γ-ray Studies

PHYS352 Experimental Modern Physics
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1 Introduction

Studies of radioactivity, first reported in 1896 by Becquerel, were at the cradle of the nuclear and particle physics at the time when internal structure of an *atom* was still a controversial topic. Investigations in the early decades of XX century soon led to a classification of radioactive materials as α, β, and γ emitters. The process of spontaneous fission was first observed much later.

α emitters tend to be isotopes of high mass nuclei (large mass no. A). Unstable in their nucleon configuration, they decay to a daughter nuclide of mass (A-4) plus an alpha particle. The α "ray" (or particle) was soon identified as a fully ionized helium nucleus which departs the scene of decay with about 5 MeV of kinetic energy.

β emitters tend to be unstable nuclear isotopes which have a low to medium nass number A. The *beta* particle was determined to be an electron (or positron).

γ emitters span the full range of the nuclear chart. γ rays from these sources represent the photons emitted in electromagnetic transitions as an excited nucleus makes a transition from an upper to a lower energy level. Formally, the electromagnetic radiation with photon energies above 100 keV is referred as γ-ray radiation.

In this lab you will investigate the interactions of γ-rays with various materials. In particular, you will study two effect that are dominant for sub-MeV γ-rays. These are *Compton scattering*, in which γ-ray scatters from an essentially free electron inside the samples, changing its energy and propagation direction; and the *photoelectric effect* in which the γ-ray is absorbed by an atom which then ejects an electron with all the available energy. You will also become familiar with the experimental methods of γ-ray spectroscopy, starting from the fundamental mechanisms that occur when a γ-ray enters a detector such as a NaI:Tl crystal (sodium iodide doped with thallium), and concluding with a Multi-Channel analyzer (MCA) that displays the amplified detector signals.

2 Compton Scattering

The Compton effect describes the process of scattering of electromagnetic waves by a charge particle, in which a portion of the electromagnetic energy is given to the charged particle in an elastic relativistic collision. The effect was discovered in 1922 by Arthur H. Compton who observed it while studying scattering of X-rays by various light metals. He was later able to explain the effect using both special relativity

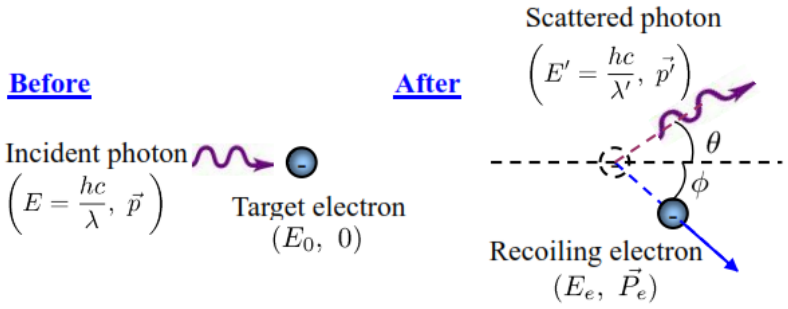

Figure 1: Compton scattering diagram showing the electron (initially at rest) to scatter a photon via an elastic collision.

and quantum mechanics - both barely developed theories at that time, - and was awarded the Nobel prize in 1927. Compton effect became one of the first solid demonstrations of quantization of electromagnetic radiation as well as relativistic energy-momentum relationships of Einstein special relativity.

To describe Compton scattering, we consider the case of a γ-ray photon with energy $E_\gamma = \hbar c/\lambda$ and momentum $\vec{p}$ ($|\vec{p}| = \hbar/\lambda$) interacting with an electron originally at rest, as shown in Fig. 1. To carry out the calculations properly one must treat the electron relativistically; so if the electron is originally at rest, then its total energy is equal to its rest energy $E_0 = m_e c^2$. After the elastic scattering, the electron can pick up some momentum $\vec{P}_e$, and thus its total energy is now given by the expression:$E_e^2 = c^2 P_e^2 + E_0^2$. However, the energy and momentum of the photon must change as well: $E'_\gamma = \hbar c/\lambda'$ and $|\vec{p'}| = \hbar/\lambda'$. Note that this means that the wavelength of the photon changes! This is a consequence of treating a photon as a particle using quantum mechanics description. If an electromagnetic wave is treated classically as a wave, it cannot change its wavelength due to scattering.

