PHYS 404/690 Quantum and Nonlinear Optics

Problem set # 5 *(due February 27)*

Each problem is 10 points. The problems marked with $*$ are required for graduate students only, and are extra credit problems for undergraduates.

P1 Solve the coupled equations valid for phase-matched forward three-wave mixing in a nonlinear crystal with absorption:
$\frac{d}{dz}E_1 = -\alpha_1 E_1 + \chi_1 E_3^*$,
$\frac{d}{dz}E_3^* = -\alpha_3^* E_3^* + \chi_3^* E_1$,
The expected answer is:
$E_1(z) = e^{-az}\left[E_1(0)cosh(\kappa z) + (-\alpha E_1(0) + \chi_1 E_3^*(0))\, sinh(\kappa z)/\kappa\right]$,
$E_3^*(z) = e^{-az}\left[E_3^*(0)cosh(\kappa z) + (\alpha E_3^*(0) + \chi_3^* E_1(0))\, sinh(\kappa z)/\kappa\right]$,
where $\alpha = (\alpha_1 - \alpha_3^*)/2$, $a = (\alpha_1 + \alpha_3^*)/2$, and $\kappa = \sqrt{\alpha^2 + \chi_1\chi_3^*}$.

P2 Using the results of previous problem, analyze the expected output of the down conversion, in which only the signal field $E_1(0)$ is present at the input, and the idler $E_3(0) = 0$. Using a computer, plot the dependencies of both fields' amplitudes on the propagation length for various values of parameters $\alpha_{1,2}$ and $\chi_{1,2}$, and analyze the observed behaviors. Is it possible to achieve 100% conversion of the input signal field into the idler field?

P3 Calculate all wavelengths generated in a $\chi^{(3)}$ nonlinear medium by a combintion of 632.8 nm and 388 nm laser light.

P4 Using the expression for probe field susceptibility under the EIT conditions for a resonant pump field ($\Delta_2 = 0$):
$\chi_P = \frac{i\wp_{13}^2}{\hbar\epsilon_0} N \frac{\Gamma_{12}}{\Gamma_{12}\Gamma_{13} + |\Omega_2|^2/4}$,
where $\Gamma_{12} = \gamma_{12} - i\delta$, $\Gamma_{13} = \gamma_{13} - i\delta$ (since in this case $\delta = \Delta_1 = \nu - \omega_{13}$, and N is the atomic number density, calculate the group velocity at exact two-photon resonance ($\delta = 0$).

P5* The coherent part of interaction between a three-level system and two optical field is identical for Λ, V and ladder configurations, however, the difference in decay values results changes the parameters of the EIT resonance. Following the steps for a Λ system, calculate the near-resonance and far-detuned susceptibilities for a ladder system, in which a weak probe field is applied between the ground and the first excited state, and two excited states are coupled by a strong pump field. Assume that atoms are stationary, the radiative decay rates of the excited states γ_2 and γ_3 are comparable, and there is no additional sources of decoherence in the system. What is the best absorption suppression is achievable in such system?