

**Physics 352, The Mössbauer experiment with ^{57}Fe .
A supplement to the manual**

The recoil corresponding to the 14.36 keV nuclear transition ($E_\gamma = p_\gamma = p_R$) is $T_R = 1/2 E_\gamma^2 / M_{^{57}\text{Fe}} \sim 0.0019$ eV (note error in manual). It occurs twice, at emission and at absorption.

- a) Show that the relative speed required to make up this difference (and enable resonance excitation) from Doppler effect is of order 50 m/s (using non-relativistic Doppler formula).
- b) Calculate the excited state level width corresponding to the mean life-time $\tau = 3.4 \times 10^{-7}$ s of the first excited state of ^{57}Fe .
- c) Calculate the Doppler shift of a 14.36 keV photon for a relative speed between source and absorber of 1 cm/s. Note that the Doppler shift (non-relativistically) is $\Delta E = \Delta h\nu = \beta h\nu$, with $\beta = v/c$; v is the velocity of the source.

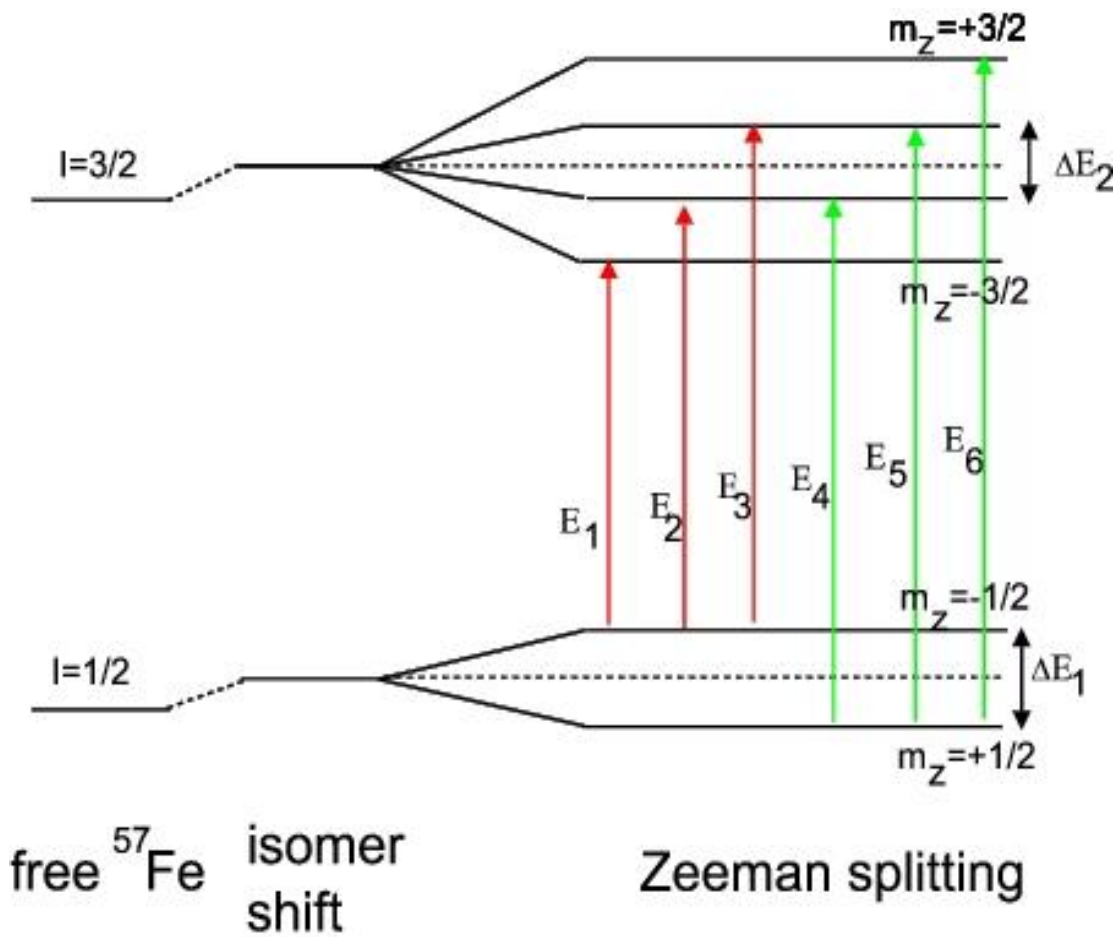
In a magnetic field the $I=1/2$ ground state of ^{57}Fe nucleus splits into 2 levels, with spacing ΔE_1 , and the $I=3/2$ excited state splits into 4 levels with spacing ΔE_2 . The order of the magnetic substates with $-I \geq m_z \geq +I$ are inverted relative to each other because the nuclear magnetic moments of these 2 states have opposite sign:

The energy shift of a level due to Zeeman effect is given by $E = -g_I \mu_{nm} m_z H$. g_I is the gyro-magnetic ratio (a number of nuclear magnetons) and μ_{nm} the nuclear magneton 3.15×10^{-14} eV T $^{-1}$. The product $g_I \mu_{nm}$ is the magnetic moment.

c) calculate the (expected values of the) two splittings ΔE_1 and ΔE_2 (see figure next page) in terms of the magnetic field H , in eV T $^{-1}$, or for the given field of 330 kG.

d) calculate the same splittings ΔE_1 and ΔE_2 from the observed line separations, converted from cm/s to eV, separately for the ground state and excited state, and calculate the average value of the magnetic field. Estimate the uncertainty on that number. See figure next page. As this is circular, namely it uses the splitting given in the table to calibrate the (mm/s per channel), and then uses that number to calculate the splitting, instead do the best determination of the spacings in number of channel, and determine the ratio of ΔE_1 and ΔE_2 , and compare with the ratio of the gyro-magnetic factors given in table. Estimate the error bars on the ratio, from an estimate

of the uncertainty on the number of channels. Is your results compatible with expectation.