To figure out the magnitude of the wavelength change one must apply the energy and momentum conservation:

$$E_\gamma + E_0 = E'_\gamma + E_e; \vec{p} = \vec{p'} + \vec{P}_e. \tag{1}$$

After somewhat tedious but straightforward calculations, one arrives to the following expression for the wavelength change of the scattered photon:

$$\Delta\lambda = \lambda' - \lambda = \frac{2\pi\hbar}{m_e c}(1 - \cos\theta), \tag{2}$$

where θ is the scattering angle of the photon, and m_e is the rest mass of the electron. The ratio $\lambda_c = \frac{2\pi\hbar}{m_e c}$ is called Compton wavelength. One remarkable property of the Compton scattering is that the absolute wavelength change does not depend on the actual wavelength of the electromagnetic radiation. That means that independently of its energy, all the photon, scattered off an electron will experience the same wavelength change. If so, you may ask yourself why we have to go through the troubles of working with γ-rays when using, for example, visible light is so much more comfortable? The answer lays in the magnitude of the Compton wavelength $\lambda_c = 2.42 \cdot 10^{-12}$m. Considering that the typical wavelength of the visible light is hundreds of nanometers ($\sim 10^{-7}$m, the effect of the Compton scattering is just too small to measure. However, for a γ-ray with $\lambda_\gamma = 1.87 \cdot 10^{-12}$m the effect is very noticeable, and can be easily measured.

Since in the experiment we will be measuring γ-ray energies, it is convenient to rewrite Eq. 2 in these terms:

$$E'_\gamma = \frac{E_\gamma}{1 + \epsilon(1 - \cos\theta)}, \tag{3}$$

where $\epsilon = E_\gamma/m_e c^2$.

Traditionally, the scattering rates in physics are defined in terms of cross-section σ: if the incoming radiation is composed of quanta or 'particles' (for example, photons or neutrons), a cross-section is a scattering rate (number of scattering events per unit time) per unit incident radiation flux, where the latter is the number of incident particles striking the target surface per unit time per unit area. For Compton scattering the scattering cross-section σ is the ratio between the number of scattered γ-quanta to the flux of incoming γ-quanta over the same time period. To calculate the probability of a γ-quanta to be scattered *in specific direction*, one can use a differential cross-section $d\sigma/d\Omega$, that is equal to the fractional relative scattering rate $d\sigma$ into the solid angle element $d\Omega$. Klein and Nishina [1] derived the following equation for Compton scattering cross-section, now usually called the Klein-Nishina cross-section:

$$\frac{d\sigma}{d\Omega} = \frac{r_c^2}{2}\left(\frac{E'_\gamma}{E_\gamma}\right)^2\left(\frac{E_\gamma}{E'_\gamma} + \frac{E'_\gamma}{E_\gamma} - \sin^2\theta\right), \tag{4}$$

where $r_c = e^2/m_e c^2$ is the classical electron radius. By substituting the expression for E'_γ from Eq. 2 one obtains:

$$\frac{d\sigma}{d\Omega} = \frac{r_c^2(1+\cos^2\theta)}{2}\left\{\frac{1}{[1+\epsilon(1-\cos\theta)]^2}\left[1 + \frac{\epsilon^2(1-\cos\theta)^2}{(1+\cos^2\theta)[1+\epsilon(1-\cos\theta)]}\right]\right\} \tag{5}$$

While nowadays one can fairly easily derive the expression of the Compton scattering cross-section using Feynman diagram, in 1928, when Klein and Nishina first published their results, this method was unknown. Moreover, their calculation was one of the first successful application of newly minted Dirac equation (describing the quantum behavior of a relativistic electron), and served as its validation. The classical alternative, described by Thompson using non-relativistic electron description, yielded the following expression:

$$\frac{d\sigma_{Th}}{d\Omega} = \frac{r_c^2(1+\cos^2\theta)}{2}. \tag{6}$$

It is easy to see that this expression matches the Klein-Nishina formula Eq. 5 for $\epsilon \ll 1$ or $E_\gamma \ll m_e c^2$. However, for this experiment $E_\gamma = 0.55\text{MeV} \simeq m_e c^2 = 0.51\text{MeV}$ there is a significant discrepancy between the two expressions, as you will be able to verify with your measurements.

