PHYS 404/690 Quantum and Nonlinear Optics

Problem set # 9 *(due April, 17)*

Each problem is 10 points. The problems marked with $*$ are required for graduate students only, and are extra credit problems for undergraduates.

P1 Let's step back to the semiclassical description of light atom interactions, and remind ourselves about classical Rabi oscillations. Assuming that an atom initially is in the excited state, and the frequency of the electromagnetic field matches the energy difference between the two states of a two-level system (resonant conditions), please write down:

- Quantum state of the system as a function of time
- Atomic inversion as a function of time
- Dipole moment as a function of time

Compare the evolution of the dipole moment with the atomic inversion and comment on any similarities and differences.

P2 Now, let's repeat the same steps for the fully quantized model of a two-level atom interacting with a quantized electromagnetic field under the resonant conditions. We start with an initial state where the atom is excited and where the filed is in a number state $|n\rangle$. Repeat the steps of the previous problem, namely please write down:

- Quantum state of the system as a function of time
- Atomic inversion as a function of time
- Dipole moment as a function of time

Is the time evolution of the atomic dipole moment in any way similar to that obtained in the semiclassical case?

P3 Obtain the expectation value of the atomic dipole moment as given by the Jaynes-Cummings model in the case where the field is initially in a coherent state. How does the result compare with the two previous cases? You should make a plot of the expectation value of the dipole moment as a function of time.

P4 In class we focused on resonant Jaynes-Cummings model, but it is possible to obtain the analytic expression for the quantum state evolution for a far-detuned laser field ($\omega - \omega_0 = \Delta$. In particular, if the initial state of the system is an atom in the superposition of the ground and excited states $(|a\rangle + |b\rangle)/\sqrt{2}$, and the optical field is in a coherent state $|\alpha\rangle$, the time-dependent wave function is described by the following expression:
$|\psi\rangle = \left(|\alpha e^{i\chi t}\rangle|a\rangle + e^{-i\chi t}|\alpha e^{-i\chi t}\rangle|b\rangle\right)/\sqrt{2}$,
where $\chi = g^2/\Delta$. If $\chi t = \pi/2$ we can obtained the following entangled state between the light and the atom:
$|\psi\rangle = (|i\alpha\rangle|a\rangle - i| - i\alpha\rangle|b\rangle)/\sqrt{2}$.
Suppose that there is some way to determine the state of the atom. What would be the state of the optical field if the atom is detected in state $|b\rangle$ or state $|a\rangle$? What if it is possible to detect the atom in the superposition states $(|a\rangle \pm |b\rangle)/\sqrt{2}$, what optical field states are generated in this case?

P5* A resonant two-photon extension of the Jayne-Cummongs model is described by the effective Hamiltonian $\hat{H}_{eff} = \hbar\eta\left(\hat{a}^2\hat{\sigma}_+ + (\hat{a}^\dagger)^2\hat{\sigma}_-\right)$. This Hamiltonian represents two-photon absorption and emission between atomic levels of like parity. The process is represented by the figure, where the broken line represents a virtual intermediate state of the opposite parity. Obtain the dressed states for this system. Obtain the atomic inversion for this model assuming the atom initially in the ground state and that the field is initially in a number state. Repeat for a coherent state.