CFP February 3, 2009

You can write $E_2 - E_1 = \Delta E_2$, $E_3 - E_2 = \Delta E_2$, $E_4 - E_3 = \Delta E_1 - \Delta E_2$, $E_5 - E_4 = \Delta E_2$, $E_6 - E_5 = \Delta E_2$. Calculate the ratio of the magnetic moments of the two states, from ΔE_1 and ΔE_2 ; compare with actual value. Can the relative sign of the magnetic moments be found from your data?

e) Find the electric field gradient at the site of the ^{57}Fe in sodium prusside.

f) Obtain the line width in stainless steel, and compare with the natural width defined by the mean life-time.

The numbers:

Kinetic energy associated with velocity 1 cm/s

$$M=1.67 \times 10^{-27}, A=57, KE=1/2MAv^2=0.5 \times 1.67 \times 10^{-27} * 57 * 10^{-4} = 4.76 \times 10^{-26} \text{ J}$$

$$1 \text{ eV} = 1.602 \times 10^{-19}: 4.76 \times 10^{-26} / 1.602 \times 10^{-19} = 2.97 \times 10^{-7} \text{ eV.}$$

CFP February 1, 2009, corrected Feb. 18, 2009

Energy of Zeeman transitions. Assume gyro-magnetic ratios are $g_2=1.6$ (excited) and $g_1=1$ (ground state) times actual value of g_1 (0.0903).

$E_0 + \mu H$ times ($-3/2 g_2 - 1/2 g_1 = -2.9$)	in order of increasing energy	1
$-1/2 g_2 - 1/2 g_1 = -1.3$		2
$+1/2 g_2 - 1/2 g_1 = +0.3$		4
$-1/2 g_2 + 1/2 g_1 = -0.3$		3
$+1/2 g_2 + 1/2 g_1 = +1.3$		5
$+3/2 g_2 + 1/2 g_1 = +2.9$		6

Now consider energy differences:

$$E_2 - E_1 = g_2 (=1.6 \text{ units of } g_1)$$

$$E_4 - E_2 = g_2 (=1.6)$$

$$E_4 - E_3 = g_2 - g_1 (=0.6)$$

$$E_3 - E_2 = g_2 (=1.6)$$

From a table of data:

$$E_1=250, E_2=360, E_4=473 \text{ and } E_3=560 \text{ (in units of MCA channels)}$$

$$E_3 - E_2 = g_1 = 200$$

$$E_4 - E_3 = g_2 - g_1 = 87 \text{ so } g_2 = (E_4 - E_3) + g_1 = 87 + 200 = 287$$

$$\text{and } g_2/g_1 = 287/200 = 1.43 \text{ (expect 1.69).}$$

How to start the experiment

First verify that the ^{57}Fe source is attached to the plunger of the drive.

Then set the driving circuit unit to “CV” for constant velocity, either symmetric hacksaw or asymmetric sawtooth shape. Set the spill to 100 on mechanical counter. Adjust the level (right bottom on drive unit) so as to have no resonating signal on the v versus t shape as seen on the digital scope. For the drive signal to be seen, the **data acquisition** (multichannel analyzer, MCA, set to “Moessbauer”) computer **must be turned on**, and a run started.

When you see that the plunger is oscillating, and the signal on the scope shows a linear variation of the velocity versus time, i.e. constant acceleration, set the MCA to multichannel, and observe the energy spectrum. If the amplifier is set to 200, you should see 3 peaks; the third may be the 14.4 keV transition. To verify that calculate the ratio third/second/first and compare with X-energy to verify that the first is the L X-ray and the second the K X-ray. You may change the amplifier gain to verify the assumption (that the third peak is the 14.4 keV transition). Then set the region of interest (ROI) around the 14.4 keV peak, at least twice FWHM. Switch back to the Moessbauer mode. Now insert one of the 3 target closest to the source, and place the detector about 4-5 cm away. Take data. You should see a constant background , with dips (1 or 2 or 3 dips depending on the target) rather quickly (10 minutes or so).

Real data taking will take a few hours per target.

The values given in table 3.2 of man10 for the gyromagnetic form factors of the ground- and excited states are wrong. According to Wertheim, $g_0=3.924$ mm/s, and $g_{ex}=2.244$ mm/s. The interpretation of the velocity spectrum becomes trivial with this inversion. The signs of the g's does not matter as long as they are different for the ground state and excited states.

CFP Feb. 26, 2008, revised April 21, 2010 and April 10, 2011