2.1 Photoelectric effect

Another important interaction of γ-rays with matter is the photoelectric absorption, which is actually the most significant for low-energy γ-rays (with energies up to several hundred keV). In the photoelectric effect, the energy of a γ-quanta is transferred to the electron by first supplying enough energy to release it from its bound state to a nucleus and then transferring the rest of its energy into kinetic energy of the freed electron. Typically, the electrons are ejected form the K shell and have binding energy of a few keV. This is relatively small amount compared to the energy of the γ-ray so that most of the incident energy is transferred to kinetic energy of the electron. Note that since the original γ-quanta are absorbed by the material, the photoelectric effect results in the reduction of the photon flux. In this case its strength is characterised by the *absorption* cross-section. In *Alpha-, Beta- and Gamma-ray spectroscopy* edited by K. Siegbahn [2], the dominant K-shell photoelectric cross-section is given as:

$$\sigma_{pe} = 1.367 \times 10^{-22}(\alpha Z)^5 \frac{m_e c^2}{E_\gamma} cm^2/atom, \tag{7}$$

where $\alpha \approx 1/137$ is the fine structure constant. For a light metal like aluminum σ_{pe} is only about 1% of the Compton cross-section and therefore can be ignored for the analysis of the Compton scattering. For

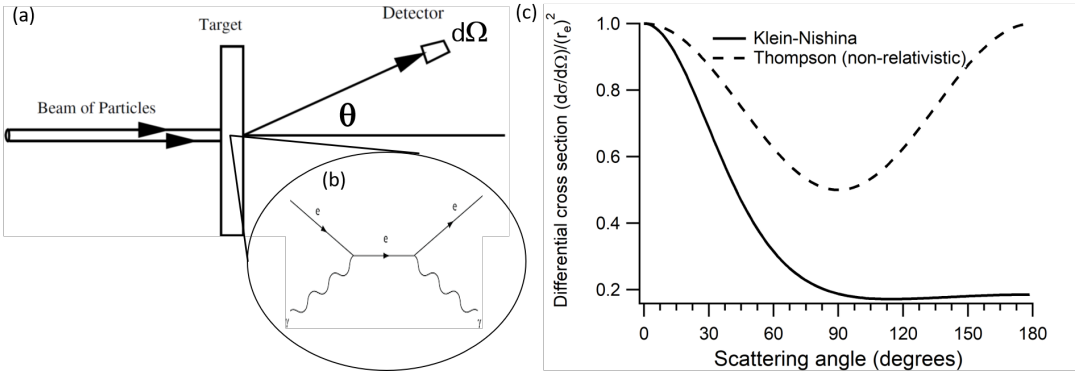

Figure 2: (a) Experimental arrangement: an incident beam of particles strikes a target and some scatter into a detector at an angle θ and ϕ subtending a solid angle dΩ. (b) Feynman diagram for Compton Scattering. A photon, represented by a wiggly line interacts with an electron, this in turn radiates a photon. What looks like a cartoon-ish sketch is actually a recipe for calculating the scattering cross-section (but you have to wait until a quantum electrodynamics course to learn its secret). (c) Relativistic (Klein-Nishina) and non-relativistic (Thompson) predictions for Compton scattering differential cross-section for $E_\gamma \simeq m_e c^2$).

lead with $Z = 82$ the photo-effect the absorption is much more significant, which explains why lead is used to screen radioactive sources.

3 Experimental Setup

3.1 Detector equipment

To study Compton scattering one must be able to measure the energy and the flux of γ rays before and after interaction with the target. To accomplish that, you will be using a scintillation detector that consists of a sodium iodide (NaI) scintillation crystal and a photomultiplier tube (PMT). In this case each detected γ quanta produces a photon inside the scintillator that is then recorded by the PMT. The produced voltage pulse is proportional to the energy deposited into the detector, and then can be measured by the Multi-channel Analyzer (MCA). The MCA output then shows the distribution of voltage pulse heights for each energy channel. The general layout of the detector is shown in Fig. 3.

