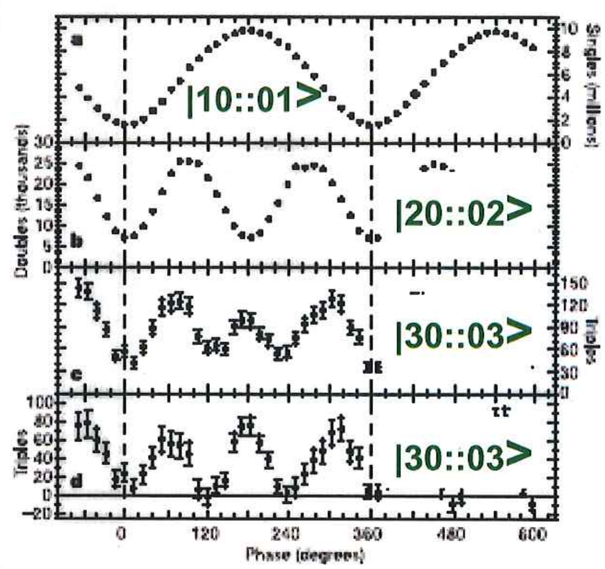
De Broglie wavelength of a non-local four-photon state

Philip Walther[1], Jian-Wei Pan[1*], Markus Aspelmeyer[1], Rupert Ursin[1], Sara Gasparoni[1] & Anton Zeilinger[1,2]

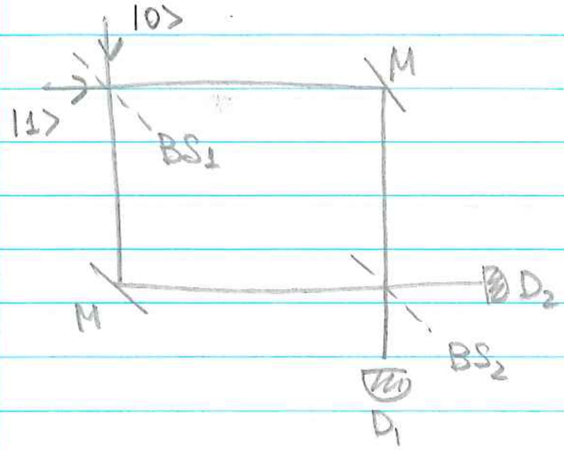


Super-resolving phase measurements with a multiphoton entangled state

M. W. Mitchell, J. S. Lundeen & A. M. Steinberg



Interaction - free measurements



θ is set to zero

After BS 1

$$\frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + i|1\rangle|0\rangle\right)$$

After BS 2

$$\frac{1}{2}\left\{\left(\cancel{|0\rangle|1\rangle} + i|1\rangle|0\rangle\right) + i\left(|1\rangle|0\rangle + \cancel{i|0\rangle|1\rangle}\right)\right\}$$

$$= |1\rangle_{1}|0\rangle_{2}$$

Only D_1 detects photons, and never D_2.

Now we put something in one of the arms

After BS 1

$$\frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + i|1\rangle|0\rangle\right)$$

the Bomb scatters all the photons, so after it (and before the BS2)

$$\frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + i|1\rangle|0\rangle\right)$$

After the second BS

$$\psi_{fin} = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{i}{2}\left(|1\rangle|0\rangle + i|0\rangle|1\rangle\right) = \frac{1}{\sqrt{2}}|0\rangle|0\rangle + \frac{i}{2}|1\rangle|0\rangle - \frac{1}{2}|0\rangle|1\rangle$$

Possible outcomes: nothing - 50% (but bomb is set off!)
D1 clicks - 25% (no information)
D2 clicks - 25% (bomb is there!)
even though the photon took the other path