To understand the spectrum you will work with a little better, one has to learn a few more details about the detector components.

NaI scintillator is based on the photoelectron effect: when an incoming γ-quanta is absorbed by an atom, it kicks out a highly-energetic photoelectron. Because the binding energy of an ejected electron in the scitillator is relatively low ($\approx$ 2 keV, almost all energy of the gamma quanta E_γ is transferred into the kinetic energy of the electron. Upon the ejection, the photoelectron very quickly (within 1mm for our experimental conditions) looses its energy as it collides with other atoms, creating ionization and excitation along its path. Finally, photons of visible light are produced as these excited atoms return to their ground states, and the amount of light is proportional to the energy of the photoelectron and, hence, to the energy of the original γ-ray.

The role of the *photomultiplier tube (PMT)* is to measure the amount of light emitted by the scitillator. PMTs are widely used in a variety of experiments and equipment, and you can find more details about them in Ref. [3]. Very generally, their operational principle is the following: each visible photon interacts with

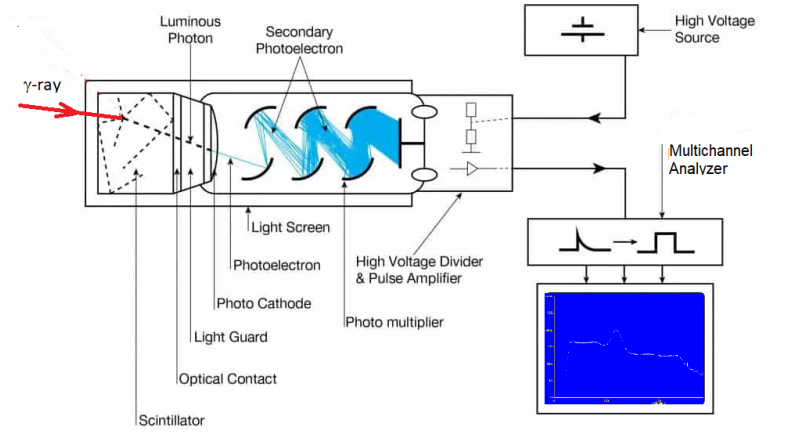

Figure 3: Main components of the γ-ray detection and analysis [3].

the photocathode, ejecting a photoelectron; this electron is then accelerated toward a series of dynodes, where in each stage each electron produces several secondary electrons, thus strongly amplifying the initial signal and making it possible to detect it using regular electronics. In principle, the amplitude of the electric pulse at the end is proportional to the number of visible photons emitted by the scintillator, and thus to the energy of the original γ-ray.

A multichannel analyzer (MCA) is used to measure voltage distributions (and hence the energy distributions of the γ-rays) by dividing the range of possible voltages into a number of channels. For each voltage pulse from the detector, the MCA electronics assigns the channel number, based on the assigned value range for each channel. If the pulse value falls within the right range, one count is added to the spectrum for this particular channel. The final spectrum is produced by accumulating the counts from many different channels. More details on its operation can be found in the manufacturer's manual [4].

3.2 Recorded spectra

In the ideal case of mono-energetic γ-rays we would expect to see a single narrow peak at the channel corresponding to the energy E_γ, as shown in red in Fig. 4. However, the actual signal is more complex. First of all, since each γ-quanta of the same energy does not produce exact same number of visible photons in the scintillator, the schitillator photons do not produce the exact same number of photoelectrons in the PMT. Moreover, the number of secondary electrons produced at each PMT dynode are also not exactly the same. As a result, even γ-rays with precisely defined energy produce a distribution of voltage pulses, that looks like a Gaussian distribution of counts spread over several channels around the mean value. This results in finite width of the measured photopeak. While many parameters can contribute to the actual width of the peak, we can assume that most of these processes have Poisson statistics, and thus the full width half maximum (FWHM) ΔE_{FWHM} of the detected photopeak, shown in Fig. 4, should be approximately proportional to the square root of the average measured energy $\langle E_\gamma \rangle$, or $\Delta E_{FWHM} \propto \sqrt{\langle E_\gamma \rangle}$.

So far we discussed only the possibility of electrons being produced inside the scintillator via the photoelectric effect. However, we cannot neglect the possibility of electrons to be released through Compton scattering. The main difference between the two contributions is that nearly full γ-ray energy is transferred to photoelectrons, while only the fraction of this energy is transferred to the Compton-scattered electrons. We can, in fact, find this energy using Eq.(2) as the difference between the photon energies before and

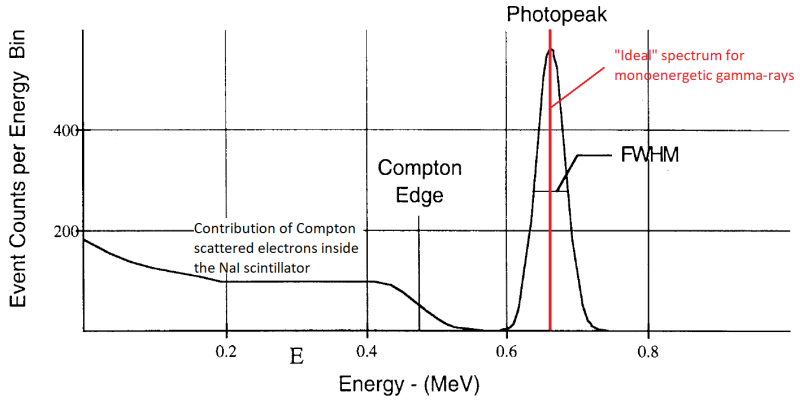

Figure 4: Pulse height distribution for a gamma-ray incident on a NaI detector. Red peak shows the "idealized" expectation for γ-rays of specific energy. The black trace shows a more realistic spectrum, showing the finite width of the photopeak, and the Compton edge due to contributions of the electrons scattered inside the scintillator material.

after the scattering:

$$E_e = E_\gamma - E'_\gamma = E_\gamma \left(1 + \frac{1}{\epsilon(1-\cos\theta)}\right)^{-} 1. \tag{8}$$

Clearly, the energy of such scattered electron depends on the scattering angle: for forward-scattering ($\theta = 0$) virtually no energy is transferred to the electron, while maximum energy electrons are produced when $\theta = 180^o$:

$$E_e^{(max)} = E_\gamma \frac{2\epsilon}{2\epsilon + 1} = \frac{2E_\gamma^2}{2E_\gamma + m_e c^2}. \tag{9}$$

These scattered electrons will give a continuous contribution to the recorded spectra for energies below $E_e^{(max)}$, which is typically referred to as "Compton edge", as shown in Fig. 4.

4 Experimental Procedure

4.1 (

Detector calibration] Your first task is to calibrate your spectrometer: i.e., to establish the correspondence between the number of each photo-peak channel and the energy it is responsible to detect. For that you need to measure the emission of three different radioactive sources, emitting γ-rays at different energies. In particular, you should be able to record the two peaks for each ^{60}Co and ^{22}Na sources, and a single peak of ^{137}Cs. Use the known energies from Fig. 5. and make a plot of the recorded E_γ values versus channel number. Fit this to a straight line: $E_\gamma = a + b \cdot n$. Determine the calibration constants a and b (with their uncertainties). You will use this calibration to determine the shifted energy of the Compton scattering peak.

4.2 Signal analysis

As you measure the peaks produced by different sources, you may also identify and analyze the main features of the recorded spectra, discussed above. In particular,

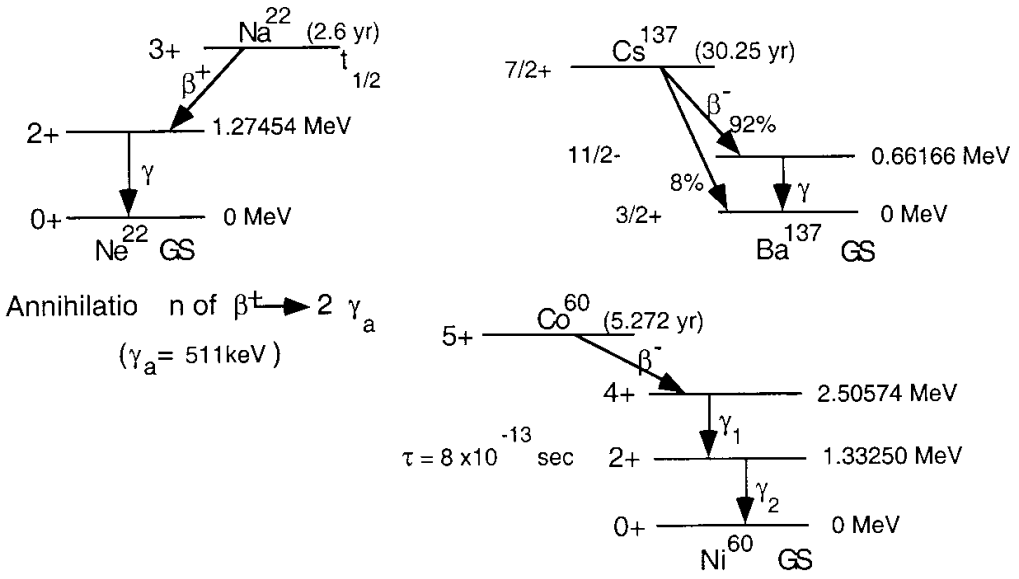

Figure 5: Nuclear states and their decay characteristics.

Compton Edges Determine the channel number for the Compton edges (see Fig. 4). From these and your calibration determine the γ-ray energies of these edges. Compare to the theoretical values given by Eq.(3.2).

Line Widths Determine the full width at half maximum ΔE_{FWHM}, for each peak. You can use the region of interest feature to do this. Make a plot of ΔE_{FWHM} vs. $\sqrt{\langle E_\gamma \rangle}$ for all the peaks. Does your graph support the assumption that the recorded number of counts follows the Poisson statistics?

4.3 Absorption coefficients

For the rest of the measurements you will be using only the ^{137}Cs source, since it is the strongest. Pick a set of samples made of two different materials (we have plastic, aluminum, copper, and lead), and place it between the source and the detector. Because some of the γ-rays are now absorbed or scattered, the number of photocounts per same time interval should drop. By measuring the reduction of detected counts for various thicknesses of the target materials, you will be able to measure their absorption coefficients, as described below.

Let N_{inc} be the number incident on the target and N_{det} be the number detected. If the target is not present then we assume $N_{det} = N_{inc}$. If the target is very thin, almost all the γ ray make it through $N_{det} \approx N_{inc}$, and the small reduction in the number of detections can be written as $dN_{det} = -PN_{inc} \approx -\kappa N_{det}$, where κ is the probability for the γ ray to be absorbed or scattered per unit length, also known as an *absorption coefficient*. We can rewrite this in a form of a differential equation:

$$\frac{dN_{det}}{dx} = -\kappa N_{det}, \tag{10}$$

where x is the coordinate describing the depth of the scattering target. Using this, one can predict the exponential drop in the number of detections for a thicker material:

$$N_{det}(x) = N_{inc} e^{-\kappa x}. \tag{11}$$

Assuming that each target atom contributes equally into the absorption process, we can find the relationship between the absorption coefficient and the absorption cross-section σ_{abs}:

$$\kappa = N_{tgts} \cdot \sigma_{abs}, \tag{12}$$

where N_{tgts} is the number of the where $N_{tgts}(cm^{-2})$ is the number density of the target maerial. Look up the number density for at least one of your target materials, and compare the experimentally measured cross-section with the theoretical value for the photoelectric effect, given by Eq.(7). Do you expect these numbers to match well?

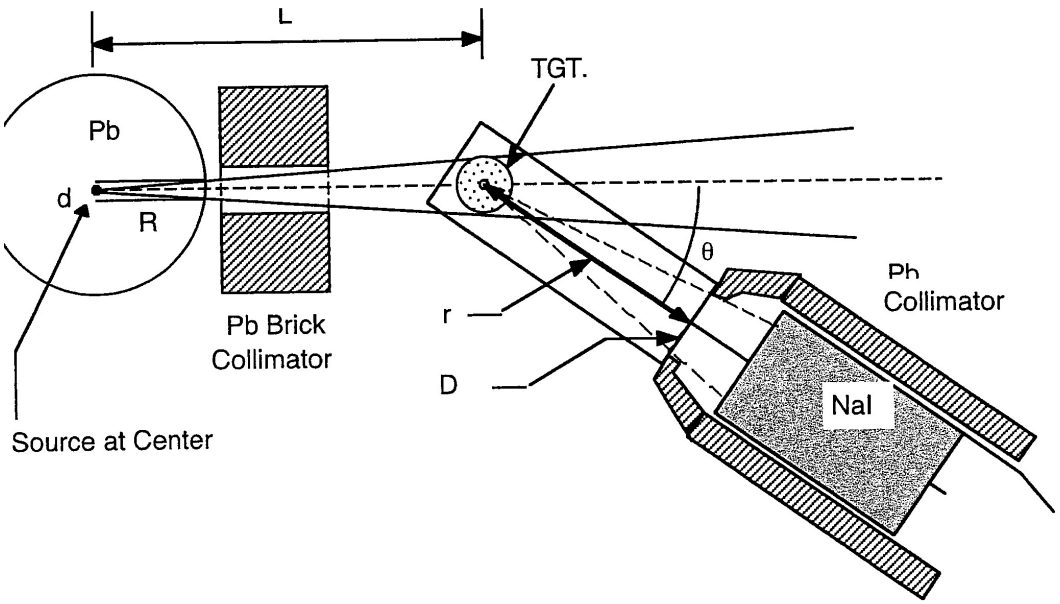

Figure 6: The experimental arrangement for the Compton scattering measurements.

4.4 Compton effect measurements

In these portion of the experiment your goal will be to determine the energy shift and the differential scattering cross-section of the scattered γ-rays as functions of the scattering angle. You will be using an aluminum cylinder as your target.

Since these measurements will likely take a significant amount of time, it may be a good idea to plan ahead by considering possible experimental challenges. For example: since the original beam is not very well collimated, it is likely that some fraction of unscattered γ-ray will reach the detector, especially for the small angles θ. How will this affect the measured spectrum? It may be also useful to have a look at Fig. 2 for a realistic guess on the size of the expected signal and how many different angles you want to test. Do you expect being able to accurately measure the scattering parameter for $\theta > 90^o$? Also, think about the time you allot for each measurement: does it make sense to adjust it for different angles?

Once you decided on how to record the data, your goal will be to verify Compton effect predictions by measuring the energy shift and the differential scattering cross-section as functions of the scattering angle.

- For each scattering angle record the energy spectrum for fixed live-time as a function of angle with the aluminum cylinder in place. Repeat the same measurement with the target cylinder removed (for background subtraction). You will need to determine the mean energy of the scattered γ-rays $\langle E'_\gamma \rangle$, as well as the number of counts N_{det} for each angle.

- Plot $\langle E'_\gamma \rangle$ as a function of angle and compare it with the expected values for E'_γ from Eq.(2). To make the comparison easier, consider plotting $1/E'_\gamma - 1/E_\gamma$ vs $(1 - \cos\theta)$ and compare the slope of the resulting linear fit with the expected $1/m_e c^2$.

- Calculate the ratio of γ-rays scattered at the angle θ and estimate the fractional cross-section $\frac{d\sigma(\theta)}{d\Omega}$. Plot this as a function of angle together with the theoretical predictions of Eq.(4). How well the theoretical predictions work? Describe and try to explain possible discrepancies. Based on your experimental data, can you rule out the Thompson prediction given by Eq.(6)?

Final thought: think carefully about experimental uncertainties. Since there is a complex experiment, with many variables, it is very hard to accurately account for *all* possible errors. Thus, it is important to identify the leading source(s) of error for each types of measurements, and use it in your data analysis. This makes a good discussion topic with your lab partner and the course instructor.

References

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